Fundamentals of CALCULUS

Carla C. Morris • Robert M. Stark

with website

WILEY

FUNDAMENTALS OF CALCULUS

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Published by John Wiley & Sons, Inc., Hoboken, New Jersey
Published simultaneously in Canada

Library of Congress Cataloging-in-Publication Data:

Morris, Carla C.
Fundamentals of Calculus / Carla C. Morris, Robert M. Stark.
pages cm
Includes bibliographical references and index.
ISBN 978-1-119-01526-0 (cloth)
1. Calculus–Textbooks. I. Stark, Robert M., 1930- II. Title.
QA303.2.M67 2015
515–dc23

2014042182

Printed in the United States of America

10 9 8 7 6 5 4 3 2 1

Preface

A fundamental calculus course is a staple for students in Business, Economics, and the Natural, Social, and Environmental Sciences, among others. Most topics within this book parallel conventional texts, and they appear here directly, unimpeded by lengthy examples, explanations, data, irrelevances, and redundancies. Examples are the primary means to illustrate concepts and techniques as students readily respond to them. While there are ample Exercises and Supplementary Exercises, distracting abundance is avoided. Arrow symbols interspersed in the text, UP (↑) and DOWN (↓), are used to signal student tips, insights, and general information much as an instructor might to students in class to help their understanding. UP arrows modestly increase text depth while DOWN arrows expand aid to students.

Students often question the importance and usefulness of calculus, and some find math courses confusing and difficult. To address such issues, one goal of the text is for students to understand that calculus techniques involve basic rules used in combinations to solve complex problems. The challenge to students is to disassemble problems into manageable components. For example, a derivative of $[(f(x)g(x))/h(x)]^r$. The text encourages students to use power, quotient, and product rules for solutions. Another goal is to encourage students to understand calculus as the mathematics of change. To help, text examples guide students in modeling skills.

The elements of finite calculus lacking in most texts is a feature in this book, which serves multiple purposes. First, it offers an easier introduction by focusing on "change" and enables students to compare corresponding topics with differential calculus. Many may argue that it is the more natural calculus for social and other sciences. Besides, and equally important, finite calculus lends itself to modeling and spreadsheets. In Chapters 2 and 5, finite calculus is applied to marginal economic analysis, finance, growth, and decay.

Each chapter begins with an outline of sections, main topics of discussion, and examples. The outline displays topical chapter coverage and aids in finding items of particular interest. The Historical Notes sketch some of the rich 4000-year history of mathematics and its people.

Some students may skip Chapter 1, **Linear Equations and Functions** while others find it a useful review.

In Chapter 2, **The Derivative** finite differences are introduced naturally in forming derivatives and is a topic in its own right. Usually absent from applied calculus texts, finite calculus emphasizes understanding calculus as the "mathematics of change" (not simply rote techniques) and is an aid to popular spreadsheet modeling.

In Chapter 3, **Using the Derivative** students' newly acquired knowledge of a derivative appears in everyday contexts including marginal economic analysis. Early on, it shows students an application of calculus.

Chapter 4, **Exponential and Logarithmic Functions** delineates their principles. This chapter appears earlier than in other texts for two reasons. One, it allows for more complex derivatives to be discussed in Chapter 5 to include exponentials and logarithms. Two, it allows for the discussion on finite differences in Chapter 5 to be a lead in for integration in Chapter 6, **Integral Calculus**.

Chapter 5, **Techniques of Differentiation** treats the derivatives of products and quotients, maxima, and minima. Finite differences (or finite calculus) appear again in a brief section of this chapter. The anti-differences introduced in Chapter 5 anticipate the basics of integration in Chapter 6.

Chapters 7 and 8, **Integration Techniques** and **Functions of Several Variables** respectively, typify most texts. An exception is our inclusion of partial fractions.

Chapter 9, **Series and Summations** includes important insight to applications.

Chapter 10, **Applications to Probability** links calculus and probability.

The table below suggests sample topic choices for a basic calculus course:

Chapter	Traditional Course	Enhanced Course	Two-Semester Course
1 Linear Equations and Functions	Selections	Selections	Selections
2 The Derivative	✓	✓	✓
3 Using the Derivative	✓	✓	✓
4 Exponential and Logarithmic Functions	✓	✓	✓
5 Differentiation Techniques	Selections	Selections	✓
6 Integral Calculus	✓	✓	✓
7 Integration Techniques	Selections	Selections	✓
8 Functions of Several Variables	Selections	Selections	✓
9 Series and Summations	Optional	Selections	✓
10 Applications to Probability	Optional	Selections	✓

SUPPLEMENTS

A modestly priced Student Solutions Manual contains complete solutions.

SUGGESTIONS

Suggestions for improvements are welcome.

ACKNOWLEDGEMENTS

We have benefitted from advice and discussions with Professors Louise Amick, Washington College; Nancy Hall, University of Delaware Associate in Arts Program-Georgetown; Richard Schnackenberg, Florida Gulf Coast University; Robert H. Mayer, Jr, US Naval Academy, Dr Wiseley Wong, University of Maryland; and Carolyn Krause, Delaware Technical and Community College-Terry Campus. Amanda Seiwell assisted with a PowerPoint® supplement. We acknowledge the University of Delaware's Morris Library for use of its resources during the preparation of this text.

ABOUT THE COMPANION WEBSITE

This book is accompanied by a companion website:

http://www.wiley.com/go/morris/calculus

The website includes:

- Instructors' Solutions Manual
- PowerPoint® slides by chapter
- Test banks by chapter
- Teacher Commentary

About The Authors

Carla C. Morris has taught courses ranging from college algebra to calculus and statistics since 1987 at the Dover Campus of the University of Delaware. Her B.S. (Mathematics) and M.S. (Operations Research & Statistics) are from Rensselaer Polytechnic Institute, and her Ph.D. (Operations Research) is from the University of Delaware.

Robert M. Stark is Professor Emeritus of Mathematical Sciences at the University of Delaware. His undergraduate and doctoral degrees were awarded by, respectively, Johns Hopkins University and the University of Delaware in physics, mathematics, and operations research. Among his publications is the 2004 Dover Edition of Mathematical Foundations for Design with R. L. Nicholls.

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1.1 SOLVING LINEAR EQUATIONS

Mathematical descriptions, often as **algebraic expressions**, usually consist of alphanumeric characters and special symbols.

↑ The name "algebra" has fascinating origins in early Arabic language (Historical Notes).

For example, physicists describe the distance, s, that an object falls under gravity in a time, t, by $s = (1/2)gt^2$. Here, the letters s and t represent **variables** since their values may change while, g, the acceleration of gravity, is considered as constant. While any letters can represent variables, typically, later letters of the alphabet are customary. Use of x and y is generic. Sometimes, it is convenient to use a letter that is descriptive of a variable, as t for time.

Earlier letters of the alphabet are customary for fixed values or **constants**. However, exceptions are common. The equal sign, a special symbol, is used to form an **equation**. An equation equates algebraic expressions. Numerical values for variables that preserve equality are called **solutions** to the equations.

For example, $5x + 1 = 11$ is an equation in a single variable, x. It is a **conditional** equation since it is only true when $x = 2$. Equations that hold for all values of the variable are called **identities**. For example, $(x + 1)^2 = x^2 + 2x + 1$ is an identity. By solving an equation, values of the variables that satisfy the equation are determined.

An equation in which only the first powers of variables appear is a **linear equation**. Every linear equation in a single variable can be solved using some or all of these properties:

Substitution – Substituting one expression for an equivalent one does not alter the original equation. For example, $2(x - 3) + 3(x - 1) = 21$ is equivalent to $2x - 6 + 3x - 3 = 21$ or $5x - 9 = 21$.

Addition – Adding (or subtracting) a quantity to each side of an equation leaves it unchanged. For example, $5x - 9 = 21$ is equivalent to $5x - 9 + 9 = 21 + 9$ or $5x = 30$.

Multiplication – Multiplying (or dividing) each side of an equation by a non-zero quantity leaves it unchanged. For example, $5x = 30$ is equivalent to $(5x)(1/5) = (30)(1/5)$ or $x = 6$.

↓ Here are examples of linear equations: $5x - 3 = 11$, $y = 3x + 5$, $3x + 5y + 6z = 4$. They are linear in one, two, or three variables, respectively. It is the unit exponent on the variables that identifies them as linear.

↓ By "solving an equation" we generally intend the numerical values of its variables.

To Solve Single Variable Linear Equations

1. **Resolve fractions.**
2. **Remove grouping symbols.**
3. **Use addition (and/or subtraction) to have variable terms on one side of the equation.**
4. **Divide the equation by the variable's coefficient.**
5. **As a check, verify the solution in the original equation.**

Example 1.1.1 Solving a Linear Equation

Solve $(3x/2) - 8 = (2/3)(x - 2)$.

Solution:
To remove fractions, multiply both sides of the equation by 6, the least common denominator of 2 and 3. (Step 1 above)
The revised equation becomes

$$9x - 48 = 4(x - 2).$$

Next, remove grouping symbols (Step 2). That leaves

$$9x - 48 = 4x - 8.$$

Now, subtract 4x and add 48 to both sides (Step 3). Now,

$$9x - 4x - 48 + 48 = 4x - 4x - 8 + 48 \text{ or } 5x = 40.$$

Finally, divide both sides by the coefficient 5 (Step 4). One obtains $x = 8$.
The result, $x = 8$, *is checked by substitution in the original equation (Step 5):*

$$3(8)/2 - 8 = (2/3)(8 - 2)$$

$$4 = 4 \text{ checks!}$$

$$\text{The solution } x = 8 \text{ is correct!}$$

Equations often have more than one variable. To solve linear equations in several variables simply bring a variable of interest to one side. Proceed as for a single variable regarding the other variables as constants for the moment.

↓ If y is the variable of interest in $3x + 5y + 6z = 2$, it can be written as $y = (2 - 3x - 6z)/5$ regarding x and z as constants for now.

Example 1.1.2 Solving for y

Solve for y: $5x + 4y = 20$.

Solution:
Move terms with y to one side of the equation and any remaining terms to the opposite side. Here, $4y = 20 - 5x$. Next, divide both sides by 4 to yield $y = 5 - (5/4)x$.

Example 1.1.3 Simple Interest

"Interest equals Principal times Rate times Time" expresses the well-known Simple Interest Formula, $I = PRT$. Solve for the time, T.

Solution:
Grouping, $I = (PR)T$ so PR becomes a coefficient of T. Dividing by PR gives $T = I/PR$.

Mathematics is often called "the language of science" or "the universal language". To study phenomena or situations of interest, mathematical expressions and equations are used to create **mathematical models**. Extracting information from the mathematical model provides solutions and insights. Mathematical modeling ideas appear throughout the text. These suggestions may aid your modeling skills.

To Solve Word Problems

1. **Read problems carefully.**
2. **Identify the quantity of interest (and possibly useful formulas).**
3. **A diagram may be helpful.**
4. **Assign symbols to variables and other unknown quantities.**
5. **Use symbols as variables and unknowns to translate words into an equation(s).**
6. **Solve for the quantity of interest.**
7. **Check your solution and whether you have answered the proper question.**

Example 1.1.4 Investment

Ms. Brown invests \$5000 at 6% annual interest. Model her resulting capital for one year.

Solution:
Here the principal (original investment) is \$5000. The interest rate is 0.06 (expressed as a decimal) and the time is 1 year.
Using the simple interest formula, $I = PRT$, Ms. Brown's interest is

$$I = (\$5000)(0.06)(1) = \$300.$$

After one year a model for her capital is $P + PRT = \$5000 + \$300 = \$5300$.

Example 1.1.5 Gasoline Prices

Recently East Coast regular grade gasoline was priced about \$3.50 per gallon. West Coast prices were about \$0.50/gallon higher.

a) What was the average regular grade gasoline price on the East Coast for 10 gallons?
b) What was the average regular grade gasoline price on the West Coast for 15 gallons?

Solution:

a) On average, a model for the East Coast cost of ten gallons was (10)(3.50) = \$35.00.
b) On average, a model for the West Coast of fifteen gallons was (15)(\$4.00) = \$60.00.

◆ Consumption as a function of disposable income can be expressed by the linear relation $C = mx + b$, where C is consumption (in \$); x, disposable income (in \$); m, marginal propensity to consume and b, a scaling constant. This consumption model arose in Keynesian economic studies popular during The Great Depression of the 1930s.

EXERCISES 1.1

In Exercises 1–6 identify equations as an identity, a conditional equation, or a contradiction.

1. $3x + 1 = 4x - 5$
2. $2(x + 1) = x + x + 2$
3. $5(x + 1) + 2(x - 1) = 7x + 6$
4. $4x + 3(x + 2) = x + 6$
5. $4(x + 3) = 2(2x + 5)$
6. $3x + 7 = 2x + 4$

In Exercises 7–27 solve the equations.

7. $5x - 3 = 17$
8. $3x + 2 = 2x + 7$
9. $2x = 4x - 10$
10. $x/3 = 10$
11. $4x - 5 = 6x - 7$
12. $5x + (1/3) = 7$
13. $0.6x = 30$
14. $(3x/5) - 1 = 2 - (1/5)(x - 5)$
15. $2/3 = (4/5)x - (1/3)$
16. $4(x - 3) = 2(x - 1)$
17. $5(x - 4) = 2x + 3(x - 7)$
18. $3x + 5(x - 2) = 2(x + 7)$
19. $3s - 4 = 2s + 6$
20. $5(z - 3) + 3(z + 1) = 12$
21. $7t + 2 = 4t + 11$
22. $(1/3)x + (1/2)x = 5$
23. $4(x + 1) + 2(x - 3) = 7(x - 1)$
24. $1/3 = (3/5)x - (1/2)$
25. $\dfrac{x + 8}{2x - 5} = 2$
26. $\dfrac{3x - 1}{7} = x - 3$
27. $8 - \{4[\mathrm{x} - (3\mathrm{x} - 4) - \mathrm{x}] + 4\}$
$= 3(\mathrm{x} + 2)$

In Exercises 28–35 solve for the indicated variable.

28. Solve: $5x - 2y + 18 = 0$ for y.
29. Solve: $6x - 3y = 9$ for x.
30. Solve: $y = mx + b$ for x.
31. Solve: $3x + 5y = 15$ for y.
32. Solve: $A = P + PRT$ for P.
33. Solve: $V = LWH$ for W.
34. Solve: $C = 2\pi r$ for r.
35. Solve: $Z = \dfrac{x - \mu}{\sigma}$ for x.

Exercises 36–45 feature mathematical models.

36. The sum of three consecutive positive integers is 81. Determine the largest integer.
37. Sally purchased a used car for \$1300 and paid \$300 down. If she plans to pay the balance in five equal monthly installments, what is the monthly payment?
38. A suit, marked down 20%, sold for \$120. What was the original price?
39. If the marginal propensity to consume is $m = 0.75$ and consumption, C, is \$11 when disposable income is \$2, develop the consumption function.
40. A new addition to a fire station costs \$100,000. The annual maintenance cost increases by \$2500 with each fire engine housed. If \$115,000 has been allocated for the addition and maintenance next year, how many additional fire engines can be housed?

41. Lightning is seen before thunder is heard as the speed of light is much greater than the speed of sound. The flash's distance from an observer can be calculated from the time between the flash and the sound of thunder.
    The distance, d (in miles), from the storm can be modeled as $d = 4.5t$ where time, t, is in seconds.
    a) If thunder is heard two seconds after lightning is seen, how far is the storm?
    b) If a storm is 18 miles distant, how long before thunder is heard?
42. A worker has forty hours to produce two types of items, A and B. Each unit of A takes three hours to produce and each item of B takes two hours. The worker made eight items of B and with the remaining time produced items of A. How many of item A were produced?
43. An employee's Social Security Payroll Tax was 6.2% for the first \$87,000 of earnings and was matched by the employer. Develop a linear model for an employee's portion of the Social Security Tax.
44. An employee works 37.5 hours at a \$10 hourly wage. If Federal tax deductions are 6.2% for Social Security, 1.45% for Medicare Part A, and 15% for Federal taxes, what is the take-home pay?
45. The body surface area (BSA) and weight (Wt) in infants and children weighing between 3 kg and 30 kg has been modeled by the linear relationship
    $\text{BSA} = 1321 + 0.3433\text{Wt}$ (where BSA is in square centimeters and weight in grams)
    a) Determine the BSA for a child weighing 20 kg.
    b) A child's BSA is 10,320 cm^2. Estimate its weight in kilograms.

    Current, J.D.,"A Linear Equation for Estimating the Body Surface Area in Infants and Children.", The Internet Journal of Anesthesiology 1998:Vol2N2.

1.2 LINEAR EQUATIONS AND THEIR GRAPHS

Mathematical models express features of interest. In the managerial, social, and natural sciences and engineering, linear equations often relate quantities of interest. Therefore, a thorough understanding of linear equations is important.

The standard form of a linear equation is $ax + by = c$ where a, b, and c are real valued constants. It is characterized by the first power of the exponents.

Standard Form of a Linear Equation

$$ax + by = c$$

a, b, c are real numbered constants; a and b, not both zero