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Engineering Mechanics of Deformable Solids

A Presentation with Exercises

SANJAY GOVINDJEE

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To my teachers who showed me the beauty of learning,
To my parents who led me to academia,
To Arjun and Rajiv for loving all things technical,
To Marilyn for always being there for all of us

Preface

This text was developed for a Strength of Materials course I have taught at the University of California, Berkeley for more than 15 years. The students in this course are typically second-semester Sophomores and first-semester Juniors. They have already studied one semester of mechanics in the Physics Department and had a separate two-unit engineering course in statics, and most have also completed or are concurrently completing a four-semester mathematics sequence in calculus, linear algebra, and ordinary and partial differential equations. Additionally they have already completed a laboratory course on materials. With regard to this background, the essential prerequisites for this text are the basic physics course in mechanics and the mathematics background (elementary one- and multi-dimensional integration, linear ordinary differential equations with constant coefficients, introduction to partial differentiation, and concepts of matrices and eigen-problems). The additional background is helpful but not required. While there is a wealth of texts appropriate for such a course, they uniformly leave much to be desired by focusing heavily on special techniques of analysis overlaid with a dizzying array of examples, as opposed to focusing on basic principles of mechanics. The outlook of such books is perfectly valid and serves a useful purpose, but does not place students in a good position for higher studies.

The goal of this text is to provide a self-contained, concise description of the main material of this type of course in a modern way. The emphasis is upon kinematic relations and assumptions, equilibrium relations, constitutive relations, and the construction of appropriate sets of equations in a manner in which the underlying assumptions are clearly exposed. The preparation given puts weight upon model development as opposed to solution technique. This is not to say that problem-solving is not a large part of the material presented, but it does mean that “solving a problem” involves two key items: the formulation of the governing equations of a model, and then their solution. A central motivation for placing emphasis upon the formulation of governing equations is that many problems, and especially many interesting problems, first require modeling before solution. Often such problems are not amenable to hand solution, and thus they are solved numerically. In well-posed numerical computations one needs a clear definition of a complete set of equations with boundary conditions. For effective further studies in mechanics

this viewpoint is essential, and thus the presentation, in this regard, is strongly influenced by the need to adequately prepare students for further study in modern methods.

Sanjay Govindjee
Berkeley and Zürich

*Mechanics is the paradise of mathematical science,
because here we come to the fruits of mathematics*

Leonardo da Vinci

Theory is the captain, practice the soldier

Leonardo da Vinci

Mechanics is not a spectator sport

Sanjay Govindjee

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