# Automatic Control Systems

Third Edition

BENJAMIN C. KUO

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# Preface

The first edition of this book, published in 1962, was characterized by having chapters on sampled-data and nonlinear control systems. The treatment of the analysis and design of control systems was all classical.

The two major changes in the second edition, published in 1967, were the inclusion of the state variable technique and the integration of the discrete-data systems with the continuous data system. The chapter on nonlinear systems was eliminated in the second edition to the disappointment of some users of that text. At the time of the revision the author felt that a comprehensive treatment on the subject of nonlinear systems could not be made effectively with the available space.

The third edition is still written as an introductory text for a senior course on control systems. Although a great deal has happened in the area of modern control theory in the past ten years, preparing suitable material for a modern course on introductory control systems remains a difficult task. The problem is a complicated one because it is difficult to teach the topics concerned with new developments in modern control theory at the undergraduate level. The unique situation in control systems has been that many of the practical problems are still being solved in the industry by the classical methods. While some of the techniques in modern control theory are much more powerful and can solve more complex problems, there are often more restrictions when it comes to practical applications of the solutions. However, it should be recognized that a modern control engineer should have an understanding of the classical as well as the modern control methods. The latter will enhance and broaden one's perspective in solving a practical problem. It is the author's opinion that one should strike a balance in the teaching of control systems theory at the beginning and intermediate levels. Therefore in this current edition equal emphasis is placed on the classical methods and the modern control theory.

A number of introductory books with titles involving modern control theory have been published in recent years. Some authors have attempted to unify and integrate the classical control with the modern control, but according to the critics and reviews, most have failed. Although such a goal is highly desirable, if only from the standpoint of presentation, there does not seem to be a good solution. It is possible that the objective may not be achieved until new theories and new techniques are developed for this purpose. The fact remains that control systems, in some way, may be regarded as a science of learning how to solve one problem—control, in many different ways. These different ways of solution may be compared and weighed against each other, but it may not be possible to unify all the approaches. The approach used in this text is to present the classical method and the modern approach independently, and whenever possible, the two approaches are considered as alternatives, and the advantages and disadvantages of each are weighed. Many illustrative examples are carried out by both methods.

Many existing text books on control systems have been criticized for not including adequate practical problems. One reason for this is, perhaps, that many text book writers are theorists, who lack the practical background and experience necessary to provide real-life examples. Another reason is that the difficulty in the control systems area is compounded by the fact that most real-life problems are highly complex, and are rarely suitable as illustrative examples at the introductory level. Usually, much of the realism is lost by simplifying the problem to fit the nice theorems and design techniques developed in the text material. Nevertheless, the majority of the students taking a control system course at the senior level do not pursue a graduate career, and they must put their knowledge to immediate use in their new employment. It is extremely important for these students, as well as those who will continue, to gain an actual feel of what a real control system is like. Therefore, the author has introduced a number of practical examples in various fields in this text. The homework problems also reflect the attempt of this text to provide more real-life problems.

The following features of this new edition are emphasized by comparison with the first two editions:

1. Equal emphasis on classical and modern control theory.
2. Inclusion of sampled-data and nonlinear systems.
3. Practical system examples and homework problems.

The material assembled in this book is an outgrowth of a senior-level control system course taught by the author at the University of Illinois at Urbana-Champaign for many years. Moreover, this book is written in a style adaptable for self-study and reference.

Chapter 1 presents the basic concept of control systems. The definition of feedback and its effects are covered. Chapter 2 presents mathematical foundation and preliminaries. The subjects included are Laplace transform, $z$-transform, matrix algebra, and the applications of the transform methods. Transfer function and signal flow graphs are discussed in Chapter 3. Chapter 4 introduces the state variable approach to dynamical systems. The concepts and definitions of controllability and observability are introduced at the early stage. These subjects are later being used for the analysis and design of linear control systems. Chapter 5 discusses the mathematical modeling of physical systems. Here, the emphasis is on electromechanical systems. Typical transducers and control systems used in practice are illustrated. The treatment cannot be exhaustive as there are numerous types of devices and control systems. Chapter 6 gives the time response considerations of control systems. Both the classical and the modern approach are used. Some simple design considerations in the time domain are pointed out. Chapters 7, 8, and 9 deal with topics on stability, root locus, and frequency response of control systems.

In Chapter 10, the design of control systems is discussed, and the approach is basically classical. Chapter 11 contains some of the optimal control subjects which, in the author's opinion, can be taught at the undergraduate level if time permits. The text does contain more material than can be covered in one semester.

One of the difficulties in preparing this book was the weighing of what subjects to cover. To keep the book to a reasonable length, some subjects, which were in the original draft, had to be left out of the final manuscript. These included the treatment of signal flow graphs and time-domain analysis, of discrete-data systems, the second method of Liapunov's stability method, describing function analysis, state plane analysis, and a few selected topics on implementing optimal control. The author feels that the inclusion of these subjects would add materially to the spirit of the text, but at the cost of a higher price.

The author wishes to express his sincere appreciation to Dean W. L. Everitt (emeritus), Professors E. C. Jordan, O. L. Gaddy, and E. W. Ernst, of the University of Illinois, for their encouragement and interest in the project. The author is grateful to Dr. Andrew Sage of the University of Virginia and Dr. G. Singh of the University of Illinois for their valuable suggestions. Special thanks also goes to Mrs. Jane Carlton who typed a good portion of the manuscript and gave her invaluable assistance in proofreading.

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# 1

# Introduction

## 1.1 Control Systems

In recent years, automatic control systems have assumed an increasingly important role in the development and advancement of modern civilization and technology. Domestically, automatic controls in heating and air conditioning systems regulate the temperature and the humidity of modern homes for comfortable living. Industrially, automatic control systems are found in numerous applications, such as quality control of manufactured products, automation, machine tool control, modern space technology and weapon systems, computer systems, transportation systems, and robotics. Even such problems as inventory control, social and economic systems control, and environmental and hydrological systems control may be approached from the theory of automatic control.

The basic control system concept may be described by the simple block diagram shown in Fig. 1-1. The objective of the system is to control the variable $c$ in a prescribed manner by the actuating signal $e$ through the elements of the control system.

In more common terms, the controlled variable is the *output* of the system, and the actuating signal is the *input*. As a simple example, in the steering control of an automobile, the direction of the two front wheels may be regarded as the controlled variable $c$, the output. The position of the steering wheel is the input, the actuating signal $e$. The controlled process or system in this case is composed of the steering mechanisms, including the dynamics of the entire automobile. However, if the objective is to control the speed of the automobile, then the amount of pressure exerted on the accelerator is the actuating signal, with the speed regarded as the controlled variable.


**Fig. 1-1.** Basic control system.

There are many situations where several variables are to be controlled simultaneously by a number of inputs. Such systems are referred to as *multivariable systems*.

***Open-Loop Control Systems (Nonfeedback Systems)***

The word *automatic* implies that there is a certain amount of sophistication in the control system. By automatic, it generally means that the system is usually capable of adapting to a variety of operating conditions and is able to respond to a class of inputs satisfactorily. However, not any type of control system has the automatic feature. Usually, the automatic feature is achieved by feeding the output variable back and comparing it with the command signal. When a system does not have the feedback structure, it is called an *open-loop system*, which is the simplest and most economical type of control system. Unfortunately, open-loop control systems lack accuracy and versatility and can be used in none but the simplest types of applications.

Consider, for example, control of the furnace for home heating. Let us assume that the furnace is equipped only with a timing device, which controls the on and off periods of the furnace. To regulate the temperature to the proper level, the human operator must estimate the amount of time required for the furnace to stay on and then set the timer accordingly. When the preset time is up, the furnace is turned off. However, it is quite likely that the house temperature is either above or below the desired value, owing to inaccuracy in the estimate. Without further deliberation, it is quite apparent that this type of control is inaccurate and unreliable. One reason for the inaccuracy lies in the fact that one may not know the exact characteristics of the furnace. The other factor is that one has no control over the outdoor temperature, which has a definite bearing on the indoor temperature. This also points to an important disadvantage of the performance of an open-loop control system, in that the system is not capable of adapting to variations in environmental conditions or to external disturbances. In the case of the furnace control, perhaps an experienced person can provide control for a certain desired temperature in the house; but if the doors or windows are opened or closed intermittently during the operating period, the final temperature inside the house will not be accurately regulated by the open-loop control.

An electric washing machine is another typical example of an open-loop system, because the amount of wash time is entirely determined by the judgment and estimation of the human operator. A true automatic electric washing machine should have the means of checking the cleanliness of the clothes continuously and turn itself off when the desired degree of cleanliness is reached.

Although open-loop control systems are of limited use, they form the basic elements of the closed-loop control systems. In general, the elements of an open-loop control system are represented by the block diagram of Fig. 1-2. An input signal or command $r$ is applied to the controller, whose output acts as the actuating signal $e$; the actuating signal then actuates the controlled process and hopefully will drive the controlled variable $c$ to the desired value.

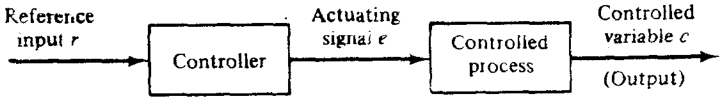



Fig. 1-2. Block diagram of an open-loop control system.

### *Closed-Loop Control Systems (Feedback Control Systems)*

What is missing in the open-loop control system for more accurate and more adaptable control is a link or feedback from the output to the input of the system. In order to obtain more accurate control, the controlled signal $c(t)$ must be fed back and compared with the reference input, and an actuating signal proportional to the difference of the output and the input must be sent through the system to correct the error. A system with one or more feedback paths like that just described is called a *closed-loop system*. Human beings are probably the most complex and sophisticated feedback control system in existence. A human being may be considered to be a control system with many inputs and outputs, capable of carrying out highly complex operations.

To illustrate the human being as a feedback control system, let us consider that the objective is to reach for an object on a desk. As one is reaching for the object, the brain sends out a signal to the arm to perform the task. The eyes serve as a sensing device which feeds back continuously the position of the hand. The distance between the hand and the object is the error, which is eventually brought to zero as the hand reaches the object. This is a typical example of closed-loop control. However, if one is told to reach for the object and then is blindfolded, one can only reach toward the object by estimating its exact position. It is quite possible that the object may be missed by a wide margin. With the eyes blindfolded, the feedback path is broken, and the human is operating as an open-loop system. The example of the reaching of an object by a human being is described by the block diagram shown in Fig. 1-3.

As another illustrative example of a closed-loop control system, Fig. 1-4

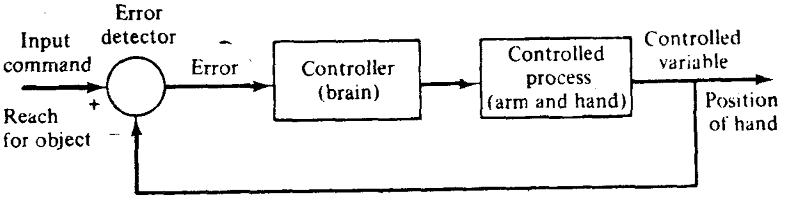



Fig. 1-3. Block diagram of a human being as a closed-loop control system.

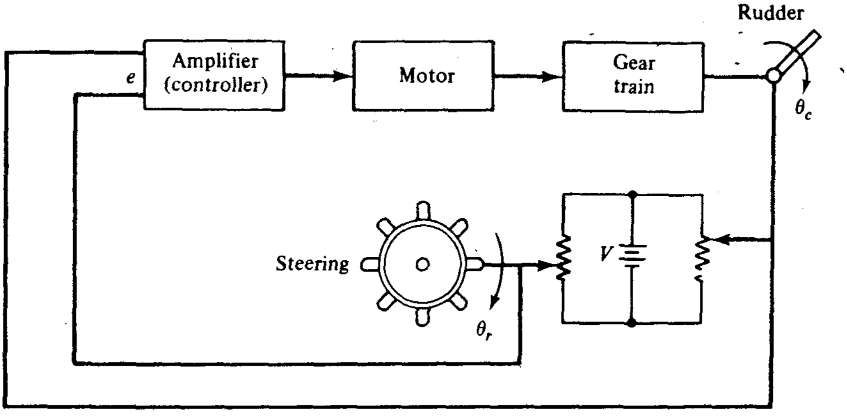



**Fig. 1-4.** Rudder control system.

shows the block diagram of the rudder control system of a ship. In this case the objective of control is the position of the rudder, and the reference input is applied through the steering wheel. The error between the relative positions of the steering wheel and the rudder is the signal, which actuates the controller and the motor. When the rudder is finally aligned with the desired reference direction, the output of the error sensor is zero. Let us assume that the steering wheel position is given a sudden rotation of $R$ units, as shown by the time signal in Fig. 1-5(a). The position of the rudder as a function of time, depending upon the characteristics of the system, may typically be one of the responses shown in Fig. 1-5(b). Because all physical systems have electrical and mechanical inertia, the position of the rudder cannot respond instantaneously to a step input, but will, rather, move gradually toward the final desired position. Often, the response will oscillate about the final position before settling. It is apparent that for the rudder control it is desirable to have a nonoscillatory response.

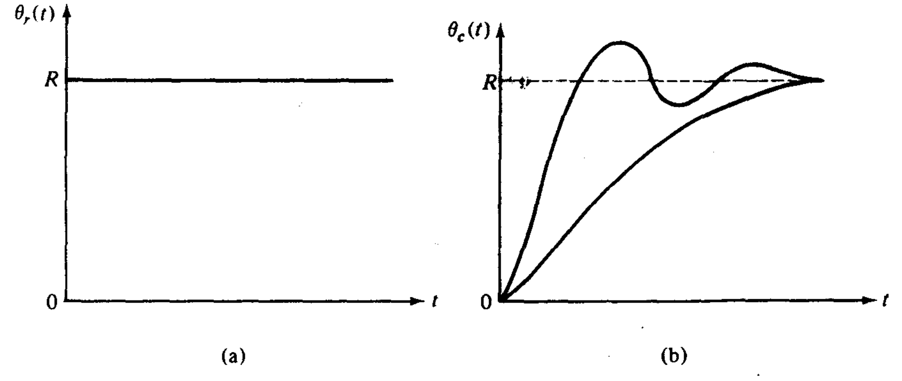



**Fig. 1-5.** (a) Step displacement input of rudder control system. (b) Typical output responses.

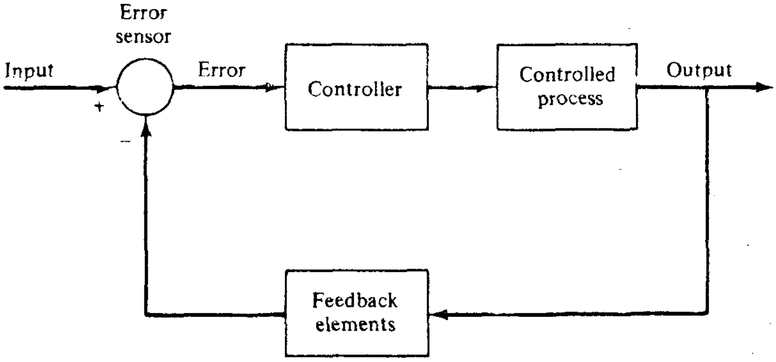



**Fig. 1-6.** Basic elements of a feedback control system.

The basic elements and the block diagram of a closed-loop control system are shown in Fig. 1-6. In general, the configuration of a feedback control system may not be constrained to that of Fig. 1-6. In complex systems there may be a multitude of feedback loops and element blocks.

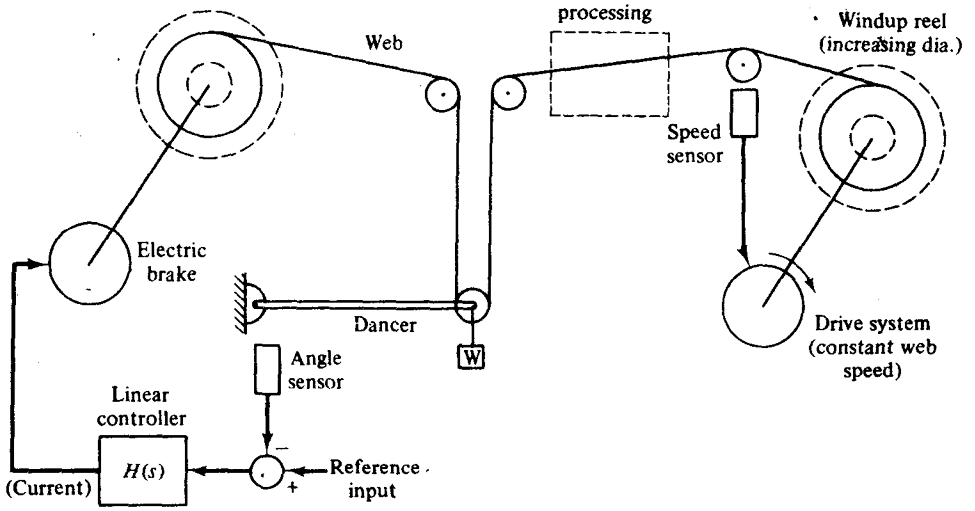
Figure 1-7(a) illustrates the elements of a tension control system of a windup process. The unwind reel may contain a roll of material such as paper or cable which is to be sent into a processing unit, such as a cutter or a printer, and then collects it by winding it onto another roll. The control system in this case is to maintain the tension of the material or web at a certain prescribed tension to avoid such problems as tearing, stretching, or creasing.

To regulate the tension, the web is formed into a half-loop by passing it down and around a weighted roller. The roller is attached to a pivot arm, which allows free up-and-down motion of the roller. The combination of the roller and the pivot arm is called the *dancer.*

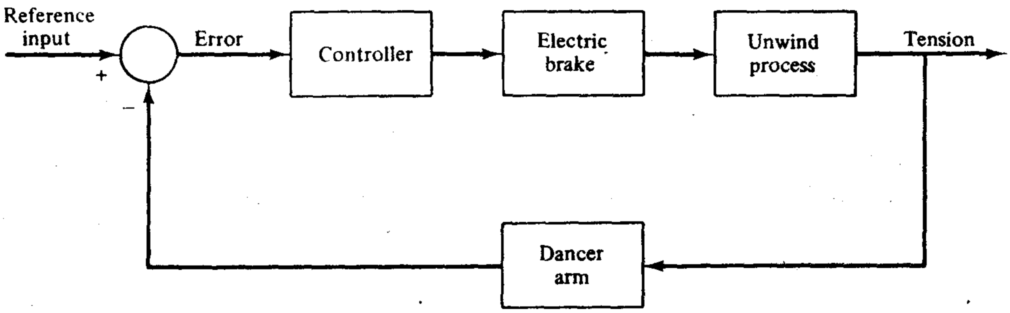
When the system is in operation, the web normally travels at a constant speed. The ideal position of the dancer is horizontal, producing a web tension equal to one-half of the total weight $W$ of the dancer roll. The electric brake on the unwind reel is to generate a restraining torque to keep the dancer in the horizontal position at all times.

During actual operation, because of external disturbances, uncertainties and irregularities of the web material, and the decrease of the effective diameter of the unwind reel, the dancer arm will not remain horizontal unless some scheme is employed to properly sense the dancer-arm position and control the restraining braking torque.

To obtain the correction of the dancing-arm-position error, an angular sensor is used to measure the angular deviation, and a signal in proportion to the error is used to control the braking torque through a controller. Figure 1-7(b) shows a block diagram that illustrates the interconnections between the elements of the system.




(a)




(b)

**Fig. 1-7.** (a) Tension control system. (b) Block diagram depicting the basic elements and interconnections of a tension control system.

## 1.2 What Is Feedback and What Are Its Effects?

The concept of feedback plays an important role in control systems. We demonstrated in Section 1.1 that feedback is a major requirement of a closed-loop control system. Without feedback, a control system would not be able to achieve the accuracy and reliability that are required in most practical applications. However, from a more rigorous standpoint, the definition and the significance of feedback are much deeper and more difficult to demonstrate than the few examples given in Section 1.1. In reality, the reasons for using feedback carry far more meaning than the simple one of comparing the input with the output in order to reduce the error. The reduction of system error is merely one of the many effects that feedback may bring upon a system. We shall now show that feedback also has effects on such system performance characteristics as stability bandwidth, overall gain, impedance, and sensitivity.

To understand the effects of feedback on a control system, it is essential that we examine this phenomenon with a broad mind. When feedback is deliberately introduced for the purpose of control, its existence is easily identified. However, there are numerous situations wherein a physical system that we normally recognize as an inherently nonfeedback system may turn out to have feedback when it is observed in a certain manner. In general we can state that whenever a closed sequence of *cause-and-effect relation* exists among the variables of a system, feedback is said to exist. This viewpoint will inevitably admit feedback in a large number of systems that ordinarily would be identified as nonfeedback systems. However, with the availability of the feedback and control system theory, this general definition of feedback enables numerous systems, with or without physical feedback, to be studied in a systematic way once the existence of feedback in the above-mentioned sense is established.

We shall now investigate the effects of feedback on the various aspects of system performance. Without the necessary background and mathematical foundation of linear system theory, at this point we can only rely on simple static system notation for our discussion. Let us consider the simple feedback system configuration shown in Fig. 1-8, where $r$ is the input signal, $c$ the output signal, $e$ the error, and $b$ the feedback signal. The parameters $G$ and $H$ may be considered as constant gains. By simple algebraic manipulations it is simple to show that the input–output relation of the system is

$$M = \frac{c}{r} = \frac{G}{1 + GH} \tag{1-1}$$

Using this basic relationship of the feedback system structure, we can uncover some of the significant effects of feedback.

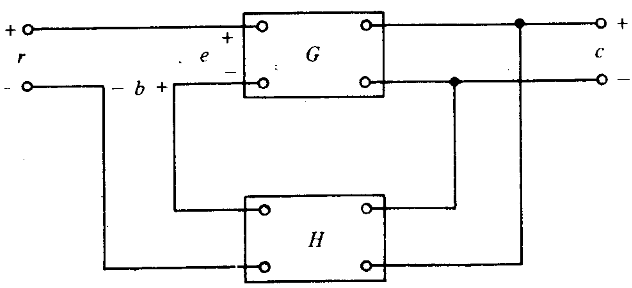



**Fig. 1-8.** Feedback system.

***Effect of Feedback on Overall Gain***

As seen from Eq. (1-1), feedback affects the gain $G$ of a nonfeedback system by a factor of $1 + GH$. The reference of the feedback in the system of Fig. 1-8 is negative, since a minus sign is assigned to the feedback signal. The quantity $GH$ may itself include a minus sign, so the general effect of feedback is that it may increase or decrease the gain. In a practical control system, $G$ and $H$ are functions of frequency, so the magnitude of $1 + GH$ may be greater than 1 in one frequency range but less than 1 in another. Therefore, feedback could increase the gain of the system in one frequency range but decrease it in another.

### *Effect of Feedback on Stability*

Stability is a notion that describes whether the system will be able to follow the input command. In a nonrigorous manner, a system is said to be unstable if its output is out of control or increases without bound.

To investigate the effect of feedback on stability, we can again refer to the expression in Eq. (1-1). If $GH = -1$, the output of the system is infinite for any finite input. Therefore, we may state that feedback can cause a system that is originally stable to become unstable. Certainly, feedback is a two-edged sword; when it is improperly used, it can be harmful. It should be pointed out, however, that we are only dealing with the static case here, and, in general $GH = -1$ is not the only condition for instability.

It can be demonstrated that one of the advantages of incorporating feedback is that it can stabilize an unstable system. Let us assume that the feedback system in Fig. 1-8 is unstable because $GH = -1$. If we introduce another feedback loop through a negative feedback of $F$, as shown in Fig. 1-9, the input–output relation of the overall system is

$$\frac{c}{r} = \frac{G}{1 + GH + GF} \tag{1-2}$$

It is apparent that although the properties of $G$ and $H$ are such that the inner-loop feedback system is unstable, because $GH = -1$, the overall system can be stable by properly selecting the outer-loop feedback gain $F$.

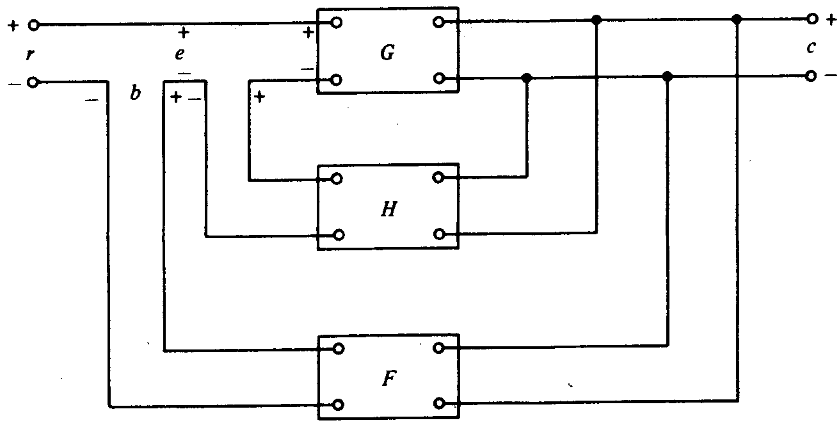



**Fig. 1-9.** Feedback system with two feedback loops.

### *Effect of Feedback on Sensitivity*

Sensitivity considerations often play an important role in the design of control systems. Since all physical elements have properties that change with environment and age, we cannot always consider the parameters of a control system to be completely stationary over the entire operating life of the system. For instance, the winding resistance of an electric motor changes as the temperature of the motor rises during operation. In general, a good control system should be very insensitive to these parameter variations while still able to follow the command responsively. We shall investigate what effect feedback has on the sensitivity to parameter variations.

Referring to the system in Fig. 1-8, we consider $G$ as a parameter that may vary. The sensitivity of the gain of the overall system $M$ to the variation in $G$ is defined as

$$S_G^M = \frac{\partial M/M}{\partial G/G} \tag{1-3}$$

where $\partial M$ denotes the incremental change in $M$ due to the incremental change in $G$; $\partial M/M$ and $\partial G/G$ denote the percentage change in $M$ and $G$, respectively. The expression of the sensitivity function $S_G^M$ can be derived by using Eq. (1-1). We have

$$S_G^M = \frac{\partial M}{\partial G}\frac{G}{M} = \frac{1}{1+GH} \tag{1-4}$$

This relation shows that the sensitivity function can be made arbitrarily small by increasing $GH$, provided that the system remains stable. It is apparent that in an open-loop system the gain of the system will respond in a one-to-one fashion to the variation in $G$.

In general, the sensitivity of the system gain of a feedback system to parameter variations depends on where the parameter is located. The reader may derive the sensitivity of the system in Fig. 1-8 due to the variation of $H$.

#### *Effect of Feedback on External Disturbance or Noise*

All physical control systems are subject to some types of extraneous signals or noise during operation. Examples of these signals are thermal noise voltage in electronic amplifiers and brush or commutator noise in electric motors.

The effect of feedback on noise depends greatly on where the noise is introduced into the system; no general conclusions can be made. However, in many situations, feedback can reduce the effect of noise on system performance.

Let us refer to the system shown in Fig. 1-10, in which $r$ denotes the command signal and $n$ is the noise signal. In the absence of feedback, $H = 0$, the output $c$ is

$$c = G_1G_2e + G_2n \tag{1-5}$$

where $e = r$. The signal-to-noise ratio of the output is defined as

$$\frac{\text{output due to signal}}{\text{output due to noise}} = \frac{G_1G_2e}{G_2n} = G_1\frac{e}{n} \tag{1-6}$$

To increase the signal-to-noise ratio, evidently we should either increase the magnitude of $G_1$ or $e$ relative to $n$. Varying the magnitude of $G_2$ would have no effect whatsoever on the ratio.

With the presence of feedback, the system output due to $r$ and $n$ acting

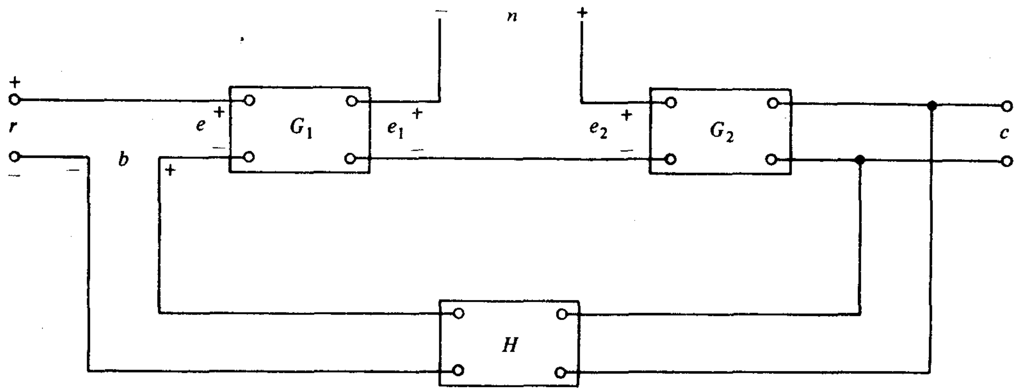



**Fig. 1-10.** Feedback system with a noise signal.

simultaneously is

$$c = \frac{G_1 G_2}{1 + G_1 G_2 H} r + \frac{G_2}{1 + G_1 G_2 H} n \tag{1-7}$$

Simply comparing Eq. (1-7) with Eq. (1-5) shows that the noise component in the output of Eq. (1-7) is reduced by the factor $1 + G_1 G_2 H$, but the signal component is also reduced by the same amount. The signal-to-noise ratio is

$$\frac{\text{output due to signal}}{\text{output due to noise}} = \frac{G_1 G_2 r/(1 + G_1 G_2 H)}{G_2 n/(1 + G_1 G_2 H)} = G_1 \frac{r}{n} \tag{1-8}$$

and is the same as that without feedback. In this case feedback is shown to have no direct effect on the output signal-to-noise ratio of the system in Fig. 1-10. However, the application of feedback suggests a possibility of improving the signal-to-noise ratio under certain conditions. Let us assume that in the system of Fig. 1-10, if the magnitude of $G_1$ is increased to $G'_1$ and that of the input $r$ to $r'$, with all other parameters unchanged, the output due to the input signal acting alone is at the same level as that when feedback is absent. In other words, we let

$$c|_{n=0} = \frac{G'_1 G_2 r'}{1 + G'_1 G_2 H} = G_1 G_2 r \tag{1-9}$$

With the increased $G_1$, $G'_1$, the output due to noise acting alone becomes

$$c|_{r=0} = \frac{G_2 n}{1 + G'_1 G_2 H} \tag{1-10}$$

which is smaller than the output due to $n$ when $G_1$ is not increased. The signal-to-noise ratio is now

$$\frac{G_1 G_2 r}{G_2 n/(1 + G'_1 G_2 H)} = \frac{G_1 r}{n}(1 + G'_1 G_2 H) \tag{1-11}$$

which is greater than that of the system without feedback by a factor of $(1 + G'_1 G_2 H)$.

In general, feedback also has effects on such performance characteristics as bandwidth, impedance, transient response, and frequency response. These effects will become known as one progresses into the ensuing material of this text.

## 1.3 Types of Feedback Control Systems

Feedback control systems may be classified in a number of ways, depending upon the purpose of the classification. For instance, according to the method of analysis and design, feedback control systems are classified as linear and nonlinear, time varying or time invariant. According to the types of signal found in the system, reference is often made to continuous-data and discrete-data systems, or modulated and unmodulated systems. Also, with reference to the type of system components, we often come across descriptions such as electromechanical control systems, hydraulic control systems, pneumatic systems, and biological control systems. Control systems are often classified according to the main purpose of the system. A positional control system and a velocity control system control the output variables according to the way the names imply. In general, there are many other ways of identifying control systems according to some special features of the system. It is important that some of these more common ways of classifying control systems are known so that proper perspective is gained before embarking on the analysis and design of these systems.

### *Linear Versus Nonlinear Control Systems*

This classification is made according to the methods of analysis and design. Strictly speaking, linear systems do not exist in practice, since all physical systems are nonlinear to some extent. Linear feedback control systems are idealized models that are fabricated by the analyst purely for the simplicity of analysis and design. When the magnitudes of the signals in a control system are limited to a range in which system components exhibit linear characteristics (i.e., the principle of superposition applies), the system is essentially linear. But when the magnitudes of the signals are extended outside the range of the linear operation, depending upon the severity of the nonlinearity, the system should no longer be considered linear. For instance, amplifiers used in control systems often exhibit saturation effect when their input signals become large; the magnetic field of a motor usually has saturation properties. Other common nonlinear effects found in control systems are the backlash or dead play between coupled gear members, nonlinear characteristics in springs, nonlinear frictional force or torque between moving members, and so on. Quite often, nonlinear characteristics are intentionally introduced in a control system to improve its performance or provide more effective control. For instance, to achieve minimum-time control, an on–off (bang-bang or relay) type of controller is used. This type of control is found in many missile or spacecraft control systems. For instance, in the attitude control of missiles and spacecraft, jets are mounted on the sides of the vehicle to provide reaction torque for attitude control. These jets are often controlled in a full-on or full-off fashion, so a fixed amount of air is applied from a given jet for a certain time duration to control the attitude of the space vehicle.

For linear systems there exists a wealth of analytical and graphical techniques for design and analysis purposes. However, nonlinear systems are very difficult to treat mathematically, and there are no general methods that may be used to solve a wide class of nonlinear systems.

### *Time-Invariant Versus Time-Varying Systems*

When the parameters of a control system are stationary with respect to time during the operation of the system, we have a time-invariant system. Most physical systems contain elements that drift or vary with time to some extent. If the variation of parameter is significant during the period of operation, the system is termed a time-varying system. For instance, the radius of the unwind reel of the tension control system in Fig. 1-7 decreases with time as the material is being transferred to the windup reel. Although a time-varying system without nonlinearity is still a linear system, its analysis is usually much more complex than that of the linear time-invariant systems.

### *Continuous-Data Control Systems*

A continuous-data system is one in which the signals at various parts of the system are all functions of the continuous time variable $t$. Among all continuous-data control systems, the signals may be further classified as ac or dc. Unlike the general definitions of ac and dc signals used in electrical engineering, ac and dc control systems carry special significances. When one refers to an ac control system it usually means that the signals in the system are modulated by some kind of modulation scheme. On the other hand, when a dc control system is referred to, it does not mean that all the signals in the system are of the direct-current type; then there would be no control movement. A dc control system simply implies that the signals are unmodulated, but they are still ac by common definition. The schematic diagram of a closed-loop dc control system is shown in Fig. 1-11. Typical waveforms of the system in response to a step

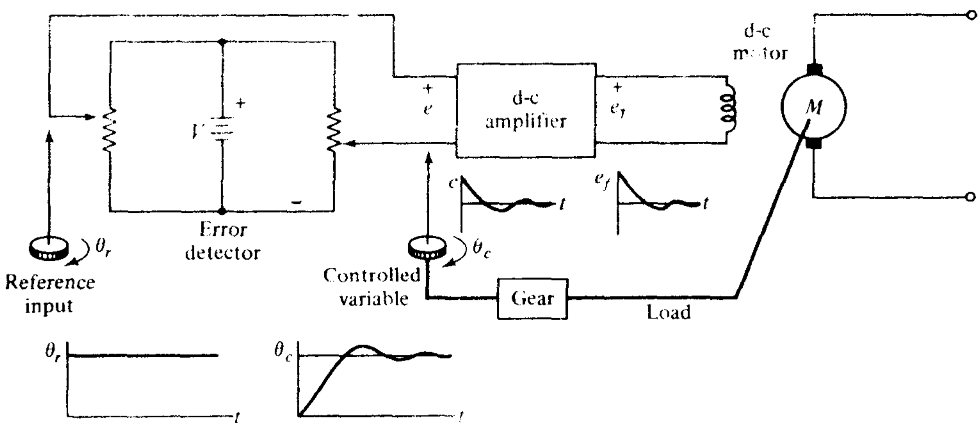



**Fig. 1-11.** Schematic diagram of a typical dc closed-loop control system.