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Introduction to the Theory of Coherence and Polarization of Light

光的相干与偏振理论导论

（影印版）

〔美〕沃尔夫（E. Wolf） 著

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序　　言

物理学是研究物质、能量以及它们之间相互作用的科学。她不仅是化学、生命、材料、信息、能源和环境等相关学科的基础，同时还是许多新兴学科和交叉学科的前沿。在科技发展日新月异和国际竞争日趋激烈的今天，物理学不仅囿于基础科学和技术应用研究的范畴，而且在社会发展与人类进步的历史进程中发挥着越来越关键的作用。

我们欣喜地看到，改革开放三十多年来，随着中国政治、经济、教育、文化等领域各项事业的持续稳定发展，我国物理学取得了跨越式的进步，做出了很多为世界瞩目的研究成果。今日的中国物理正在经历一个历史上少有的黄金时代。

在我国物理学科快速发展的背景下，近年来物理学相关书籍也呈现百花齐放的良好态势，在知识传承、学术交流、人才培养等方面发挥着无可替代的作用。从另一方面看，尽管国内各出版社相继推出了一些质量很高的物理教材和图书，但系统总结物理学各门类知识和发展，深入浅出地介绍其与现代科学技术之间的渊源，并针对不同层次的读者提供有价值的教材和研究参考，仍是我国科学传播与出版界面临的一个极富挑战性的课题。

为有力推动我国物理学研究、加快相关学科的建设与发展，特别是展现近年来中国物理学者的研究水平和成果，北京大学出版社在国家出版基金的支持下推出了“中外物理学精品书系”，试图对以上难题进行大胆的尝试和探索。该书系编委会集结了数十位来自内地和香港顶尖高校及科研院所的知名专家学者。他们都是目前该领域十分活跃的专家，确保了整套丛书的权威性和前瞻性。

这套书系内容丰富，涵盖面广，可读性强，其中既有对我国传统物理学发展的梳理和总结，也有对正在蓬勃发展的物理学前沿的全面展示；既引进和介绍了世界物理学研究的发展动态，也面向国际主流领域传播中国物理的优秀专著。可以说，“中外物理学精品书系”力图完整呈现近现代世界和中国物理

科学发展的全貌，是一部目前国内为数不多的兼具学术价值和阅读乐趣的经典物理丛书。

“中外物理学精品书系”另一个突出特点是，在把西方物理的精华要义“请进来”的同时，也将我国近现代物理的优秀成果“送出去”。物理学科在世界范围内的重要性不言而喻，引进和翻译世界物理的经典著作和前沿动态，可以满足当前国内物理教学和科研工作的迫切需求。另一方面，改革开放几十年来，我国的物理学研究取得了长足发展，一大批具有较高学术价值的著作相继问世。这套丛书首次将一些中国物理学者的优秀论著以英文版的形式直接推向国际相关研究的主流领域，使世界对中国物理学的过去和现状有更多的深入了解，不仅充分展示出中国物理学研究和积累的“硬实力”，也向世界主动传播我国科技文化领域不断创新的“软实力”，对全面提升中国科学、教育和文化领域的国际形象起到重要的促进作用。

值得一提的是，“中外物理学精品书系”还对中国近现代物理学科的经典著作进行了全面收录。20 世纪以来，中国物理界诞生了很多经典作品，但当时大都分散出版，如今很多代表性的作品已经淹没在浩瀚的图书海洋中，读者们对这些论著也都是“只闻其声，未见其真”。该书系的编者们在这方面下了很大工夫，对中国物理学科不同时期、不同分支的经典著作进行了系统的整理和收录。这项工作具有非常重要的学术意义和社会价值，不仅可以很好地保护和传承我国物理学的经典文献，充分发挥其应有的传世育人的作用，更能使广大物理学人和青年学子切身体会我国物理学研究的发展脉络和优良传统，真正领悟到老一辈科学家严谨求实、追求卓越、博大精深的治学之美。

温家宝总理在 2006 年中国科学技术大会上指出，“加强基础研究是提升国家创新能力、积累智力资本的重要途径，是我国跻身世界科技强国的必要条件”。中国的发展在于创新，而基础研究正是一切创新的根本和源泉。我相信，这套“中外物理学精品书系”的出版，不仅可以使所有热爱和研究物理学的人们从中获取思维的启迪、智力的挑战和阅读的乐趣，也将进一步推动其他相关基础科学更好更快地发展，为我国今后的科技创新和社会进步做出应有的贡献。

“中外物理学精品书系”编委会 主任
中国科学院院士，北京大学教授
王恩哥
2010 年 5 月于燕园

Introduction to the Theory of Coherence and Polarization of Light

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Dedicated to the memory of Leonard Mandel,
a dear friend and colleague of more than forty years.

Thomas Young[1]
(1773–1829)

Gabriel Stokes[2]
(1819–1903)

Frits Zernike[3]
(1888–1966)

The three pioneers who laid the foundations of the theory presented in this book

[1] Reproduced from the Gallery of Legendary Optical Scientists, The Institute of Optics, University of Rochester.
[2] Reproduced with the courtesy of AIP Emilio Segrè Visual Archives.
[3] Reproduced with the courtesy of the Universiteitsmuseum Groningen.

Preface

> *"... the image that will be formed in a photographic camera – i.e. the distribution of intensity on the sensitive layer – is present in an invisible, mysterious way in the aperture of the lens, where the intensity is equal at all points."*
>
> F. Zernike, discussing coherence in a lecture published in *Proc. Phys. Soc.* (London), **61** (1948), 158.

The optical coherence and polarization phenomena with which this book is concerned have their origin in the fact that all optical fields, whether encountered in nature or generated in a laboratory, have some random fluctuations associated with them. The monochromatic sources and monochromatic fields discussed in most optics textbooks are not encountered in real life.

For thermal light, the fluctuations are mainly due to spontaneous emission of radiation from atoms which generate the field. For laser light the fluctuations are due to uncontrollable causes such as mechanical vibrations of the mirrors at the end of the cavity, temperature fluctuations and, again, arise also because contributions from spontaneous emission are always present. For a well-stabilized laser these effects are largely manifested in phase fluctuations rather than in amplitude fluctuations and also in the chaotic behavior of the laser output that may be detected over a sufficiently long time. In the optical region of the electromagnetic spectrum, the field fluctuations are too rapid to be directly measurable. The theory of coherence and polarization involves average quantities that can be measured. Consequently, the theory deals with observable quantities.

When I started writing this book my intention was to provide an introductory text on the subject of classical optical coherence theory alone. Although there are now several books and book chapters devoted to coherence, none of them seems to me to treat the subject at a level appropriate for a reader who has a reasonable knowledge of elementary optics and none presents the basic concepts and results of coherence theory in a sufficiently sound and yet not too abstract manner. By the time I had written several chapters, a new development had taken place, namely the discovery that coherence and polarization of light are two aspects of statistical optics which are intimately related and can be treated in a unified manner. Until then coherence and polarization had been considered as being essentially independent of each other – the only apparent link was the term "coherency matrix," a 2×2 correlation matrix which has been used in the analysis of elementary polarization problems since the 1930s. This term is actually a misnomer, because such a matrix has nothing to do with coherence, as the term is now understood. Coherence is essentially a consequence of correlations between some components of the fluctuating electric field at two (or more) points and is manifested by the sharpness of fringes in Young's interference experiment. Polarization, on the other hand, is a manifestation of correlations involving components of the fluctuating electric field at a single point and may be determined with the help of polarizers, rotators and phase plates. Both concepts reflect "degrees of order" in an electromagnetic field, but they pertain to somewhat different statistical aspects of it. The theories of coherence and polarization are, however, concerned not only with order and disorder in optical fields. Their basic tools are correlation functions and correlation matrices which, unlike some directly measurable quantities such as the spectrum of light, obey precise propagation laws. With the help of these laws one may determine, for example, spectral and polarization changes as light propagates, whether in free space or in a medium, which may be either deterministic (e.g. a glass fiber) or random (e.g. the turbulent atmosphere). Consequences of these laws are among the most useful aspects of the theory.

Until very recently, coherence phenomena have been studied largely on the basis of scalar theory, whereas polarization requires a vector treatment. It was actually a generalization of the concept of coherence from scalar fields to electromagnetic vector fields, introduced only a few years ago, that has made it evident that coherence and polarization of light, whilst distinct phenomena, are just two closely related aspects of statistical optics; and that many features of fluctuating electromagnetic fields can be fully understood only when they are treated in close partnership.[1] This discovery has not only enriched both subjects, but also has already provided new insights into many aspects of statistical optics. This development, which is discussed in the concluding chapter, is likely to find useful applications, for example, in connection with optical communication, with imaging by laser radar and in medical diagnostics, but undoubtedly other applications will be forthcoming.

In order to provide a treatment of the subject which is not too demanding mathematically and which will help the reader to acquire a working knowledge of it, detailed proofs are sometimes omitted. Most of them can be found in M. Born and E. Wolf, *Principles of Optics* (Cambridge University Press, Cambridge, 7th (expanded) edition, 1999) and in L. Mandel and E. Wolf, *Optical Coherence and Quantum Optics* (Cambridge University Press,

[1] Developments leading to the recognition that there exists an intimate relationship between the phenomena of coherence and polarization are discussed in an article by E. Wolf, "Young's interference experiment and its influence on the development of statistical optics" in volume 50 of *Progress in Optics* (Amsterdam, Elsevier, 2007).

Cambridge, 1995),[1] which also contain more detailed accounts of the subject, with full references. The historical development of coherence theory is outlined in an article in *Selected Works of Emil Wolf with Commentary* (World Scientific, Singapore, 2001), pp. 620–633. Accounts of the development of the theory of polarization can be found in E. Collett, *Polarized Light* (M. Dekker, New York, 1993) and in C. Brosseau, *Fundamentals of Polarized Light* (J. Wiley, New York, 1998).

Some readers may note that parts of the presentation resemble fairly closely the treatments given in B&W and M&W. This is mainly due to the fact that I had difficulties in providing a different formulation, but it should be clear that this book uses more elementary and less rigorous analysis, aimed at non-specialists, especially teachers and students, who might perhaps also find it helpful that problems are included at the end of each chapter. Additional problems can be found in M&W.

I am grateful to Dr. Gale Gant and Dr. Don Nicolson for the photograph of the 20-foot Michelson stellar interferometer at Mount Wilson Observatory, built in the 1920s. The photograph, reproduced as Fig. 3.12, was taken around the year 2000.

In writing this book I have greatly benefited from the assistance of many colleagues and students who read drafts of the manuscript and helped in weeding out errors and improving the text. I would particularly like to acknowledge substantial advice that I received from Professor Taco Visser and also useful suggestions from Prof. Jannick Rolland, Mrs. Nicole Carlson-Moore, Dr. David Fischer, Dr. Olga Korotkova, Dr. Mircea Mujat, Mr. Jonathan Petruccelli, Mr. Mohamed Salem, Mr. Thijs Stegeman, Dr. Tomohiro Shirai, Mr. Mayukh Lahiri and Mr. Thomas van Dijk. I am also obliged to Mr. Mohamed Salem and Dr. Sergei Volkov for help with checking the proofs.

The staff of the Physics–Optics–Astronomy Library of the University of Rochester provided much help, especially with locating articles and checking references. I am much obliged to Mrs. Patricia Sulouff, the Head Librarian, and to Mrs. Sandra Cherin and Mrs. Miriam Margala for their assistance.

I am greatly obliged to Dr. Greg Gbur for preparing the excellent line drawings of most of the figures and also for the beautiful figure which appears on the front cover.

Some of the research described in this book, especially that connected with the unified theory of coherence and polarization discussed in Chapter 9, was supported by the Air Force Office of Scientific Research (AFOSR). I am much indebted to Dr. Arje Nachman of AFOSR for his continued support over many years and for his interest in our work.

I acknowledge, with many thanks, the very substantial help provided by my secretary, Mrs. Ellen Calkins, who, without any complaints, typed and re-typed numerous versions of the manuscript and also prepared the author index.

I wish to express my appreciation to my patient wife, Marlies, who spent long periods in solitude whilst I was preparing the manuscript.

I presented much of the material contained in this book in courses to Physics and Optics graduate students at the University of Rochester and at the University of Central Florida;

[1] References to these books are abbreviated in the present work as B&W and M&W, respectively.

but a good part of the text is an expanded version of notes that I prepared for a course which I taught for many years at Annual Meetings of the Optical Society of America. I am indebted to Dr. Simon Capelin, the Publishing Director for Physical Sciences of Cambridge University Press, for suggesting that I expand the notes into a book and for encouraging me to do so.

Finally I wish to express my appreciation to the staff of Cambridge University Press and, particularly, to J. Bottrill, the production editor, K. Howe, the production manager and to Dr. S. Holt who copy-edited the manuscript, for their cooperation and for having transformed an imperfect manuscript into a beautiful end product.

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Spring 2007

Contents

Preface *page* xi

1. Elementary coherence phenomena **1**
1.1 Interference and statistical similarity 1
1.2 Temporal coherence and the coherence time 4
1.3 Spatial coherence and the coherence area 5
1.4 The coherence volume 8
Problems 10

2. Mathematical preliminaries **11**
2.1 Elementary concepts of the theory of random processes 11
2.2 Ergodicity 17
2.3 Complex representation of a real signal and the envelope of a narrow-band signal 19
2.4 The autocorrelation and the cross-correlation functions 22
2.4.1 The autocorrelation function of a finite sum of periodic components with random amplitudes 24
2.5 The spectral density and the Wiener–Khintchine theorem 25
Problems 29

3. Second-order coherence phenomena in the space–time domain **31**
3.1 Interference law for stationary optical fields. The mutual coherence function and the complex degree of coherence 31
3.2 Generation of spatial coherence from an incoherent source. The van Cittert–Zernike theorem 37
3.3 Illustrative examples 46
3.3.1 Michelson's method for measuring stellar diameters 46
3.3.2 Michelson's method for determining energy distribution in spectral lines 51
3.4 Propagation of the mutual intensity 54
3.5 Wave equations for the propagation of mutual coherence in free space 56
Problems 58

4. Second-order coherence phenomena in the space–frequency domain **60**
4.1 Coherent-mode representation and the cross-spectral density as a correlation function 60
4.2 The spectral interference law and the spectral degree of coherence 63
4.3 An illustrative example: spectral changes on interference 69
4.4 Interference of narrow-band light 73
Problems 76

5. Radiation from sources of different states of coherence **79**
5.1 Fields generated by sources with different coherence properties 79
5.2 Correlations and the spectral density in the far field 81
5.3 Radiation from some model sources 88
5.3.1 Schell-model sources 88
5.3.2 Quasi-homogeneous sources 90
5.4 Sources of different states of spatial coherence which generate identical distributions of the radiant intensity 95
5.5 Coherence properties of Lambertian sources 97
5.6 Spectral changes on propagation. The scaling law 102
Problems 108

6. Coherence effects in scattering **111**
6.1 Scattering of a monochromatic plane wave on a deterministic medium 111
6.2 Scattering of partially coherent waves on a deterministic medium 115
6.3 Scattering on random media 118
6.3.1 General formulas 118
6.3.2 Examples 121
6.3.3 Scattering on a quasi-homogeneous medium 123
Problems 127

7. Higher-order coherence effects **129**
7.1 Introduction 129
7.2 Intensity interferometry with radio waves 131
7.3 The Hanbury Brown–Twiss effect and intensity interferometry with light 134
7.4 Einstein's formula for energy fluctuations in blackbody radiation and the wave–particle duality 140
7.5 Mandel's theory of photoelectric detection of light fluctuations 143
7.5.1 Mandel's formula for photocount statistics 143
7.5.2 The variance of counts from a single photodetector 145
7.5.3 Correlation between count fluctuations from two detectors 147
7.6 Determination of statistical properties of light from photocount measurements 149
Problems 151

8. Elementary theory of polarization of stochastic electromagnetic beams **154**
8.1 The 2×2 equal-time correlation matrix of a quasi-monochromatic electromagnetic beam 154
8.2 Polarized, unpolarized and partially polarized light. The degree of polarization 158
8.2.1 Completely polarized light 158
8.2.2 Natural (unpolarized) light 160

8.2.3 Partially polarized light and the degree of polarization 161
8.2.4 The geometrical significance of complete polarization. The Stokes parameters of completely polarized light. The Poincaré sphere 165
Problems 171

9. Unified theory of polarization and coherence **174**
9.1 The 2 × 2 cross-spectral density matrix of a stochastic electromagnetic beam 174
9.2 The spectral interference law, the spectral degree of coherence and the spectral degree of polarization of stochastic electromagnetic beams 175
9.3 Determination of the cross-spectral density matrix from experiments 179
9.4 Changes in random electromagnetic beams on propagation 181
9.4.1 Propagation of the cross-spectral density matrix of a stochastic electromagnetic beam – general formulas 181
9.4.2 Propagation of the cross-spectral density matrix of an electromagnetic Gaussian Schell-model beam 183
9.4.3 Examples of correlation-induced changes in stochastic electromagnetic beams on propagation 186
9.4.4 Coherence-induced changes of the degree of polarization in Young's interference experiment 191
9.5 Generalized Stokes parameters 194
Problems 197

Appendices 202
I Cells of phase space and the degeneracy parameter 202
(a) Cells of phase space of a quasi-monochromatic light wave (Section 1.4) 202
(b) Cells of phase space of radiation in a cavity (Sections 7.4 and 7.5) 204
(c) The degeneracy parameter 206
II Derivation of Mandel's formula for photocount statistics [Eq. (2) of Section 7.5.1] 208
III The degree of polarization of an electromagnetic Gaussian Schell-model source 210
IV Some important probability distributions 212
(a) The binomial (or Bernoulli) distribution and some of its limiting cases 212
(b) The Bose–Einstein distribution 214

Author index 216
Subject index 220

1

Elementary coherence phenomena

1.1 Interference and statistical similarity

The simplest manifestation of coherence between light vibrations at different points in an optical field is provided by the phenomenon of interference. In fact, as we will learn later, some features of the interference pattern provide a quantitative measure of the coherence between light vibrations at two points in space and at the two instants of time.

Let us first consider light vibrations at a point P in an optical field. For the sake of simplicity we will ignore, to begin with, the polarization properties of light and we may then represent the light vibrations at a point in the field by a scalar, say $U(t)$. If the light were monochromatic, it would be expressed as

$$U(t) = a\cos(\phi - \omega t), \tag{1}$$

where a and ϕ are the (constant) amplitude and phase, respectively, ω is the frequency and t denotes the time. However, as we have already noted, monochromatic light is an idealization which is never encountered in nature or in a laboratory. Light that in some respects imitates monochromatic light most closely is so-called *quasi-monochromatic* light. It is defined by the property that its effective bandwidth, $\Delta\omega$ is much smaller than its mean frequency $\bar{\omega}$, i.e. that

$$\frac{\Delta\omega}{\bar{\omega}} \ll 1. \tag{2}$$

For such light the amplitude and the phase are no longer constant and its vibrations at a point in space may be represented by a generalization of Eq. (1), viz.,

$$U(t) = a(t)\cos[\phi(t) - \bar{\omega}t], \tag{3}$$

where the amplitude $a(t)$ and the phase $\phi(t)$ now depend on time, and generally fluctuate randomly. With the help of elementary Fourier analysis one can show (M&W, Section 3.1, especially pp. 99–100) that for quasi-monochromatic light $a(t)$ and $\phi(t)$ will vary very