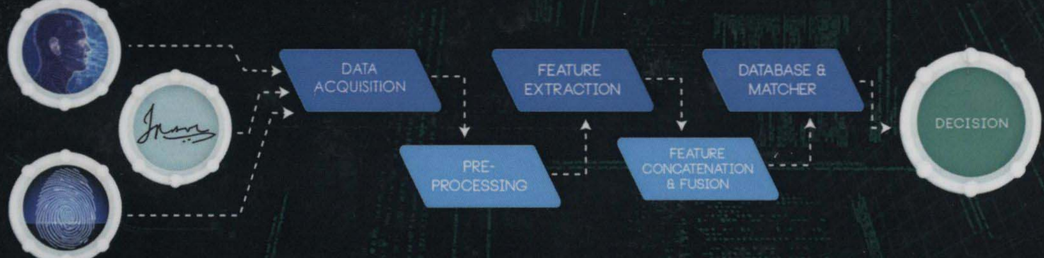
DATA FUSION MATHEMATICS

THEORY AND PRACTICE

Jitendra R. Raol



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Preface

Data fusion (DF), also known more formally as multi-sensory-source information fusion or multi-sensor data fusion (MSDF) or simply as sensor data fusion (SDF) is the process of combining data/information logically (and now more recently, intelligently, due to the inroads made by the theory and practice of artificial intelligence [AI] in many areas) from a number of different data sources or sensors to provide a robust and complete description of an environment, process or an object of interest (that we are monitoring), such that the overall total information after the fusion (i.e. of the fusion system) is supposed to have increased significantly as measured by some information (-based) metric. Thus, DF is of a special significance and importance in many applications where a large amount of data or information must be combined and then fused intelligently. The sensor or data aggregation (DAG), which is not quite as the same as the DF, has a special meaning and place in wireless sensor networks (WSN). In a WSN there would be a very large amount of data coming from various nodes/sensors/input channels, and one should aggregate certain data before sending these data to some other nodes/outputs/sink nodes and so on – this is DAG. Ultimately SDF is carried out to obtain enhanced information of appropriate and required quality and integrity so that the decisions that are made on the basis of this integrated/fused data/information can be highly trustworthy, and have more accuracy of prediction with reduced uncertainty about the state/status of the object/process/scenario of interest. The DF should be carried out logically and with the proper understanding of the data processing and data fusion methods. Hence, the current book presents the basic mathematics related to SDF. The DF mathematics provide credence to the data fusion process (DFP). This DFP is nothing but enhanced signal processing activity with support from other well known and proven soft technologies/disciplines: optimisation, control, estimation theory and soft computing paradigms. Image fusion is at the core level image processing, which itself is the next higher level of digital signal processing. Hence, in image fusion we deal with pixel-, and feature level fusion processes and methods. Decision making and decision fusion are at core level rooted into statistical decision theory and can use probabilistic and fuzzy logic-related arguments. All levels of DF find practical applications in (i) military systems, (ii) civilian surveillance and monitoring tasks, (iii) process control, (iv) information systems, (v) robotics, (vi) WSN, (vii) security systems and (viii) medical data (image) – fusion. Many key DF methods are particularly important towards development of autonomous systems in many of these applications. Thus, DF, also sophisticatedly known as multi-source, multi-sensor information fusion (MUSSIF), is rapidly emerging as an independent discipline to reckon with and finds further applications in (i) biomedical/bioengineering, (ii) industrial automation, (iii) aerospace engineering and (iv) environmental engineering processes and systems, in addition to the usual defence applications. The SDF offers benefits of (i) more spatial coverage of the object under observation due to (spread and distributed) geographical locations of the sensor systems, (ii) redundancy of sensors/measurements due to more number of sensors, (iii) robustness of the system's performance and fault tolerance, due to the former two aspects, (iv) increased accuracy of inferences (prediction accuracy is supposed to have increased) due to the fusion process and (v) overall assured performance of the sensors-integrated systems and fusion-system's robustness due to the all of the former aspects.

A really good study and successful application of the DF process involves a good grasp of several allied and supportive disciplines: (a) probabilistic and statistical methods, (b) signal/image processing, (c) numerical methods/algorithms, (d) sensor control and optimisation, (e) soft computing paradigms and (f) system identification, parameter estimation and filtering methods. Most important aspect of this study is to have a very good understanding of the basic mathematics related directly (or indirectly) to the DF processes and methods. There are several good books on SDF, however the treatment of mathematical aspects, that is, the DF mathematics, is highly limited or not even found explicitly in many such books. The treatment of the DF mathematics in the present volume is comprehensive and covers major DF concepts, and mathematical expressions, formulae and equations as well as their derivations (where appropriate) from the related disciplines which will help in a good understanding of SDF strategies and methods. The book follows and covers largely a mathematical approach to the extent possible. Thus, the main aim of the present volume is to fill the existing gap in the discipline of MSDF. However, the practically potential aspects and uses of these methods are also highlighted and treated, where appropriate, with a few illustrative examples via numerical simulations coded in MATLAB®. The users should have access to PC-based MATLAB software and other toolboxes such as signal processing, control systems, system identification, neural networks (NWs), fuzzy logic, genetic algorithm and image processing. Some SW for Chapters 3, 6, 8 and 10 would be made available from the data fusion mathematics (DFM)-book's website (by the publisher). Listings of some other DF-related software are provided in Appendix D. Also, the solution manual for the exercises presented in the book chapters will be available to instructors from the publisher.

In this book, we cover probability and statistical methods, fuzzy logic-based mathematics, decision theory, reliability concepts, component analyses, image algebra, tracking and filtering-related mathematics and methods, WSN/DF and soft computing paradigms (neuro-fuzzy-genetic algorithms). All these mathematical methods have a direct (sometimes indirect but closely related) bearing on the proper understanding of SDF concepts and methods. Thus, this book can be viewed as (partially) an archival and a reference volume for the DF community, as well as the primary text book for introducing the SDF subject to the undergraduate (or postgraduate) in one semester class of any discipline. The readers/users are advised to use their own discretion and judgment to apply various methods/approaches/algorithms/expressions/formulae/results discussed in the book to their practical problems, though enough care has been taken in the presentation of material.

The main users of this integrated DF mathematics will be engineers/scientists/teachers/researchers from electrical systems-, aero-, mechanical-, chemical-, civil-educational institutions, R&D laboratories and the aviation and transportation/automation/robotics industries.

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Introduction

SDF encompasses the theory, methods and tools used for creating a synergy of the information acquired from multiple sources, sensors, databases and direct information through humans – this synergy is much more than the concept of DAG. The resulting fused information of the object or process, and the decision or action is (found to be or should be), in some sense, better in terms of accuracy (in fact accuracy of prediction), and robustness, than would have been possible if any of these sources were used individually without such a synergy creation.

SDF is rapidly emerging as an independent discipline and is finding an ever-increasing role and applications in many systems/disciplines: biomedical, industrial automation, aerospace and environmental engineering processes and systems. The expected benefits of DF are: (i) more spatial coverage of the object under observation, (ii) redundancy of measurements, (iii) robustness of the system's performance, (iv) increased accuracy – in fact enhanced prediction capability – of inferences and (v) overall assured functioning and performance of the sensor-data integrated and task-oriented DF systems. The entire process of SDF involves a comprehensive study of several allied and supportive disciplines: (a) image-signal processing, (b) numerical methods and algorithms, (c) statistical/probabilistic methods, (d) sensor management, control and optimisation, (e) neural NWs, fuzzy systems and genetic algorithms, (f) system identification, parameter estimation, and Kalman filtering (and other modern filtering methods) and (g) data-structure and data base management.

The most important aspect of this synergistic study is a good understanding of the mathematics directly related to DF concepts and methods. In this book, the probability and statistical methods, fuzzy logic-based mathematics, decision theory, reliability, component analyses, image algebra, tracking and filtering-related mathematics and soft computing paradigms such as neuro-fuzzy logic-genetic algorithms (ANN-FL-GA) are presented. All these mathematical methods have a direct bearing on the proper understanding of MSDF concepts and methods, and fill the existing gap of the mathematics of DF for the fusion community at large.

There are several good books on various topics SDF [1–15], however the treatment of mathematical aspects, that is, the DF mathematics per se, is highly limited or not even found explicitly in many such books (except in References 3 and 13). The treatment of the DF mathematics in the current book is very comprehensive and covers major mathematical expressions, formulae and equations as well as their derivations (where appropriate) from the related disciplines and hence, this book can be viewed as an archival and reference volume, and in addition will also be very useful for practicing scientists and engineers. Certain other concepts of (the mathematics of) DF are briefly outlined in Appendix B. It is also emphasised here, that the treatment of mathematics in Chapters 2 to 10 is not based on theorem–proof–corollary style, because the mathematics mainly comes from several disciplines, and an attempt at unification would be a huge and a challenging task. The present approach itself is a starting point for such unification, however, there is a further scope for unification and this is left to future endeavours. A brief discussion of the chapters follows.

In Chapter 1, as a brief introduction to the topic, we discuss SDF concepts, DF models (briefly) and architectures. Several connected aspects and disciplines are briefly mentioned. More specific DF architectures in the context of decentralised systems are considered in Chapter 5.

In Chapter 2, we discuss important aspects and methods of probability and statistics. These probabilistic and information measures can be utilised in defining DF approaches, metrics and measures, and especially the weights/coefficients in the DF rule. Actually DF starts from these measures. Further, these metrics and norms are also very useful in evaluating performance of the designed and developed DF systems.

Chapter 3 discusses mainly fuzzy logic type 1 (FLT1), fuzzy implication functions and possibility theory. Fuzzy logic actually expands the scope of classical set theory and the theory of probability. Also, interval type 2 fuzzy logic (IT2FL) is discussed, which further expands the scope of type 1 fuzzy logic. We also, discuss fuzzy sets, fuzzy operators and fuzzy implication functions. The adaptive neuro-fuzzy inference system (ANFIS) is discussed. The use of fuzzy logic and ANFIS in DF is highlighted here, and in Chapters 8 and 10. Also, fusion of long wave IR and EOT images using type 1 and type 2 fuzzy (T1FL and IT2FL) logics is presented. Such applications of IT2FL for DF are perhaps novel.

In Chapter 4, we cover the mathematical treatment of many types of filtering algorithms, and target-tracking methods, and also, discuss kinematic DF methods. Single and multi-sensor tracking and fusion mathematics is presented. Gating and data association concepts and related filtering methods are described. Information filtering approaches are also presented. H-infinity filters are now being used for DF and hence these algorithms are also presented. The aspects of handling randomly missing measurements in the Kalman filter and the KF like-filtering algorithms are treated and some ramifications are derived and simulated results are presented. Two factorisation filtering algorithms for target-tracking-cum-data fusion are also briefly discussed and some practical results presented. In Chapter 5, we study decentralised DF and related filtering/estimation approaches for DF. Information filtering and Bayesian approaches are discussed. Decentralised DF architectures are very important for monitoring of large-scale structures (buildings), and health monitoring of aerospace systems, vehicles and large industrial automation plants. Various DF architectures and DF rules are given. The square root information filter for decentralised DF is considered and some numerical simulation results are presented.

Chapter 6 discusses the component analysis methods including wavelets and related methods, for example, discrete cosine transform. These concepts and methods are very useful for DF, especially for image fusion. The component analysis is also fundamentally used in many signal processing methods and the extension thereof to image fusion is a natural one. Also discussed is the curvelet method for image fusion. Also, multi-resolution singular value decomposition approach for image fusion is illustrated. A few approaches are evaluated with simulated image-data using codes written in MATLAB. Image processing and image fusion are now crucial technologies for complex modern day aerospace and other systems, for example, enhanced synthetic vision, their newer integrated versions, situation assessment requirements and field/medical robotics. Thus, understanding of image algebra and fusion mathematics is of paramount importance. These aspects are extensively dealt with in Chapter 7 by Dr. Mrs S. Sethu Selvi (professor, and head, Department of Electronics and Communications Engineering, MSRIT, Bangalore). Also, some examples of fusion in biometric systems are presented.

In Chapter 8, we present briefly decision theory and its use in the process of SDF, mainly the Bayesian approach. Decision fusion is treated as a higher level fusion method and is crucial in all applications of sensor/DF including the WSN. An approach of decision making/fusion using fuzzy logic type 1 for several aviation scenarios is presented using simulated data. Various decision fusion rules are also presented in this chapter. In Chapter 9, we discuss WSN and associated SDF and DAG approaches. These networks (NWs) have applications in structural health monitoring (including large aircraft wings, large spacecraft structures) and many surveillance systems. Distributed sensing, detection, estimation and DF are important technologies and approaches that will accrue definite benefits to the users of WSN-based utility systems, for example, security systems. Also, the signals generated by sensors of different types of modalities (principles) need to be sampled, filtered, compressed, transmitted, fused and stored. These WSNs are an important part of smart environments (in buildings, utilities, industrial places, homes, on board ships, and in the automation of transportation systems and security systems).

Finally, Chapter 10 discusses three soft computing paradigms which find increasing applications in multi-sensory DF approaches and applications: artificial neural networks (ANNs) for learning/adaptation, fuzzy logic for decision making and modelling vagueness (in data, and in rules), and genetic algorithms for global optimisation (of ANNs weights, etc.). These are very promising soft computing approaches with enormous scope for DF applications at all levels of SDF: kinematic, image and decision levels. Some hybrid methods centred on ANNs/FL/GA are also important to derive benefits from the merits and capabilities (such as learning, optimisation) of any two individual soft computing methods, and hence are briefly treated.

In Appendixes A, B, C, D, E and F we provide, respectively: (i) a few more algorithms and derivations of some filters (A), (ii) some other methods for DF and image fusion performance metrics (B), (iii) a brief note on an automatic

DF (C), (iv) some important information on commercial and other software for target tracking and sensor data/image fusion (D), (v) several definitions of sensor/DF (E) and (vi) some current research topics in the area of data/image fusion and target tracking (F).

Where appropriate, some numerical simulation examples using MATLAB are given (in Chapters 3 to 8 and 10). The end users of this integrated technology of SDF mathematics will be systems-aero-educational institutions, R&D laboratories, aerospace and other industries, the transportation/automation/robotics industries, all engineering disciplines and some branches of science, for example, the explanation of some evolutionary mechanism (of a biological species) using fuzzy logic!

Summary: Can Unified Data Fusion Mathematics Emerge?

Fundamentally the process of DF starts with combining some information (after some analysis of the original data) from individual sensors/sources using some formulae or rule, a simple rule being the average of the two 'informations', or a weighted average of these 'informations'. This information/s is either in the form of information factor or information matrix. Then we need to determine the optimal weights for this fusion rule. The Kalman filter gives the information on these weights automatically in the form of predicated covariance matrices coming from, say, two Kalman filters processing the data from two individual sensor channels. Also since an information matrix is the inverse of its covariance matrix, the information quantities are also used as weights in a DF rule. These covariance matrices are originally based on the probability definitions of uncertain phenomena/events. These events are affected by random noise processes. The covariance matrices are regarded as the statistics of the signals coming from several sensor channels. Thus, the statistics, and probability aspects of the sensor signals are connected measures, which are then used as weights for a DF rule. Since the Kalman filter can be basically derived from the Bayesian rule of probability, the latter itself then independently, becomes the basis of a DF rule. However, since vagueness (another kind of uncertainty) can be truthfully and usefully modelled by using fuzzy logic, DF rules can also be derived using the weights from fuzzy logic-based analysis. Hence, statistics, probability and fuzzy logic become naturally connected, especially because fuzzy logic generalises the concept of probability. In addition, fuzzy logic being a rule-based logic, it allows the incorporation of the human/expert's experience and intuition into this rule base. So, a designer of the SDF system can enhance the performance of the system using fuzzy logic-based analysis and control approaches. Interval type 2 fuzzy logic can further enhance the scope and performance of such a DF system. The component analysis (Fourier, discrete cosine transform, principal component analysis, wavelets, etc.) is currently very much in use for (sensor) signal processing, and system identification. Since image processing and image fusion are higher levels of signal processing aspects, the component analysis has a natural extension for image (-data) fusion. Image algebra is also an extension of signal algebra, and is useful for understanding image–data fusion approaches, and to undertake analysis of such fusion methods. Decision fusion is a higher level fusion process over kinematic DF and image fusion. Several concepts of the basic levels of DF are applicable to decision fusion also. Decision fusion involves determining statistics, obtaining the (state-) estimates from the sensor/image signal, and the use of some logic, and hence, from this point of view as well, it is a higher level of fusion activity. Also, decision fusion can be regarded as a symbol level fusion. Bayesian rule and fuzzy logic can play a very important role in decision fusion. WSN involve all the types of basic signal processing activities, and hence, many of the foregoing concepts are applicable to SDF and DAG in WSN. Many of the DF rules in decision fusion and WSN are based on probability and information measures. The neuro-fuzzy-GA paradigms are based on three distinct aspects of: (i) our own nature, (ii) our observation of natural phenomena and (iii) the evolutionary mechanism of nature, respectively. The ANNs are modelled on the basis of the biological neural networks, and ANNs have the ability to learn and adapt to the environment. Fuzzy logic models the vagueness and imperfections in the nature of our observations. Genetic algorithms (GA) are based on the evolutionary system of nature. These three soft computing paradigms are also collectively the ingredients of AI. We can then effectively build intelligent DF systems by utilising these three basic aspects of AI. ANNs use training algorithms that are rooted in some optimisation principles and techniques. GAs themselves use direct search methods for optimisation. ANFIS uses ANNs for learning the parameters of the fuzzy membership functions from the presented data. The learning algorithms can be based on classical optimisation methods or GA itself. Much of the basic mathematics of ANNs (training algorithms), fuzzy inference systems and GA is not very complicated and is rooted in basic function theory (FT), vector-space norms (VSN), classical logic chains (CLC/forward and backward) and optimisation criteria (OC). All these four aspects, FT, VSN, CLC and OC also form the basic mathematical ingredients of the DF mathematics. So, we see here that some unification and uniformity is beginning to emerge for SDF from the diverse concepts of mathematics related to statistical/probabilistic signal/image processing, filtering, component analysis, image algebra, decision making and neuro-FL–GA paradigms.

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