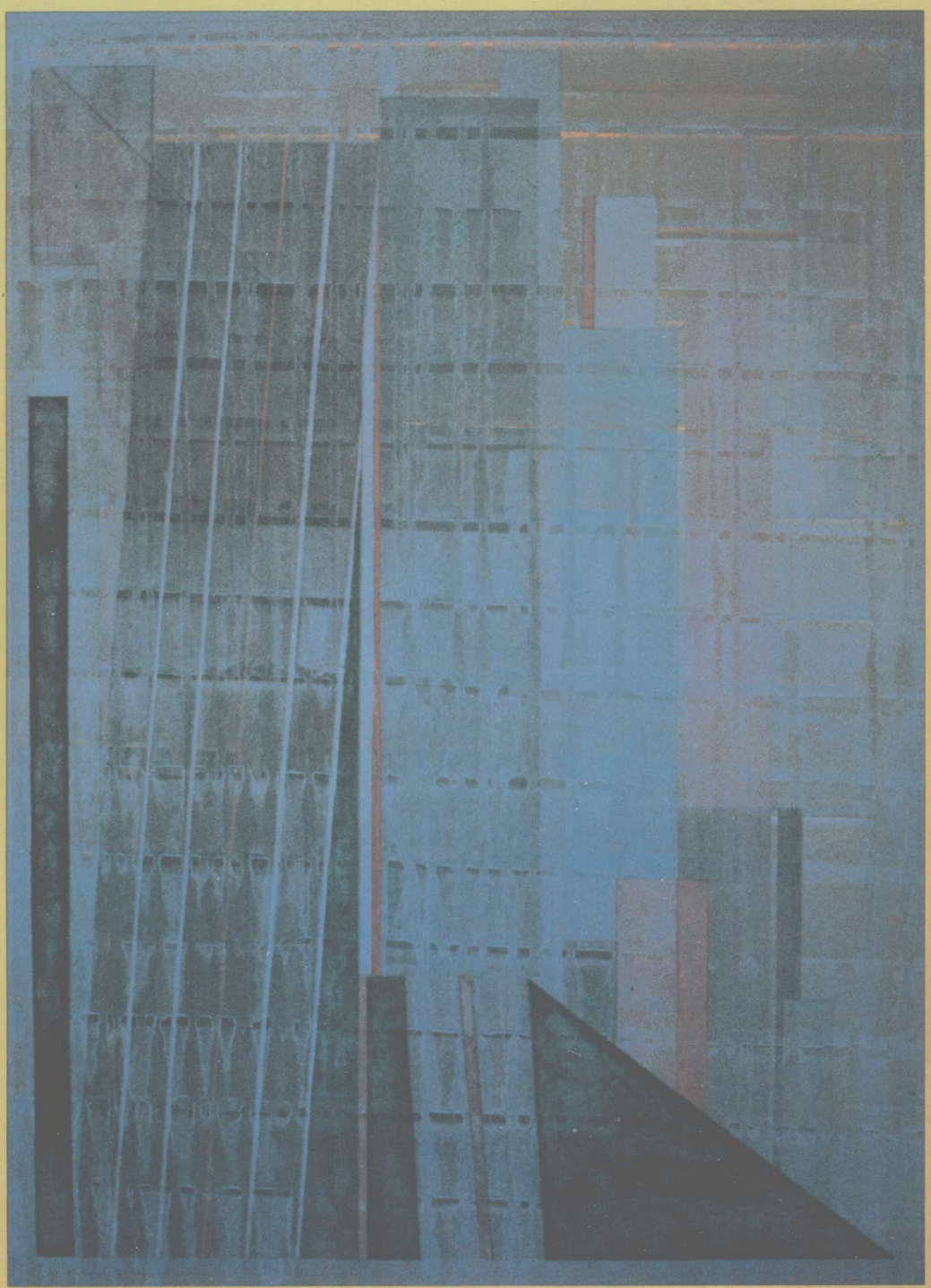
Linda Pulsinelli Patricia Hooper

Second Edition

INTRODUCTORY ALGEBRA

AN INTERACTIVE APPROACH



Introductory Algebra

An Interactive Approach

SECOND EDITION

Linda Pulsinelli
Patricia Hooper
Department of Mathematics
Western Kentucky University

MACMILLAN PUBLISHING COMPANY
NEW YORK

Collier Macmillan Publishers
London

For our children
Gary and Elizabeth
Jim W., Regina, Bob, and Mel

Macmillan Publishing Company
866 Third Avenue, New York, New York 10022

Collier Macmillan Canada, Inc.

Library of Congress Cataloging in Publication Data

Pulsinelli, Linda Ritter.
Introductory algebra.
Includes index.
1. Algebra. I. Hooper, Patricia (Patricia I.)
II. Title.
QA152.2.P85 1987 512 86-2795
ISBN 0-02-396940-7

Printing: 8 Year: 0 1 2 3 4 5

ISBN 0-02-396940-7

Preface

As its title indicates, *Introductory Algebra: An Interactive Approach* is intended to teach students the fundamental concepts and skills of algebra by involving them continually in the learning process, allowing them to see their own progress and therefore motivating them to continue.

The vocabulary level of a book of this type is crucial to its success and we have attempted to word our explanations in simple, straightforward language. The writing style is informal without being condescending, and important ideas are summarized in table form rather than paragraph form whenever appropriate.

Our experience with students at this level leads us to believe that they must practice each new skill as soon as it has been presented. For this reason, the text includes several unique features that are designed to provide maximum reinforcement.

Each chapter in the book follows the same basic structure.

Motivational Applied Problem

At the beginning of each chapter we have presented an applied problem that can be solved after the student has mastered the skills in that chapter. Its solution appears within the chapter.

Explanations

We have tried to avoid the ''cookbook'' approach to algebra by including a straightforward and readable explanation of each new concept. Realizing that students at the introductory level become easily bogged down in reading lengthy explanations, we have attempted to make our explanations as brief as possible without sacrificing rigor.

Highlighting

Definitions, properties, theorems, and formulas are highlighted in boxes throughout the book for easy student reference. In most cases a rephrasing of a generalization in words accompanies the symbolic statement, and it is also highlighted in a box.

Examples

Immediately following the presentation of a new idea, several completely worked-out examples appear together with several partially worked-out examples to be finished by the student. These examples are completed correctly at the end of each section and the student is advised to check his or her work immediately.

Trial Runs

Sprinkled throughout each section are several short Trial Runs, a list of six or eight problems to check on the student's grasp of a new skill. The answers appear at the end of the section.

Exercise Sets
Each section concludes with an extensive Exercise Set in which each odd-numbered problem corresponds closely to the following even-numbered problem.

Stretching the Topics
At the end of each Exercise Set there are several problems designed to challenge the better students by extending to the next level of difficulty the skills learned in the chapter.

Checkups
Following each Exercise Set, a list of about 10 problems checks on the student's mastery of the most important concepts in the section. Each Checkup problem is keyed to comparable examples in the section for restudy if necessary.

Problem Solving
One section of almost every chapter involves switching from words to algebra. By including such a section in each chapter we are attempting to treat problem solving as a natural outgrowth of acquiring algebraic skills.

Chapter Summaries
Each chapter concludes with a summary in which the important ideas are again highlighted, in tables when possible. New concepts are presented in symbolic form and verbal form, accompanied by a typical example.

Speaking the Language of Algebra
Following the summary, we have included a group of sentences to be completed *with words* by the student. Algebra students (especially those in self-paced programs) often lack the opportunity to "speak mathematics." We hope that these short sections will help them develop a better mathematics vocabulary.

Review Exercises
A list of exercises reviewing all the chapter's important concepts serves to give the student an overview of the content. Each problem is keyed to the appropriate section and examples.

Practice Test
A Practice Test is included to help the student prepare for a test over the material in the chapter. Once again, each problem is keyed to the appropriate chapter sections and examples.

Sharpening Your Skills
Finally, we have included a short list of exercises that will provide a cumulative review of concepts and skills from earlier chapters. Retention seems to be a very real problem with students at this level and we hope that these exercises will serve to minimize that problem. Each cumulative review exercise is keyed to the appropriate chapter and section.

Throughout the book we have adhered to a rather standard order of topics, making an attempt to connect new concepts to old ones whenever appropriate. This modified spiraling technique is designed to help students maintain an overview of the content. Success in future courses seems to us to hinge on students' seeing that algebra is a logical progression of ideas rather than a set of unrelated skills to be memorized and forgotten.

Chapter 1 contains a review of the arithmetic of whole numbers, fractions, decimals, and percents. Students who have recently completed an arithmetic course or whose arithmetic skills are not weak may be allowed to skip this chapter. All the properties of real numbers are included in Chapter 2, so the omission of Chapter 1 will not handicap the student who is proficient in computation with fractional and decimal numbers. Problems involving such numbers appear throughout the remaining chapters.

A section on negative exponents is included in Chapter 5 for those instructors who consider this an appropriate point at which to introduce this topic to introductory algebra students. For those who do not, the section can be omitted or inserted at a later time without disturbing the continuity of the remainder of the book.

The answers to the odd-numbered exercises in the Exercise Sets appear in the back of the book together with answers for *all* items in Stretching the Topics, Checkups, Speaking the Language of Algebra, Review Exercises, Practice Tests, and Sharpening Your Skills.

More assistance for students and instructors can be found among the supplementary materials that accompany this book.

Instructor's Manual with Test Bank

The Instructor's Manual contains the answers for all exercises in the Exercise Sets and Stretching the Topics. In addition there are eight Chapter Tests (four open-ended and four multiple choice) for each chapter and three Final Examinations (two open-ended and one multiple choice). Answers to these tests and examinations also appear in the Instructor's Manual. The chapter tests are also available on computer disks.

Student's Solutions Manual

The Student's Solutions Manual, written by Rebecca Stamper, contains step-by-step solutions for the even-numbered exercises in the Exercise Sets and for *all* items in the Review Exercises, Practice Tests, Sharpening Your Skills, and exercises involving word problems. Using the same style as appears in the text, these solutions emphasize the procedure as well as the answer.

Video Tapes

A series of 10 video tapes (each 20 to 30 minutes in length) provides explanations for some of the more difficult topics in the course.

Audio Tapes

A series of 10 audio cassettes (each 20 to 30 minutes in length) also offer explanations for the more difficult topics. Keyed to examples in the text, these cassettes encourage students to work along.

Computerized Test Generator

A set of computer-generated tests is available for producing either a 10-item test for each chapter *or* tests of any length from objective-referenced items. Cumulative tests and final exams may also be constructed using the objective-referenced items.

Interactive Software Program

This computer assisted program is available in Apple and IBM and is a series of lessons including problems at differing levels of difficulty.

Acknowledgments

The writing of this book would not have been possible without the assistance of many people. We express our appreciation to our indefatigable typist Maxine Worthington, to Rebecca Stamper for carefully working all our problems, to our families for tolerating our obsessive work schedules, to our Mathematics Editors Gary Ostedt and Bob Clark for their enthusiastic support, and to our Production Supervisor Elaine Wetterau for her efficiency and expertise.

We also thank our reviewers: Mary Jean Brod, University of Montana; Donald R. Johnson, Scottsdale Community College; Adele Le Gere, Oakton Community College; Gerald J. LePage, Bristol Community College; Lois Miller, El Camino College; Harold M. Nerr, University of Wisconsin; Dan Streeter, Portland State University; and Jack W. Rotman, Lansing Community College; for their careful scrutiny and helpful comments.

L. R. P.
P. I. H.

Contents

Working with the Numbers of Arithmetic

1

Each month, Estelle saves $\frac{1}{10}$ of her paycheck in her savings bank and $\frac{1}{9}$ of her paycheck in her credit union. What fractional part of her paycheck does Estelle save each month?

When you have completed this chapter, you will be able to solve this problem using the numbers of arithmetic. Although we assume that your basic arithmetic facts are in good shape, we must be sure that you recall the techniques required for working with whole numbers, fractions, and decimals.

In this chapter we

1. Perform operations with whole numbers.
2. Observe some properties of zero and 1.
3. Work with fractions.
4. Work with decimals and percents.
5. Switch from word expressions to number expressions.

1.1 Working with Whole Numbers

As a child, you learned to count "one, two, three," and so on. These counting numbers are also called **natural numbers**.

Natural numbers: $\{1, 2, 3, 4, 5, \ldots\}$

The dots after the 5 mean "and so on"; in other words, this set continues indefinitely. If we include the number zero in this set, we obtain a new set of numbers called the **whole numbers**.

Whole numbers: $\{0, 1, 2, 3, 4, 5, \ldots\}$

The Number Line

To picture the whole numbers, we shall use a **number line**. We draw a line and choose a zero point and a length to represent 1 unit. Then all points spaced 1 unit apart to the right of zero are labeled with the whole numbers in order. Many points between the whole numbers are named by fractions and decimal numbers.

The arrow shows that this line goes on indefinitely, so that numbers such as 24 and 793 also correspond to points.

To show a whole number on the number line, we put a solid dot at the point corresponding to that number. This is called **plotting a point** on the number line. The point is called the **graph** of the number and the number is called the **coordinate** of the point.

Example 1. Graph 4 on the number line.

Solution

You try Example 2.

Example 2. Graph 0 on the number line.

Solution

Check your work on page 13. ▶

The number line also gives us a handy way of comparing numbers. For example, if we look at the graph of the number 3, we notice that all the numbers to the right of 3 are greater than 3. All numbers to the left of 3 are less than 3.

Mathematicians use symbols to compare numbers.

Symbol	Meaning	Example
$>$	is greater than	$4 > 1$
$<$	is less than	$6 < 20$

Such statements are called **inequalities** and we summarize them as follows, for any numbers A and B.

Less than:	$A < B$	if A lies to the left of B on the number line
Greater than:	$A > B$	if A lies to the right of B on the number line

Example 3. Place a $<$ or $>$ symbol between the numbers.

$12 \_\_\_\_ 3$

$0 \_\_\_\_ 1$

Solution

$12 > 3$ because 12 lies to the right of 3

$0 < 1$ because 0 lies to the left of 1

You try Example 4.

Example 4. Place a $<$ or $>$ symbol between the numbers.

$5 \_\_\_\_ 7$

$110 \_\_\_\_ 99$

Solution

$5 \_\_\_\_ 7$

$110 \_\_\_\_ 99$

Check your work on page 13. ▶

Trial Run

1. Graph 6 on the number line.

Compare the numbers using the symbols $<$ or $>$.

2. $7 \_\_\_\_ 5$

3. $0 \_\_\_\_ 2$

4. $186 \_\_\_\_ 187$

5. $2 \_\_\_\_ 0$

Answers are on page 15.

Performing Operations with Whole Numbers

The four basic operations of arithmetic are *addition*, *subtraction*, *multiplication*, and *division*. It will help you in the study of algebra if you learn to name the parts of each of the four types of arithmetic problems now.

Addition

$$\underset{\text{term}}{12} + \underset{\text{term}}{3} = \underset{\text{sum}}{15}$$

Subtraction

$$\underset{\text{term}}{12} - \underset{\text{term}}{3} = \underset{\text{difference}}{9}$$

Multiplication

$$\underset{\text{factor}}{12} \cdot \underset{\text{factor}}{3} = \underset{\text{product}}{36}$$

Division

$$\underset{\text{dividend}}{12} \div \underset{\text{divisor}}{3} = \underset{\text{quotient}}{4}$$

Example 5. Find the sum of 10 and 9.

Solution

$$10 + 9 = 19$$

The sum is 19.

Example 6. Find the product of 10 and 9.

Solution

$$10 \cdot 9 = 90$$

The product is 90.

If you ever used your fingers to find a sum, you should agree that *addition is just a form of counting*. When you learned your basic addition facts in elementary school, you mastered a shortcut for finding sums without counting.

To learn your basic subtraction facts, you discovered that *every subtraction statement corresponds to an addition statement*.

$$15 - 8 = 7 \quad \text{because} \quad 15 = 8 + 7$$
$$9 - 0 = 9 \quad \text{because} \quad 9 = 0 + 9$$

Relating Subtraction to Addition. For whole numbers A, B, and C

$$A - B = C \quad \text{means} \quad A = B + C$$

There are several ways to indicate that two numbers are to be multiplied. For example, to multiply 4 times 3 we may write

$$4 \text{ X } 3 \qquad 4 \cdot 3 \qquad 4(3) \qquad (4)3 \qquad (4)(3)$$

Each of these expressions says the same thing, but we shall not use the first notation 4 X 3, because of the confusion that might occur when we begin to use the letter X to mean something else.

One way to find the product of two numbers is to realize that *multiplication is just repeated addition*. Your knowledge of addition helped you learn your multiplication facts.

$$4 \cdot 3 \quad \text{means} \quad 3 + 3 + 3 + 3 \quad \text{so} \quad 4 \cdot 3 = 12$$
$$3 \cdot 4 \quad \text{means} \quad 4 + 4 + 4 \quad \text{so} \quad 3 \cdot 4 = 12$$
$$2 \cdot 5 \quad \text{means} \quad 5 + 5 \quad \text{so} \quad 2 \cdot 5 = 10$$

Relating Multiplication to Addition. For whole numbers A and B

$$A \cdot B \quad \text{means} \quad \underbrace{B + B \cdot \cdots + B}_{A \text{ terms}}$$

Example 7. Write the addition statement that corresponds to the subtraction statement $25 - 12 = 13$.

Solution

$$25 - 12 = 13$$

means $25 = 12 + 13$

You try Example 8.

Example 8. Write the product $5 \cdot 7$ as a sum.

Solution

$5 \cdot 7 =$ ____ + ____ + ____ + ____ + ____

Check your answer on page 13. ▶

There are several ways to indicate the operation of division. To show 6 divided by 3, we may write

$$6 \div 3 \qquad 3\overline{)6} \qquad 6/3 \qquad \frac{6}{3}$$

Each of these expressions is equal to 2.

To understand the operation of division, you should remember that *every division statement corresponds to a multiplication statement.*

$$\frac{6}{3} = 2 \quad \text{because} \quad 6 = 3 \cdot 2$$

$$\frac{40}{8} = 5 \quad \text{because} \quad 40 = 8 \cdot 5$$

$$\frac{19}{1} = 19 \quad \text{because} \quad 19 = 1 \cdot 19$$

Relating Division to Multiplication. For whole numbers A, B, and C

$$\frac{A}{B} = C \quad \text{means} \quad A = B \cdot C$$

where B does not equal zero ($B \neq 0$).

Example 9. Write the multiplication statement that corresponds to the division statement $\frac{0}{15} = 0$.

Solution

$$\frac{0}{15} = 0$$

means $0 = 15 \cdot 0$

You complete Example 10.

Example 10. Write the multiplication statement that corresponds to the division statement $\frac{23}{23} = 1$.

Solution

$$\frac{23}{23} = 1$$

means ____ = ____ · ____

Check your answer on page 13. ▶

Trial Run

_____ 1. Write the addition statement that corresponds to the subtraction statement $11 - 0 = 11$.

_____ 2. Write the product $4 \cdot 2$ as a sum.

_____ 3. Write the multiplication statement that corresponds to the division statement $\frac{18}{6} = 3$.

_____ 4. Write the multiplication statement that corresponds to the division statement $\frac{0}{3} = 0$.

Answers are on page 15.

Working with Zero and One

When we operate with the whole numbers zero and 1, we observe some interesting facts. Consider the following sums:

$$13 + 0 = 13$$
$$0 + 7 = 7$$

Notice that

If zero is added to any whole number, the answer is that whole number.

This law is called the **addition property of zero** (or the **identity property for addition**).

Addition Property of Zero. For any whole number A

$$A + 0 = A$$
$$0 + A = A$$

We call zero the **identity for addition** because when zero is added to any number, the answer is identical to the original number.

Similarly, the differences

$$17 - 0 = 17$$
$$1 - 0 = 1$$

should help you see that

If zero is subtracted from any whole number, the answer is that whole number.

We call this law the **subtraction property of zero**.

Subtraction Property of Zero. For any whole number A

$$A - 0 = A$$

It is also interesting to note here that if any whole number is subtracted from itself, the answer is zero.

$$7 - 7 = 0 \quad \text{because} \quad 7 = 7 + 0$$
$$96 - 96 = 0 \quad \text{because} \quad 96 = 96 + 0$$

For any whole number
$$A - A = 0$$

What happens when we multiply any number times zero? Recalling that multiplication is repeated addition, look at this product:

$$5 \cdot 0 = 0 + 0 + 0 + 0 + 0$$
$$= 0$$

If any whole number is multiplied times zero, the answer is 0.

This law is called the **multiplication property of zero**.

Multiplication Property of Zero. For any whole number A
$$A \cdot 0 = 0$$
$$0 \cdot A = 0$$

Use the properties of zero to complete Example 11.

Example 11. Complete each statement.

$13 + 0 =$ ____ $\quad 167 - 0 =$ ____

$73 +$ ____ $= 73 \quad 29 \cdot 0 =$ ____

$19 -$ ____ $= 19 \quad 0 \cdot 15 =$ ____

$19 -$ ____ $= 0 \quad 1 \cdot$ ____ $= 0$

Check your work on page 13. ▶

Let's investigate what happens when zero appears as the divisor or dividend in a division problem. First we shall try to divide a whole number *by* zero. Consider the quotient $\frac{6}{0}$. Remember that every division statement corresponds to a multiplication statement. Can we say

$$\frac{6}{0} = 0 \quad \text{because} \quad 6 = 0 \cdot 0?$$