

Exploring Elementary Mathematics

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Instructor's Manual
for
EXPLORING ELEMENTARY MATHEMATICS:

by
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San Francisco

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INTRODUCTION

If you are reading this you are probably considering using or have already decided to use Exploring Elementary Mathematics: A Small Group Approach to Teaching. Whether you are going to use this book as a primary text or as a supplement for laboratory or small group work, you will most likely have some questions about the book and how to use it effectively.

I developed this book to help me teach a course in mathematics for prospective elementary school teachers at the University of California, Santa Barbara, using a small group approach in a mathematics laboratory setting. The text, the laboratory setting and the small group approach work well together. However you may find that this text works better for you with another method or in another setting. The text is readily adaptable to different uses.

Since the small group approach is a departure from the predominant form of mathematics teaching, I have addressed this manual primarily to those who will be trying out this approach. The manual is intended to indicate how I and others have used this material in classrooms during the past six years, to answer common questions that people ask about the small group approach, and to provide comments about selected exercises and the reading material. Answers to the Review Exercises are also included.

USING A SMALL GROUP APPROACH

THE INSTRUCTOR

For many of us, the decision to use a small group approach to teach mathematics is a significant change. We have typically been taught by the lecture method and most of our colleagues teach that way. It is always challenging to change, and it is particularly challenging when there are few models to follow. In addition, we may feel comfortable with the lecture method, or at least be used to preparing lectures.

The small group approach, furthermore, usually involves more than a change in teaching technique. My own experience has been one of gradually changing my attitude about my responsibilities as a teacher. Instead of viewing my primary role as one of imparting information, I now see it as one of setting up situations that facilitate learning. In addition, I now resist the tendency to treat each student as a future mathematician. I have come to believe that the experiences my students have in class and the attitudes they develop about mathematics will be as important to them in their teaching career as the knowledge they acquire.

Furthermore, interacting with small groups is different than interacting with a whole class. The students for the most part will be helping each other learn. Our task is to notice when learning is not taking place, observe the difficulties in the groups and find creative ways of resolving them. I have found that one reliable teaching technique is to help students "discover" the answer by asking a series of appropriate (well-paced) questions. We need to resist the temptation to tell students all the answers. Questions are often more fruitful

than answers. This technique can be used by the students also and your using it will be a good model for them. To be sure, it doesn't always work, but it is often more effective than explaining the situation and then discovering that you have overwhelmed the learner with too much new information.

The change of role from "delivering information" to "setting up learning situations" does not mean that we abandon the students to "do their own thing". It means thinking to the best of our ability about each group and each individual so as to help them learn. The teacher has a crucial role to play in establishing favorable conditions for learning.

I have personally found it helpful, on the first day of class, to let the students get to know me by telling them about myself and my ideas about mathematics. Also, I've found it beneficial for me to get to know the students by spending some time with each group on the first day, discussing non-mathematical subjects. One way of doing this is mentioned in the section below entitled "Class Organization".

The small-group approach will provide you with the opportunity to encourage your students individually. It is helpful to approve of and appreciate their efforts whenever you have the opportunity. Remember that many students lack confidence in themselves and may have experienced failure and criticism in previous mathematical learning situations. By appreciating their efforts and accomplishments, you will help them overcome any negative feelings they may have.

The attitude you show about mathematics and learning is extremely important. If you are enthusiastic it will be contagious.

You will have the opportunity, if you use the small-

group approach, to observe the thinking process of students - how they learn, where they get stuck, how they misinterpret definitions and explanations. This can be frustrating for us, but it can also be informative. We can learn a lot by observing how people learn.

I have found it helpful to be somewhat aggressive when interacting with students. Students having difficulty will often become passive. This is almost always a result of experiencing criticism in previous learning situations. More criticism is not the answer, but neither is neglect. Be thoughtful in approaching students with problems, but do approach them.

Finally, encourage your students to read the section "To the Student" in the text (pp. xiii-xv). It contains useful ideas and information for student and teacher alike.

Additional information about the philosophy and practice of the small-group approach is found in my article, "Small Groups: An Alternative to the Lecture Method", which won the 1976 George Polya Award from the Mathematics Association of America. It appeared in the Two Year College Mathematics Journal and is reprinted as an appendix to this manual.

CLASS ORGANIZATION

I have found that four or five students in a group works well. The best way I've found for organizing the groups is to divide the class size by four to obtain the number of groups and then have the students number off. For example, if there are 26 students in the class there will be six groups. The students number off 1,2,3,4,5,6, 1,2,3,..., by each student saying a number in turn. Then all of the "1's" form a group, all of the "2's" form another group and so on. You will end up with six groups (two of them will have five people and four of them will

have four people.)

Groups can be reorganized (after an examination is a good time) or they can work together for an entire term. You may need to reorganize some groups if people drop the class or if there are troublesome personality conflicts. You can decide at the time of the first exam whether or not you want to reorganize the groups. If some are not working well you can assign new groups based on your judgment about who would work well together. A note of caution: It is not a good idea to reorganize groups late in the term.

It is important that the groups get to know each other before they start learning together. On the first day have them introduce themselves and say something about themselves. In fact, it is a good idea to start each class with an introductory exercise which will get the students communicating with each other. I have included in the section Remarks on the Chapters a suggested class-starter for each set of lab exercises. These class-starters or ones you invent may seem awkward at first. My experience has been, however, that if you persist both you and your students will come to recognize the value of these activities. Class-starters should be positive. If the students start the class feeling good, they will have more attention available for learning. It is a good idea for the instructor to participate in these exercises. This is particularly important on the first day. When you go to each group you can have them repeat the introductory exercise with you. On subsequent days you can attend the groups alternately during the class-starter. Your participation in these activities will help establish good relationships between you and your students.

I have found that I can handle a maximum of 32 students (eight groups of four students) in a class. However it goes better if the class has only 28 students and 24 is

ideal. During the class, I circulate from group to group. I sometimes observe how the group is functioning for awhile. Then I ask whether they have any questions. If they do I either try to get them to discover the solution or give them some information. If they do not have any questions, I will often ask them to explain certain exercises to me. In the chapter analyses in this manual certain exercises are indicated by a \surd . and are called checkpoint questions. If the students understand these exercises it is likely that they understand the main concepts in the chapter, so I usually question them about these exercises. I try to make sure than everyone in the group understands the exercise. If someone does not understand I encourage her or him to ask more questions.

It is certainly possible to combine lecturing with the small-group approach. I often give a five minute review of the previous days material at the beginning of the class period. Occasionally I will interrupt the group work to talk about a difficult concept or to summarize some ideas or add perspective. Perspective is particularly useful when you are able to show the relationship between three or four chapters. It is also possible to lecture one or two times per week and have the class meet for lab work at different times. I have used this effectively in large classes with teaching assistants supervising the lab sections. If you do continue lecturing with the small-group approach try to have the lab experiences with a particular topic come before you talk about it. This gives the students a concrete experience before being asked to understand abstract concepts. Very often they will need these experiences for their learning to proceed well. In addition, by doing the lab exercises first they will develop curiosity about the subject. After they have formulated their own questions, they will be more receptive to receiving information (See the quotation in the text on p. 69). This is the reason that the Comment section follows the lab exercises in the text. I usually have the

students work through the lab exercises in class and assign the Comment sections to be read and worked on outside of class. I then encourage the students to discuss the exercises they do at home in their small groups if they want to. I also encourage the groups to work together outside of class if possible.

THE ROOM

Any room with movable chairs will suffice, but it is ideal to have small tables for the students to sit around.

THE EQUIPMENT

I have tried to use the equipment in a way that will challenge college students. This is explained to the students on p. xv of the text, where I also explain why it is important to use the equipment. It will be helpful if you emphasize this. Many students have missed some concrete experiences which are necessary for understanding various mathematical concepts and/or they have been pressured into symbol manipulation without being allowed enough free exploration. As a result they sometimes feel reluctant to use the equipment. It is an unfamiliar tool for them in mathematics. They feel they should be able to figure it out "in their head". I have seen students staring off into space waiting for the answer to pop into their heads while the equipment lay unused in front of them. On the other hand, there will often be one or two students in each group that are eager to use the equipment. This sometimes results in the other students becoming even more passive. They will need your encouragement.

A list of equipment and sources for purchases is on pp. xvii-xviii. As you read over the equipment list, you will find that most of the equipment can be purchased individually by the student or constructed by them. Of course it is nice to have a well-stocked lab, but you can

get by on very little. The book has been used successfully with only a set of MBA-blocks in the lab. The chart below indicates the equipment needed for the various chapters and which equipment it is feasible to have the students purchase or construct.

<u>For Chapters</u>	<u>Needed in Laboratory</u>	<u>Student Purchase or Construction</u>
1-11	MBA-blocks	A-blocks Abacus Colored Rods (Cuisenair Introductory set or made from construction paper) Red & Black Counters
12	Geoblocks (Optional)	Geoboard Tangrams
13		Mira
14		Geoboard Tangrams
15		Compass
16	Meter sticks Metric tape measure (or adding machine tape) Trundle wheel (Optional)	Colored-rods
17	Pattern Blocks Scissors	
18		Compass Mira Ruler
19	Pattern Blocks	Mira
20	Dice	Coins
21		
22	Scissors Tape	C-rods Geoboard Paper

Of course it is convenient if you can supply more of the lab equipment. Many institutions have funds for instructional improvement which you may be able to use. If not an approach to a Dean or other administrative official may be necessary. It is helpful to explain that mathematics lab equipment for this course is similar to lab equipment for courses in the sciences. You can also look into borrowing equipment from your education department or your local school system. Finally, you might be able to induce some manufacturer's to donate equipment to your institution since you will be using the equipment with future elementary teachers.

For your convenience I include on p.10 two sample budgets for supplying a mathematics laboratory based on 1979 prices. Notice that Mira's are not included. They are so attractive that they tend to disappear. Also the students will need them for work outside class.

Sample Budgets
(for 32 students)

Equipment	Option 1		Option 2	
	Number	Cost	Number	Cost
Attribute Blocks	8	\$52	8	\$52
Cuisenaire Rods	18	\$54	12	\$36
Multibase Blocks	Bases 2,3,4,5	\$225	Bases 2,3,4,5	\$225
Geoboards	32	\$32	16	\$16
Pattern Blocks	8	\$144	4	\$72
Meter Sticks	8	\$12	4	\$6
Metric tape measures	8	\$24	0	0
Scissors	8	\$26	4	\$13
Dice	32	\$8	16	\$4
Abacus	16	\$12	8	\$8
Geoblocks and Acti- vity Cards ⁺ (op- tional)	2	\$120	0	0
Trundle wheel (op- tional)	2	\$28	0	0
Unforeseen Expenses and Inflation		\$100		\$50
Total		\$837		\$482

* A set containing base 2,3,4,5 and 10 is obtainable from Creative Publications. A supplementary set of base 4 and 5 is also available. ETA (Educational Teaching Aids) carries each base separately but their base 2 does not have long blocks, flat blocks and block blocks. (These can be made by gluing, however.)

+ The Activity cards seem to be available only from the Webster Division of McGraw-Hill, 1221 Avenue of the Americas, New York, NY 10020.

ATTENDANCE AND PUNCTUALITY

Punctuality and attendance is important when using the small-group approach since group progress may be slowed somewhat by absence and tardiness. This is not as big an obstacle as it may seem since helping someone "catch up" serves as a good review for the group. In addition, there are several things you can do to encourage punctual attendance:

- * emphasize its importance at the beginning of the term with stress on the cooperative nature of the class and the value of their taking responsibility for their own learning
- * talk about the importance of full participation - that the experiences with the equipment and the interaction with each other is important for learning. Tell them that they will learn best what they teach to someone else
- * remind them that they will have great difficulty if they try to learn everything the last couple of weeks

You might also include an evaluation of their participation in the class as part of the grade. This can be a subjective evaluation or based on attendance. (It is easy to pass a sign-in sheet around at the beginning of each class period). In addition, giving a five minute review at the beginning of each class period tends to improve punctuality.

CLASS DYNAMICS

Most groups work well together right from the start. Some others take a few classes before they function well. Occasionally groups will have difficulty. There is much you can do to help the groups function better.

Remind your students from time to time of the value

of thinking about the small-group process and in particular of their paying attention to their teaching. One way to do this is have the group members evaluate themselves. Have each member say how he or she contributes to the group and how he or she would like to improve. If a group is having difficulty each person can speak to the question of what is going well in the group and what needs to improve. It may seem that this will take a lot of time away from learning. However the increased effectiveness of the group makes it worthwhile.

Be aware of potential problems. Some students will need to be reminded that they have the right to ask questions; others that they have the right to take time to think before discussing an exercise or responding to a question. You may need to remind some students that they can play a good role in the group by being less visible or dominating and instead encouraging others to speak up. Men in particular will often need to be reminded to not dominate the discussion and to instead encourage women to participate fully in the groups.

Ethnic or national minorities may experience difficulty in their groups - particularly if people from the majority culture are impatient with them or unaware of how they have been mistreated in the schools. I have had good success in my class by having a group whose primary language was Spanish. They functioned very well together, usually speaking in English until a difficulty arose and then using Spanish to explain the basic ideas involved. We need a lot more experience and information on what works best in this area. If you have any information you would like to share write to me in care of the publisher.

You will sometimes receive complaints from students about their groups or the class. The best way to handle these is to LISTEN. Often this will be enough. The student will no longer feel it is a problem or see a solution

to it. After listening you can ask them "What do you want me to do?" and then "What can you do to change the situation?" Very often problems can be solved if you speak to the entire class about the difficulty in general terms. You can point out what sometimes happens and what needs to happen so that everyone learns well. If the problem persists, you will need to speak directly to the person or persons involved.

OFFICE HOURS

Encourage your students to attend office hours, particularly if they are having difficulty. If possible, hold your office hours in the classroom where the equipment is stored. It will be easy to use the equipment and it will feel safer for the student. Also they will be able to take advantage of the office hours to come in as a group and work together. If you must hold office hours in your office be sure to have some equipment there.

GRADING

Any system of grading can be used with the small-group approach. I usually give a mid-term examination and a final. I usually emphasize the concepts based on the concrete experiences and do not have the students use the equipment very much on the examination since it is inconvenient to share the equipment. I do have them use the Mira though since each student is supposed to have one.

I include as part of their final grade a subjective evaluation of their participation in their group.

When teaching a large class with teaching assistants or undergraduates as assistants I have found it useful for me to take on the role of the evaluator and have the assistants not take on this responsibility. Doing this distinguishes the roles for the students and makes it easier for them to ask for help from the assistants.