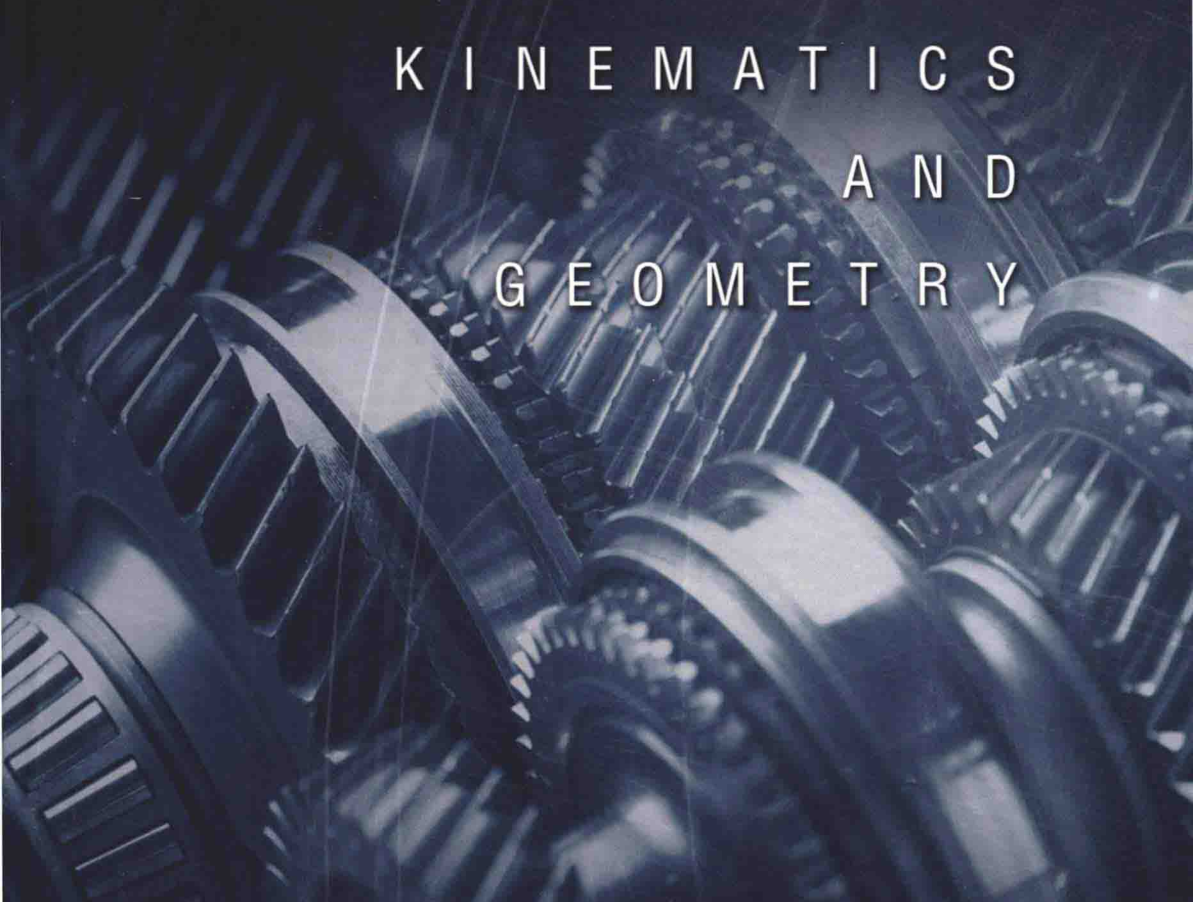
HIGH-CONFORMAL GEARING

KINEMATICS AND GEOMETRY



STEPHEN P. RADZEVICH

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This book is dedicated to my parents

Preface

The book deals with gears that feature *convex-to-concave* contact of the tooth flanks of the gear and the mating pinion. Gears of this type are commonly referred to as *conformal* gearings. *Novikov gearing** and *Wildhaber gearing* are the most widely known examples of conformal gearing. The *Bramley–Moore* (otherwise known as the *Vivkers, Bostock,* and *Bramley* gearing, or just V.B.B.-gearing) is another well-known example of conformal gearing.

Conventional external involute gearing features *convex-to-convex* contact of the tooth flanks of the gear and the mating pinion. Because of this high contact, stresses are observed when involute gears operate.

In conformal gearing, as well as in high-conformal gearing, the *convex-to-convex* contact of the tooth flanks is substituted with *convex-to-concave* contact. Due to this, the gear teeth feature higher contact strength.

The *Novikov gear* form is of the helical type but it only has face contact: There is no progressive profile contact as in the involute case. The profiles of the mating teeth at any section perpendicular to the wheel axis make contact with each other only for an instant and then separate. With the *Novikov* tooth, contact always occurs at a certain distance from the pitch point and therefore the sliding velocity is constant and unidirectional (for one particular direction of wheel rotation). A number of investigators have examined and manufactured gears of this type and load capacities of 3–6 times the corresponding involute tooth load have been reported.

Conditions to transmit a rotation smoothly from the driving to the driven shaft are outlined in this book. Much of the discussion is based on the modern theory of gearing.

In high-conformal gearing, the degree of conformity at point of contact of the tooth flanks of the gear and the mating pinion is greater than a prespecified critical value, the *threshold*. This is the main difference between conformal and high-conformal gearings. A set of conditions to meet when designing both conformal and high-conformal gears is specified.

The principal differences between conformal gearing (as well as high-conformal gearing) and *Wildhaber helical gearing* are outlined. It is shown that *Wildhaber gearing* on the one hand and *Novikov gearing* on the other hand are two completely different gear systems that cannot be combined into a common gear system. Therefore, the widely used terminologies like "*Wildhaber–Novikov gearing,*" "*W–N gearing,*" etc., are meaningless, and they need to be eliminated from use in the engineering and scientific community.

* The *Novikov gear* form is named after Colonel M.L. Novikov, Dr. Eng. Sc., who was head of a department at the Zhuokovskii Military Aero Academy in Moscow. He developed this particular gear form but it was his colleagues who published his work under his name after his death in 1958 [19].

Those who want to demonstrate their unfamiliarity with gearing in a general sense, and with *Novikov gearing* in particular, loosely use the terms "*Wildhaber–Novikov gearing*" and/or "*W–N gearing*."

It is also shown that neither conformal gears (*Novikov gears*) nor high-conformal gears can be cut in the continuously-indexing (generating) process; that is, they cannot be hobbed, ground by worm grinding wheels, shaved, etc. Only form cutting tools can be used for machining conformal and high-conformal gears: form milling cutters, form grinding wheels, etc.

It should be noted in conclusion that the discussion in this book is limited only to the kinematics and the geometry of conformal and high-conformal gearing. Other important topics such as gear accuracy, gear loading, gear wear, gear lubricating, vibration generation, and noise excitation, etc., are not addressed in this book.

Acknowledgments

I would like to share the credit for any research success with my numerous doctoral students with whom I have tested and applied the proposed ideas in industry. The contributions of many friends, colleagues, and students are overwhelming in number and cannot be acknowledged individually, and as much as my benefactors have contributed, their kindness and help must go unrecorded.

My thanks also go to those at CRC Press who took over the final stages of preparing this book and coped with the marketing and sales of the fruit of my efforts.

Author

Stephen P. Radzevich is a professor of mechanical engineering and a professor of manufacturing engineering at National Technical University of Ukraine "Kyiv Polytechnic Institute," Kyiv, Ukraine. He earned his MSc in 1976, PhD in 1982, and Dr.(Eng)Sc in 1991, all in mechanical engineering. Dr. Radzevich has extensive industrial experience in gear design and manufacture. He has developed numerous software packages dealing with computer-aided design (CAD) and computer-aided machining (CAM) of precise gear finishing for a variety of industrial sponsors. His main research interest is the kinematic geometry of part surface generation, with a particular focus on precision gear design, high-power-density gear trains, torque share in multiflow gear trains, design of special purpose gear cutting/finishing tools, and design and machine (finish) of precision gears for low-noise and noiseless transmissions of cars, light trucks, etc. Dr. Radzevich has spent over 40 years developing software, hardware, and other processes for gear design and optimization. In addition to his work in industry, he trains engineering students at universities and gear engineers in companies.

He has authored and coauthored over 30 monographs, handbooks, and textbooks. The monographs *Generation of Surfaces* (RASTAN, 2001), *Kinematic Geometry of Surface Machining* (CRC Press, 2007; 2nd Edition 2014), *CAD/CAM of Sculptured Surfaces on Multi-Axis NC Machine: The DG/K-Based Approach* (M&C Publishers, 2008), *Gear Cutting Tools: Fundamentals of Design and Computation* (CRC Press, 2010), *Precision Gear Shaving* (Nova Science Publishers, 2010), *Dudley's Handbook of Practical Gear Design and Manufacture* (CRC Press, 2012), *Geometry of Surfaces: A Practical Guide for Mechanical Engineers* (Wiley, 2013), and *Generation of Surfaces: Kinematic Geometry of Surface Machining* (CRC Press, 2014) are among his recently published volumes. He also authored and coauthored about 300 scientific papers, and holds about 250 patents on inventions in the field (the United States, Japan, Russia, Europe, Canada, Soviet Union, South Korea, Mexico, and others).

Notations

A_g	apex of the gear in intersected-axis gearing and crossed-axis gearing
A_p	apex of the pinion in intersected-axis gearing and crossed-axis gearing
A_{pa}	apex of the plane of action in intersected-axis gearing and crossed-axis gearing
C	center-distance
℄	center-line
$C_{1 \cdot g}$, $C_{2 \cdot g}$	first and second principal plane sections of the gear tooth flank, $\mathcal{G}$
$C_{1 \cdot p}$, $C_{2 \cdot p}$	first and second principal plane sections of the pinion tooth flank, $\mathcal{P}$
$Cnf_R(\mathcal{G}/\mathcal{P})$	indicatrix of conformity of the gear tooth flank, $\mathcal{G}$, and the mating pinion tooth flank, $\mathcal{P}$, at a current contact point, K
$Cnf_k(\mathcal{G}/\mathcal{P})$	converse indicatrix of conformity of the gear tooth flank, $\mathcal{G}$, and the mating pinion tooth flank, $\mathcal{P}$, at a current contact point, K
$Crv(\mathcal{G})$	curvature indicatrix at a point of the gear tooth flank $\mathcal{G}$
$Crv(\mathcal{P})$	curvature indicatrix at a point of the pinion tooth flank $\mathcal{P}$
$Dup(\mathcal{G})$	*Dupin*'s indicatrix at a point of the gear tooth flank $\mathcal{G}$
$Dup(\mathcal{P})$	*Dupin*'s indicatrix at a point of the pinion tooth flank $\mathcal{P}$
$\mathcal{E}$	characteristic line
E_g, F_g, G_g	fundamental magnitudes of the first order of the gear tooth surface, $\mathcal{G}$
E_p, F_p, G_p	fundamental magnitudes of the first order of the pinion tooth surface, $\mathcal{P}$
F	face width
F_{eff}	effective face width, or the face width of the active portion of the plane of action
$\mathcal{G}$	tooth flank of the gear
K	point of contact of the tooth flanks, $\mathcal{G}$ and $\mathcal{P}$ (or a point within a line of contact of the surfaces, $\mathcal{G}$ and $\mathcal{P}$)
LA	line of action
LA_{inst}	instant line of action
LC	line of contact
LA_{des}	desirable line of contact
L_g, M_g, N_g	fundamental magnitudes of the second order of the gear tooth flank, $\mathcal{G}$
L_p, M_p, N_p	fundamental magnitudes of the second order of the pinion tooth flank, $\mathcal{P}$

L_{pc}	length of a path of contact
N	tooth number
N_{in}	tooth count of the input member
N_{out}	tooth count of the output member
O_g	axis of rotation of the gear
O_p	axis of rotation of the pinion
$\mathscr{P}$	tooth flank of the pinion
PA	plane of action
P_c	path of contact
P_i	current pitch point
P_{pc}	pseudo path of contact
P_{ln}	axis of instant rotation of the pinion in relation to the gear (pitch line)
P_n	normal pitch
P_t	transverse pitch
$\mathbf{Rc}(PA \mapsto \mathscr{G})$	operator of rolling/sliding (the operator of transition from the plane of action, PA, to the gear, $\mathscr{G}$, in crossed-axis gearing)
$\mathbf{Rc}(PA \mapsto \mathscr{P})$	operator of rolling/sliding (the operator of transition from the plane of action, PA, to the pinion, $\mathscr{P}$, in crossed-axis gearing)
$\mathbf{Rl}_x(\varphi_y, Y)$	operator of rolling over a plane (Y-axis is the axis of rotation, X-axis is the axis of translation)
$\mathbf{Rl}_z(\varphi_y, Y)$	operator of rolling over a plane (Y-axis is the axis of rotation, Z-axis is the axis of translation)
$\mathbf{Rl}_y(\varphi_x, X)$	operator of rolling over a plane (X-axis is the axis of rotation, Y-axis is the axis of translation)
$\mathbf{Rl}_z(\varphi_x, X)$	operator of rolling over a plane (X-axis is the axis of rotation, Z-axis is the axis of translation)
$\mathbf{Rl}_x(\varphi_z, Z)$	operator of rolling over a plane (Z-axis is the axis of rotation, X-axis is the axis of translation)
$\mathbf{Rl}_y(\varphi_z, Z)$	operator of rolling over a plane (Z-axis is the axis of rotation, Y-axis is the axis of translation)
$\mathbf{Rr}_u(\varphi, Z)$	operator of rolling of two coordinate systems
$\mathbf{Rs}(A \mapsto B)$	operator of the resultant coordinate system transformation, say from a coordinate system A to a coordinate system B
$\mathbf{Rt}(\varphi_x, X)$	operator of rotation through an angle φ_x about the X-axis
$\mathbf{Rt}(\varphi_y, Y)$	operator of rotation through an angle φ_y about the Y-axis
$\mathbf{Rt}(\varphi_z, Z)$	operator of rotation through an angle φ_z about the Z-axis
$R_{1\cdot g}$, $R_{2\cdot g}$	first and second principal radii of curvature of the gear tooth flank, $\mathscr{G}$
$R_{1\cdot p}$, $R_{2\cdot p}$	first and second principal radii of curvature of the gear tooth flank, $\mathscr{P}$
$\mathbf{Sc}_x(\varphi_x, p_x)$	operator of screw motion about the X-axis
$\mathbf{Sc}_y(\varphi_y, p_y)$	operator of screw motion about the Y-axis
$\mathbf{Sc}_z(\varphi_z, p_z)$	operator of screw motion about the Z-axis
$\mathbf{Tr}(a_x, X)$	operator of translation at a distance a_x along the X-axis

$\mathbf{Tr}(a_y, Y)$	operator of translation at a distance a_y along the Y-axis
$\mathbf{Tr}(a_z, Z)$	operator of translation at a distance a_z along the Z-axis
U_g, V_g	curvilinear (*Gaussian*) coordinates of a point on the gear tooth flank, $\mathcal{G}$
U_p, V_p	curvilinear (*Gaussian*) coordinates of a point on the pinion tooth flank, $\mathcal{P}$
$\mathbf{U}_g, \mathbf{V}_g$	tangent vectors to curvilinear coordinate lines on the gear tooth flank, $\mathcal{G}$
$\mathbf{U}_p, \mathbf{V}_p$	tangent vectors to curvilinear coordinate lines on the pinion tooth flank, $\mathcal{P}$
$\mathbf{V}_\Sigma$	vector of the resultant motion of the pinion tooth flank, $\mathcal{P}$, in relation to the gear tooth flank, $\mathcal{G}$
Z_{pa}	active length of the plane of action
a	tooth addendum
b	tooth dedendum
d	pitch diameter
$d_{b \cdot g}$	base diameter of a gear
$d_{b \cdot p}$	base diameter of a pinion
$d_{o \cdot g}$	outer diameter of a gear
$d_{o \cdot p}$	outer diameter of a pinion
d_f	root diameter
d_l	start of active profile diameter
d_o	outside diameter
h_t	total tooth height
i	current point of the path of contact
$k_{1 \cdot g}, k_{2 \cdot g}$	first- and second-principal curvatures of the gear tooth flank, $\mathcal{G}$
$k_{1 \cdot p}, k_{2 \cdot p}$	first- and second-principal curvatures of the pinion tooth flank, $\mathcal{P}$
m	module
m_p	transverse (profile) contact ratio
m_F	face contact ratio
m_t	total contact ratio
$\mathbf{n}$	unit normal vector of the common perpendicular at point of contact, K, of the gear tooth flank, $\mathcal{G}$, and the pinion tooth flank, $\mathcal{P}$
$\mathbf{n}_g$	unit normal vector to the gear tooth flank, $\mathcal{G}$
$\mathbf{n}_p$	unit normal vector to the pinion tooth flank, $\mathcal{P}$
p_b	base pitch
$p_{b \cdot g}$	linear base pitch of the gear
$p_{b \cdot p}$	linear base pitch of the pinion
$p_{b \cdot op}$	operating linear base pitch of the gear pair
p_{sc}	screw parameter (reduced pitch) of instant screw motion of the pinion in relation to the gear
p_x	axial pitch of the gear teeth
$r_{b \cdot g}$	radius of base circle/cylinder of a gear

r_g	pitch radius of a gear
r_p	pitch radius of a pinion
$\mathbf{r}_g$	position vector of a point of a gear tooth flank, $\mathcal{G}$
$\mathbf{r}_p$	position vector of a point of a pinion tooth flank, $\mathcal{P}$
r_N	radius of the *boundary N-circle* in *Novikov gearing* and in parallel-axis *high-conforming gearing*
r_g	pitch radius of a gear
r_p	pitch radius of a pinion
r_{cnf}	position vector of a point of the indicatrix of conformity, $Cnf_R(\mathcal{G}/\mathcal{P})$
s	space width
s_n	normal space width
s_t	transverse space width
t	tooth thickness
t_n	normal tooth thickness
t_t	transverse tooth thickness
$\mathbf{t}_{1\cdot g}$, $\mathbf{t}_{2\cdot g}$	unit tangent vectors of principal directions on the gear tooth flank, $\mathcal{G}$
$\mathbf{t}_{1\cdot p}$, $\mathbf{t}_{2\cdot p}$	unit tangent vectors of principal directions on the gear tooth flank, $\mathcal{P}$
$\mathbf{u}_g$, $\mathbf{v}_g$	unit tangent vectors to curvilinear coordinate lines on the gear tooth flank, $\mathcal{G}$
$\mathbf{u}_p$, $\mathbf{v}_p$	unit tangent vectors to curvilinear coordinate lines on the gear tooth flank, $\mathcal{G}$
u	gear ratio
u_ω	angular velocity ratio
$\mathbf{V}_\Sigma$	unit vector of the resultant motion of the pinion tooth flank, $\mathcal{P}$, in relation to the gear tooth flank, $\mathcal{G}$
$x_g\, y_g\, z_g$	local *Cartesian* coordinate system having origin at a current point of contact of the teeth flanks, $\mathcal{G}$ and $\mathcal{P}$

Greek Symbols

Γ_l	*boundary N-cone* angle (in intersected-axis as well as crossed-axis high-conforming gearing)
Σ	crossed-axis angle (shaft angle)
$\Phi_{1\cdot g}$, $\Phi_{2\cdot g}$	first- and second-fundamental forms of the gear tooth flank, $\mathcal{G}$
$\Phi_{1\cdot p}$, $\Phi_{2\cdot p}$	first- and second-fundamental forms of the pinion tooth flank, $\mathcal{P}$
γ	specific sliding
γ_g	slide/roll ratio for the gear tooth flank, $\mathcal{G}$
γ_p	slide/roll ratio for the pinion tooth flank, $\mathcal{P}$