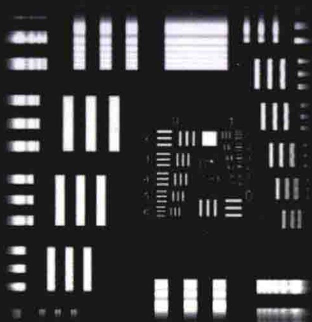
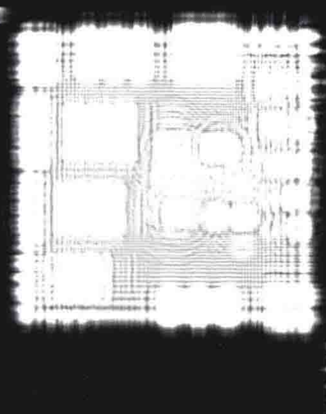
Introduction to Modern Digital Holography

with MATLAB

Ting-Chung Poon
and **Jung-Ping Liu**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INTRODUCTION TO MODERN DIGITAL HOLOGRAPHY

With MATLAB®

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INTRODUCTION TO MODERN DIGITAL HOLOGRAPHY
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Get up to speed with digital holography with this concise and straightforward introduction to modern techniques and conventions.

Building up from the basic principles of optics, this book describes key techniques in digital holography, such as phase-shifting holography, low-coherence holography, diffraction tomographic holography, and optical scanning holography. Practical applications are discussed, and accompanied by all the theory necessary to understand the underlying principles at work. A further chapter covers advanced techniques for producing computer-generated holograms. Extensive MATLAB code is integrated with the text throughout and is available for download online, illustrating both theoretical results and practical considerations such as aliasing, zero padding, and sampling.

Accompanied by end-of-chapter problems, and an online solutions manual for instructors, this is an indispensable resource for students, researchers, and engineers in the fields of optical image processing and digital holography.

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Preface

Owing to the advance in faster electronics and digital processing power, the past decade has seen an impressive re-emergence of digital holography. Digital holography is a topic of growing interest and it finds applications in three-dimensional imaging, three-dimensional displays and systems, as well as biomedical imaging and metrology. While research in digital holography continues to be vibrant and digital holography is maturing, we find that there is a lack of textbooks in the area. The present book tries to serve this need: to promote and teach the foundations of digital holography. In addition to presenting traditional digital holography and applications in Chapters 1–4, we also discuss modern applications and techniques in digital holography such as phase-shifting holography, low-coherence holography, diffraction tomographic holography, optical scanning holography, sectioning in holography, digital holographic microscopy as well as computer-generated holography in Chapters 5–7. This book is geared towards undergraduate seniors or first-year graduate-level students in engineering and physics. The material covered is suitable for a one-semester course in Fourier optics and digital holography. The book is also useful for scientists and engineers, and for those who simply want to learn about optical image processing and digital holography.

We believe in the inclusion of MATLAB® in the textbook because digital holography relies heavily on digital computations to process holographic data. MATLAB® will help the reader grasp and visualize some of the important concepts in digital holography. The use of MATLAB® not only helps to illustrate the theoretical results, but also makes us aware of computational issues such as aliasing, zero padding, sampling, etc. that we face in implementing them. Nevertheless, this text is not about teaching MATLAB®, and some familiarity with MATLAB® is required to understand the codes.

The MATLAB® codes included in this book are all available to download from the publisher at www.cambridge.org/digitalholography.

Ting-Chung Poon would like to thank his wife, Eliza, and his children, Christina and Justine, for their love. This year is particularly special to him as Christina gave birth to a precious little one – Gussie. Jung-Ping Liu would like to thank his wife, Hui-Chu, and his parents for their understanding and encouragement.

Contents

1
Wave optics

1.1 Maxwell's equations and the wave equation

In wave optics, we treat light as waves. Wave optics accounts for wave effects such as interference and diffraction. The starting point for wave optics is Maxwell's equations:

$$\nabla \cdot \boldsymbol{D} = \rho_v, \tag{1.1}$$

$$\nabla \cdot \boldsymbol{B} = 0, \tag{1.2}$$

$$\nabla \times \boldsymbol{E} = -\frac{\partial \boldsymbol{B}}{\partial t}, \tag{1.3}$$

$$\nabla \times \boldsymbol{H} = \boldsymbol{J} = \boldsymbol{J}_C + \frac{\partial \boldsymbol{D}}{\partial t}, \tag{1.4}$$

where we have four vector quantities called electromagnetic (EM) fields: the electric field strength $\boldsymbol{E}$ (V/m), the electric flux density $\boldsymbol{D}$ $(\mathrm{C/m^2})$, the magnetic field strength $\boldsymbol{H}$ (A/m), and the magnetic flux density $\boldsymbol{B}$ $(\mathrm{Wb/m^2})$. The vector quantity $\boldsymbol{J}_C$ and the scalar quantity ρ_v are the current density $(\mathrm{A/m^2})$ and the electric charge density $(\mathrm{C/m^3})$, respectively, and they are the sources responsible for generating the electromagnetic fields. In order to determine the four field quantities completely, we also need the constitutive relations

$$\boldsymbol{D} = \varepsilon \boldsymbol{E}, \tag{1.5}$$

and

$$\boldsymbol{B} = \mu \boldsymbol{H}, \tag{1.6}$$

where ε and μ are the permittivity (F/m) and permeability (H/m) of the medium, respectively. In the case of a linear, homogenous, and isotropic medium such as in vacuum or free space, ε and μ are scalar constants. Using Eqs. (1.1)–(1.6), we can derive a wave equation in $\boldsymbol{E}$ or $\boldsymbol{B}$ in free space. For example, by taking the curl of $\boldsymbol{E}$ in Eq. (1.3), we can derive the wave equation in $\boldsymbol{E}$ as

$$\nabla^2 \boldsymbol{E} - \mu\varepsilon \frac{\partial^2 \boldsymbol{E}}{\partial t^2} = \mu \frac{\partial \boldsymbol{J}_C}{\partial t} + \frac{1}{\varepsilon}\nabla \rho_v, \tag{1.7}$$

where $\nabla^2 = \partial^2/\partial x^2 + \partial^2/\partial y^2 + \partial^2/\partial z^2$ is the Laplacian operator in Cartesian coordinates. For a source-free medium, i.e., $\boldsymbol{J}_C = 0$ and $\rho_v = 0$, Eq. (1.7) reduces to the homogeneous wave equation:

$$\nabla^2 \boldsymbol{E} - \frac{1}{v^2}\frac{\partial^2 \boldsymbol{E}}{\partial t^2} = 0. \tag{1.8}$$

Note that $v = 1/\sqrt{\mu\varepsilon}$ is the velocity of the wave in the medium. Equation (1.8) is equivalent to three scalar equations, one for every component of $\boldsymbol{E}$. Let

$$\boldsymbol{E} = E_x\boldsymbol{a}_x + E_y\boldsymbol{a}_y + E_z\boldsymbol{a}_z, \tag{1.9}$$

where $\boldsymbol{a_x}$, $\boldsymbol{a_y}$, and $\boldsymbol{a_z}$ are the unit vectors in the x, y, and z directions, respectively. Equation (1.8) then becomes

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right)(E_x\boldsymbol{a_x} + E_y\boldsymbol{a_y} + E_z\boldsymbol{a_z}) = \frac{1}{v^2}\frac{\partial^2}{\partial t^2}(E_x\boldsymbol{a_x} + E_y\boldsymbol{a_y} + E_z\boldsymbol{a_z}). \tag{1.10}$$

Comparing the $\boldsymbol{a_x}$-component on both sides of the above equation gives us

$$\frac{\partial^2 E_x}{\partial x^2} + \frac{\partial^2 E_x}{\partial y^2} + \frac{\partial^2 E_x}{\partial z^2} = \frac{1}{v^2}\frac{\partial^2 E_x}{\partial t^2}.$$

Similarly, we can derive the same type of equation shown above for the E_y and E_z components by comparison with other components in Eq. (1.10). Hence we can write a compact equation for the three components as

$$\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} + \frac{\partial^2 \psi}{\partial z^2} = \frac{1}{v^2}\frac{\partial^2 \psi}{\partial t^2} \tag{1.11a}$$

or

$$\nabla^2 \psi = \frac{1}{v^2}\frac{\partial^2 \psi}{\partial t^2}, \tag{1.11b}$$

where ψ can represent a component, E_x, E_y, or E_z, of the electric field $\boldsymbol{E}$. Equation (1.11) is called the *three-dimensional scalar wave equation*. We shall look at some of its simplest solutions in the next section.

1.2 Plane waves and spherical waves

In this section, we will examine some of the simplest solutions, namely the plane wave solution and the spherical wave solution, of the three-dimensional scalar wave equation in Eq. (1.11). For simple harmonic oscillation at angular frequency ω_0 (radian/second) of the wave, in Cartesian coordinates, the plane wave solution is

$$\psi(x,y,z,t) = A\exp[j(\omega_0 t - \boldsymbol{k}_0 \cdot \boldsymbol{R})], \tag{1.12}$$

where $j=\sqrt{-1}$, $\boldsymbol{k}_0 = k_{0x}\boldsymbol{a}_x + k_{0y}\boldsymbol{a}_y + k_{0z}\boldsymbol{a}_z$ is the propagation vector, and $\boldsymbol{R} = x\boldsymbol{a}_x + y\boldsymbol{a}_y + z\boldsymbol{a}_z$ is the position vector. The magnitude of $\boldsymbol{k}_0$ is called the wave number and is $|\boldsymbol{k}_0| = k_0 = \sqrt{k_{0x}^2 + k_{0y}^2 + k_{0z}^2} = \omega_0/v$. If the medium is free space, $v = c$ (the speed of light in vacuum) and k_0 becomes the wave number in free space. Equation (1.12) is a *plane wave* of amplitude A, traveling along the $\boldsymbol{k}_0$ direction. The situation is shown in Fig. 1.1.

If a plane wave is propagating along the positive z-direction, Eq. (1.12) becomes

$$\psi(z,t) = A\exp[j(\omega_0 t - k_0 z)], \tag{1.13}$$

which is a solution to the one-dimensional scalar wave equation given by

$$\frac{\partial^2\psi}{\partial z^2} = \frac{1}{v^2}\frac{\partial^2\psi}{\partial t^2}. \tag{1.14}$$

Equation (1.13) is a complex representation of a plane wave. Since the electromagnetic fields are real functions of space and time, we can represent the plane wave in real quantities by taking the real part of ψ to obtain

$$\mathrm{Re}\{\psi(z,t)\} = A\cos[(\omega_0 t - k_0 z)]. \tag{1.15}$$

Another important solution to the wave equation in Eq. (1.11) is a spherical wave solution. The spherical wave solution is a solution which has spherical symmetry, i.e., the solution is not a function of ϕ and θ under the spherical coordinates shown in Fig. 1.2. The expression for the Laplacian operator, ∇^2, is

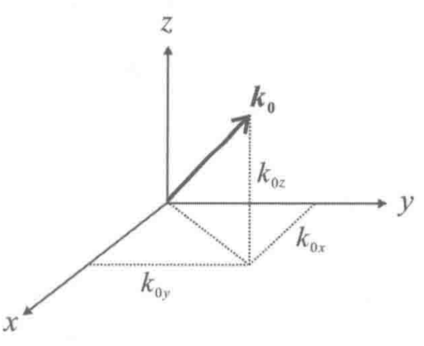

Figure 1.1 Plane wave propagating along the direction $\boldsymbol{k}_0$.

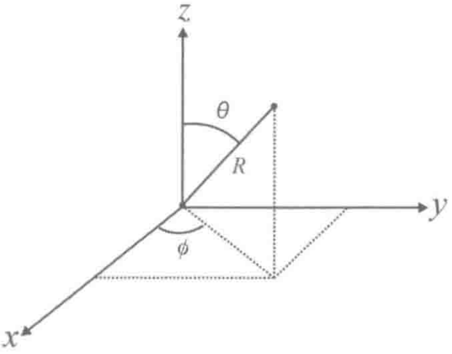

Figure 1.2 Spherical coordinate system.

$$\nabla^2 = \frac{\partial^2}{\partial R^2} + \frac{2}{R}\frac{\partial}{\partial R} + \frac{1}{R^2 \sin^2\theta}\frac{\partial^2}{\partial \phi^2} + \frac{1}{R^2}\frac{\partial^2}{\partial \theta^2} + \frac{\cot\theta}{R^2}\frac{\partial}{\partial \theta}.$$

Hence Eq. (1.11b), under spherical symmetry, becomes

$$\frac{\partial^2 \psi}{\partial R^2} + \frac{2}{R}\frac{\partial \psi}{\partial R} = \frac{1}{v^2}\frac{\partial^2 \psi}{\partial t^2}. \tag{1.16}$$

Since

$$R\left(\frac{\partial^2 \psi}{\partial R^2} + \frac{2}{R}\frac{\partial \psi}{\partial R}\right) = \frac{\partial^2 (R\psi)}{\partial R^2},$$

we can re-write Eq. (1.16) to become

$$\frac{\partial^2 (R\psi)}{\partial R^2} = \frac{1}{v^2}\frac{\partial^2 (R\psi)}{\partial t^2}. \tag{1.17}$$

By comparing the above equation with Eq. (1.14), which has a solution given by Eq. (1.13), we can construct a simple solution to Eq. (1.17) as

$$R\psi(R,t) = A\,\exp[j(\omega_0 t - k_0 R)],$$

or

$$\psi(R,t) = \frac{A}{R}\exp[j(\omega_0 t - k_0 R)]. \tag{1.18}$$

The above equation is a *spherical wave* of amplitude A, which is one of the solutions to Eq. (1.16). In summary, plane waves and spherical waves are some of the simplest solutions of the three-dimensional scalar wave equation.

1.3 Scalar diffraction theory

For a plane wave incident on an aperture or a diffracting screen, i.e., an opaque screen with some openings allowing light to pass through, we need to find the field distribution exiting the aperture or the diffracted field. To tackle the diffraction problem, we find the solution of the scalar wave equation under some initial condition. Let us assume the aperture is represented by a transparency with *amplitude transmittance*, often called *transparency function*, given by $t(x, y)$, located on the plane $z = 0$ as shown in Fig. 1.3.

A plane wave of amplitude A is incident on the aperture. Hence at $z = 0$, according to Eq. (1.13), the plane wave immediately in front of the aperture is given by $A\exp(j\omega_0 t)$. The field distribution immediately after the aperture is $\psi(x, y, z = 0, t) = At(x, y)\exp(j\omega_0 t)$. In general, $t(x, y)$ is a complex function that modifies the field distribution incident on the aperture, and the transparency has been assumed to be infinitely thin. To develop $\psi(x, y, z = 0, t)$ further mathematically, we write

$$\begin{aligned}\psi(x,y,z=0,t) &= At(x,y)\exp(j\omega_0 t) = \psi_p(x,y;z=0)\exp(j\omega_0 t)\\ &= \psi_{p0}(x,y)\exp(j\omega_0 t).\end{aligned} \tag{1.19}$$

The quantity $\psi_{p0}(x, y)$ is called the *complex amplitude* in optics. This complex amplitude is the initial condition, which is given by $\psi_{p0}(x, y) = A \times t(x, y)$, the amplitude of the incident plane wave multiplied by the transparency function of the aperture. To find the field distribution at z away from the aperture, we model the solution in the form of

$$\psi(x,y,z,t) = \psi_p(x,y;z)\exp(j\omega_0 t), \tag{1.20}$$

where $\psi_p(x, y; z)$ is the unknown to be found with initial condition $\psi_{p0}(x, y)$ given. To find $\psi_p(x, y; z)$, we substitute Eq. (1.20) into the three-dimensional scalar wave equation given by Eq. (1.11a) to obtain the *Helmholtz equation* for $\psi_p(x, y; z)$,

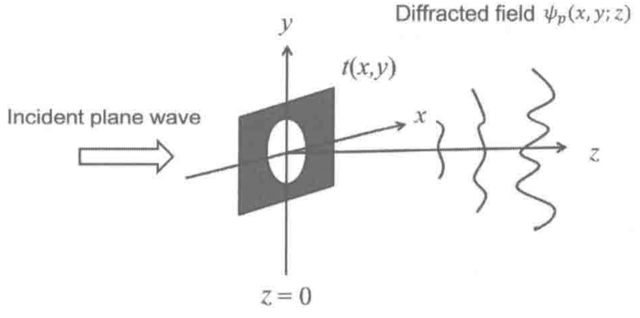

Figure 1.3 Diffraction geometry: $t(x, y)$ is a diffracting screen.

$$\frac{\partial^2\psi_p}{\partial x^2}+\frac{\partial^2\psi_p}{\partial y^2}+\frac{\partial^2\psi_p}{\partial z^2}+k_0^2\psi_p=0. \tag{1.21}$$

To find the solution to the above equation, we choose to use the Fourier transform technique. The two-dimensional Fourier transform of a spatial signal $f(x, y)$ is defined as

$$\mathcal{F}\{f(x,y)\}=F(k_x,k_y)=\iint\limits_{-\infty}^{\infty} f(x,y)\exp(jk_xx+jk_yy)dx\,dy, \tag{1.22a}$$

and the inverse Fourier transform is

$$\mathcal{F}^{-1}\{F(k_x,k_y)\}=f(x,y)=\frac{1}{4\pi^2}\iint\limits_{-\infty}^{\infty} F(k_x,k_y)\exp(-jk_xx-jk_yy)dk_x\,dk_y, \tag{1.22b}$$

where k_x and k_y are called spatial radian frequencies as they have units of radian per unit length. The functions $f(x, y)$ and $F(k_x, k_y)$ form a Fourier transform pair. Table 1.1 shows some of the most important transform pairs.

By taking the two-dimensional Fourier transform of Eq. (1.21) and using transform pair number 4 in Table 1.1 to obtain

$$\begin{aligned}\mathcal{F}\left\{\frac{\partial^2\psi_p}{\partial x^2}\right\}&=(-jk_x)^2\Psi_\mathrm{p}(k_x,k_y;z)\\ \mathcal{F}\left\{\frac{\partial^2\psi_p}{\partial y^2}\right\}&=(-jk_y)^2\Psi_\mathrm{p}(k_x,k_y;z),\end{aligned} \tag{1.23}$$

where $\mathcal{F}\{\psi_p(x,y;z)\}=\Psi_\mathrm{p}(k_x,k_y;z)$, we have a differential equation in $\Psi_\mathrm{p}(k_x, k_y; z)$ given by

$$\frac{d^2\Psi_\mathrm{p}}{dz^2}+k_0^2\left(1-\frac{k_x^2}{k_0^2}-\frac{k_y^2}{k_0^2}\right)\Psi_\mathrm{p}=0 \tag{1.24}$$

subject to the initial known condition $\mathcal{F}\{\psi_p(x,y;z=0)\}=\Psi_\mathrm{p}(k_x,k_y;z=0)=\Psi_\mathrm{p0}(k_x,k_y)$. The solution to the above second ordinary differential equation is straightforward and is given by

$$\Psi_\mathrm{p}(k_x,k_y;z)=\Psi_\mathrm{p0}(k_x,k_y)\exp\left[-jk_0\sqrt{(1-k_x^2/k_0^2-k_y^2/k_0^2)}z\right] \tag{1.25}$$

as we recognize that the differential equation of the form

$$\frac{d^2y(z)}{dz^2}+\alpha^2y(z)=0$$