Hot Working Guide

A Compendium of Processing Maps

Second Edition

Edited by
Y.V.R.K. Prasad
K.P. Rao
S. Sasidhara

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Preface to the Second Edition

Hot Working Guide: A Compendium of Processing Maps *is a unique source book with flow stress data for hot working, processing maps with metallurgical interpretation and optimum processing conditions for metals, alloys, intermetallics and metal matrix composites. The use of this book replaces the expensive and time consuming trial and error methods in process design and product development. In the first edition, which was published by ASM International in 1997, processing maps for 162 materials were presented. Since that time, processing maps for another 130 materials with different initial conditions have been published in the literature which motivated updating of the first edition. In the second edition, significant additions of maps on stainless steels, magnesium alloys, titanium alloys and nickel alloys have been made. In compiling the second edition, stress-strain curves were not included since their shapes do not lead to clear conclusions on the mechanisms. However, the flow stress data are included since they are valuable in formulating constitutive equations required for finite element simulation. In this book, the available information is compiled in such a way that the processing industry will find it easy to use.*

In the first chapter, information on typical microstructures that help in interpreting the processing maps have been presented along with literature updates on the review articles on processing maps. In Chapters 2-9, processing maps developed on the basis of data extracted from published papers have been given along with interpretations. It may not be considered as an exhaustive coverage of literature since some of the data may not have been included when the test matrix was found to be insufficient. The compilation will help researchers get started on this topic.

Information on many commercial alloys has been included in this reference book. It is believed that this will cater to the needs of the bulk metal working industry in improving the yield by process optimization and in achieving better product quality. Researchers in this area will find the compilation as a ready reference in pursuing further work in correlating the material chemistry and processing history with the hot workability. The complications in processing intermetallics and metal-matrix composite are clearly revealed and the advantage of using processing maps for these materials is easily recognizable. In materials like titanium alloys and magnesium Alloys, the importance of texture in processing is clearly seen in the maps.

Historically, the foundation to processing maps was laid in 1984 in the Materials Laboratory of Wright-Patterson Air Force Base, OH, USA. This was further pursued at the Department of Metallurgy, Indian Institute of Science, Bangalore, India, with the support of the Department of Science and Technology, Government of India, and contributions from several national laboratories. After the year 2000, research efforts on processing maps were taken up at the City University of Hong Kong, and to a large extent the effort was focused on magnesium materials. Many commercial magnesium alloys as well as new experimental alloys have been characterized and the hot workability was correlated with microstructural mechanisms including texture. This edition reflects these important contributions to hot working research.

The help and contributions to the second edition of Dr. K. Suresh, Senior Research Associate at the City University of Hong Kong, is gratefully acknowledged. His hard work and commitment to perfection is truly exemplary.

Y.V.R.K. Prasad
K.P. Rao
S. Sasidhara

Preface to the First Edition

Hot Working Guide: A Compendium of Processing Maps *is a unique source book with flow stress data for hot working, processing maps with metallurgical interpretation and recommendation of optimum processing conditions. The use of this book replaces the expensive and time consuming trial and error methods in process design and product development.*

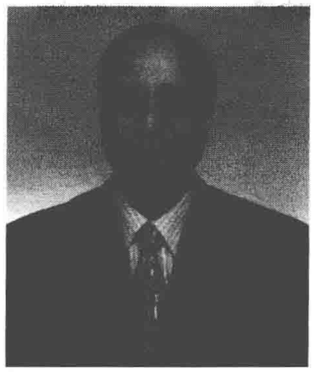
In the first chapter, the issues involved in the design and manufacturing are discussed in relation to hot working. Hot workability is defined and the concept of processing maps for o0ptimising material workability and control of microstructure are explained along with the details of the method of generating processing maps. Guidelines are given for its interpretation and application to industrial hot working process with the help of typical examples. In Chapters 2–10, processing maps and flow stress data are compiled for over 160 materials which include metals of different purity, conventional alloys used in industry like aluminum alloys, copper alloys and steels, advanced materials like superalloys, titanium and zirconium alloys, and never materials like titanium aluminides and metal matrix composites.

The contents of this book will benefit bulk metal working (rolling, forging, and extrusion) industry in improving the yield by process optimization and in achieving better quality by avoiding defects. Practicing engineers and R & D specialists may use the data base for FEM simulations and process design, and undergraduate students may utilize the material for understanding the science of processing.

The foundation for the Dynamic Materials Model leading to the concept of processing maps was laid in 1984 in the Materials Laboratory of Wright-Patterson Air Force Base (WPAFB), OH, USA. Subsequently, work in this area took two directions: WPAFB along with Ohio University and Universal Energy Systems Inc., integrated the concept with the FEM simulation model and developed process design and control concepts for use in industrial processes. Simultaneously, at Department of Metallurgy, Indian Institute of Science, Bangalore, India, the metallurgical interpretation of the processing map leading to science of mechanical processing was pursued. With the support of Department of Science and Technology, Government of India, hot compression testing (custom built by DARTEC, UK) and computational facilities were established. Several research laboratories and educational institutions in India participated in this effort, and major contributors are Defense Metallurgical Research Laboratory at Hyderabad, Bhabha Atomic Research Center at Bombay, Indira Gandhi Center for Atomic Research at Kalpakkam. Several of the processing maps developed in this effort are also industrially validated both in terms of optimizing the existing process to improve productivity and in designing newer processes. In this book, the available information is compiled in such a way that processing industry will find it easy to use. The reference material was not meant to be a complete review of the work on the topic but to help the reader to get introduced.

Y.V.R.K. Prasad
S. Sasidhara

About the Editors ....


Y.V.R.K. Prasad | **K.P. Rao** | **S. Sasidhara**

Y.V.R.K. Prasad obtained his Ph.D. degree in Mechanical Metallurgy from the Indian Institute of Science, Bangalore, India in 1971, where he taught and conducted research for four decades in dislocation dynamics, microstructure-mechanical property correlations, and mechanical processing of materials. In the early seventies, he was a post-doctoral fellow at the University of Maryland, College Park, and Drexel University, Philadelphia, USA. In the early eighties and late nineties, he was a Senior Research Associate of the U.S. National Research Council at the Materials Laboratory of Wright-Patterson Air Force Base, Dayton, Ohio, USA, where he was part of the team that developed the Dynamic Materials Model that uses processing maps to optimize hot working processes. Two U.S. Patents have been awarded for this invention. He developed the processing science laboratory at the Indian Institute of Science where he retired as Professor in 2003. Later, he continued his research at the City University of Hong Kong where he is an Academic Visitor and a consultant on processing maps. He is a Fellow of the Indian National Science Academy, Indian Academy of Sciences and Indian National Academy of Engineering.

K.P. Rao obtained his Ph.D. degree in Metal Forming from the Indian Institute of Technology, Madras (Chennai), India in 1983 and an MBA in Technology Management from LaTrobe University, Australia in 2003. After post-doctoral assignments at the University of New Brunswick, Fredericton, and the University of British Columbia, Vancouver, Canada, he joined the faculty of the City University of Hong Kong in 1990, firstly in the Department of Manufacturing Engineering and Engineering Management, and then in the Department of Mechanical and Biomedical Engineering, where he is now a Professor and Associate Head of the Department. He has extensive experience in the simulation, design, and development of metal forming processes and his recent research interests encompass the formability and processing maps for hot working of new magnesium alloys, thermo-mechanical processing of magnesium alloys for bioimplants, and processing of bulk magnesium alloy composites with nano-dispersions. His earlier research included the evaluation of hot working mechanisms in aluminum alloys, pure copper and titanium aluminide, and its *in-situ* composite produced by powder metallurgy.

S. Sasidhara obtained his B.E. degree in Mechanical Engineering from Bangalore University in 1972 after getting his B.Sc. degree from Mysore University. He joined the Department of Metallurgy of the Indian Institute of Science in 1973 to head up several universal mechanical testing and processing equipment areas. He specialized in designing tooling for specialized testing requirements, most importantly the constant true strain rate high temperature compression. He is credited with conducting more than 10,000 hot compression tests on a wide range of materials including metals, alloys, intermetallics, and metal-matrix composites, the data on which formed the basis for developing the processing maps presented in this volume. He has visited the works of Instron Limited, Dartec Limited, England, and British Alcan, and the laboratories of Imperial College, University of Sheffield, and University of Cambridge, U.K. He has been the mentor for many students that graduated from IISc with Doctoral and Masters Degrees over four decades. At present, he is a technical consultant in the Department of Materials Engineering at the Indian Institute of Science, Bangalore, India.

Contents

1. Introduction

Mechanical processing is an essential step in shaping materials into engineering components which require not only dimensional accuracy but also specified microstructures and mechanical properties. The techniques of mechanical processing involve bulk metal working using rolling, forging or extrusion which are generally conducted at elevated temperatures in order that large strains may be imposed in a single step of the operation without the onset of fracture. The secondary metal working processes generally use cold working which ensures good surface finish, high dimensional tolerance and better strength. However, these involve smaller strains and require a large number of steps with intermediate annealing to restore the ductility. Processes like sheet metal working, cold forging, impact extrusion, coining, wire and tube drawing are some examples of this category. In recent years, with the advent of rapid solidification processing and atomization techniques for producing powders of desired shape and size, powder metallurgy (PM) has assumed a significant role in shape making. Using this technique it is now possible to make complicated shapes in exotic alloys for many critical applications like gas turbine components.

Among all the mechanical processing methods, the bulk metal working stage is considered to be of primary importance for two reasons: Firstly, in this stage, major microstructural changes occur and these have a profound influence on the subsequent processing steps. Secondly, in view of the large tonnage of material being processed by bulk metal working, any improvement in processing techniques has a multiplying effect on the overall productivity in manufacturing. Thus, considerable effort has gone into developing techniques for the design and optimization of bulk metal working processes. The ultimate objective is to manufacture components with controlled microstructure and properties, without macro or microstructural defects, on a repeatable basis in a manufacturing environment. Hitherto, this is done using trial and error techniques which are expensive as well as time consuming and may not always lead to a successful solution or optimization, particularly for advanced materials like superalloys, intermetallics and metal matrix composites. In recent years, however, the trial and error techniques are replaced by modeling techniques, which are developed on the basis of science-based principles. These techniques address the following design and manufacturing issues involved.

The design requirements are:

- arriving at optimum processing conditions
- controlling the microstructure in the component
- designing optimum die shapes or preform geometry without resorting to shop floor trials
- obtaining the process limits for the design of control systems

The manufacturing issues revolve around:

- the reduction of lead time in manufacturing
- increasing the productivity without sacrificing the product quality
- reducing the rejects to improve yield
- ensuring the repeatability in manufacturing

The starting condition of the material has an immense effect on its behavior during mechanical processing and, in particular, the ingot should be free from macro and micro structural defects. The following processes are used to produce input materials for metal working:

***Metal Casting*:** Conventionally cast ingots have columnar and dendritic microstructures with heavy segregation of alloying elements. There could be a non-uniformity of microstructure from center (equiaxed) to the surface (columnar). Unless careful control is exercised in casting, macro and micro defects may occur. The importance of homogenization of the cast ingot need not be overemphasized. In recent years, ferrous materials are being produced by continuous casting route and the billets or slabs are continuously rolled in order to increase the productivity and save energy by avoiding reheating of ingots. In such a case, it is important to note that there could be steep temperature gradients from surface to center. The surface temperature will be lower than the interior, since solidification proceeds in that direction. This is in contrast to billets which are reheated. Another common casting process used for non-ferrous alloys is direct chill (DC) casting. When long freezing range alloys are DC cast, microporosity may occur unless a hot top is used.

Wrought Structures: These are either recovered (stress relieved) or recrystallized (fully annealed) microstructures and have better workability than as-cast microstructures. However, improper primary metal working may introduce microstructural damage, flow instabilities or peripheral grain growth in the wrought microstructures. Also, materials may develop preferred orientation (crystallographic texture) or mechanical fibering which needs to be controlled for further processing, for example, sheet metal working.

Heat Treated Microstructures: In several alloy systems like titanium alloys and zirconium alloys, a variety of preform microstructures may be produced ranging from acicular (β-quenched) to equiaxed (α+β annealed). The preform microstructure has a profound influence on the workability of the material.

Powder Compacts: With advances in the rapid solidification processing of alloy powders and mechanical alloying, it is possible to put in more alloy content without causing heavy segregation. Various compacting routes like cold isostatic pressing and sintering, hot isostatic pressing, vacuum hot pressing and compaction by blind extrusion have been developed. However, the presence of prior particle boundary (PPB) defects, discrete particle effects (effect of hard and soft particles in a statistical distribution of particle sizes and their individual microstructures) and the high surface reactivity of powders have made the mechanical processing of PM compacts a highly specialized technology. For advanced materials, processes like gatorizing (Pratt and Whitney Co.) and billet conditioning (Wright-Patterson Air Force Base) have been patented for this purpose.

1.1 What is Workability?

The engineering parameter that is of importance in mechanical processing is commonly termed as "workability" which refers to the ease with which a material can be shaped by plastic flow without the onset of fracture. This general term includes all other terms like forgeability, rollability, extrudability and formability (sheet metal working). A fundamental understanding of workability is essential for developing science-based techniques in mechanical processing. Detailed reviews on the influence of metallurgical parameters on workability and the various standard workability tests are available [1,2]. It is clear that the workability is influenced not only by the microstructures of the material, applied temperature, strain rate and strain but also by the stress state in the deformation zone. For example, the tensile elongation of a material may be enhanced (or necking may be delayed) by the application of an external hydrostatic compression (classical Bridgeman experiment) or by slow speed deformation of a specimen with a stable fine grained structure at higher temperatures (superplastic deformation). It is therefore convenient to consider workability to consist of two independent parts: state-of-stress (SOS) workability and intrinsic workability.

1.1.1 State-of-Stress Workability

SOS workability depends upon the geometry of the deformation zone in which the work-piece is subjected to a three-dimensional stress state. This is represented as a stress tensor with nine components or six independent components of which three shear stress components contribute to the plastic flow of the material while the hydrostatic components decide the workability. For example, if the hydrostatic components are tensile, any weak interface in the material will open up and cause internal fractures. For good SOS workability, therefore, the hydrostatic components should be essentially compressive. The SOS is controlled by the nature of the applied stress and the geometry of the deformation zone both of which are different for different metal working processes [3]. The SOS workability is thus specific to the mechanical working process and is independent of the material behavior. For example, it may be optimized in rolling by roll pass design, in forging by preform (blocking die) design and in extrusion by the design of the geometry of the die cavity. For a given geometry of the component, the available variations for the roll pass design or forging preform design are restricted. However, in extrusion, for a given container geometries and a product geometry there is considerable scope for innovation in the die design and thus die geometry like shear, conical, parabolic and streamlined dies are developed [4]. A suitable die design may be selected for a controlled SOS in the deformation zone. For example, in case of difficult-to-work materials like metal matrix composites or PM superalloys, streamlined die design for extrusion is recommended since this avoids rigid body rotation and ensures hydrostatic compressive state of stress in the die cavity. One of the accepted methods of die design for metal working processes is to use CAD/CAM techniques [5] incorporating a realistic simulation model so that expensive and time consuming shop floor trials are minimized.

Commercial FEM codes are available for this purpose and these accept experimental constitutive equations for a given work-piece material.

1.1.2 Intrinsic Workability

The intrinsic workability depends upon the initial microstructure as decided by the alloy chemistry and prior processing history and its response to the applied temperature, strain rate and strain in processing. This response is embedded *implicitly* in the flow stress variation with temperature, strain rate and strain and is represented mathematically as a constitutive equation. However, as a part of the *explicit* response of the material to the imposed process parameters, certain microstructural changes (mechanisms) occur within the material and these will have to be characterized. For example, under certain conditions, the response may be in terms of microstructural damage as the flow may be unstable or localized. Alternately, the microstructure may undergo a favorable reconstitution like dynamic recrystallization. For obtaining good intrinsic workability, it is essential to choose processing conditions that avoid microstructural damage and instability during processing.

The shapes of stress-strain curves implicitly contain information related to the mechanisms of hot deformation. For example, flow softening type of stress-strain behavior with an initial peak stress or oscillations suggests dynamic recrystallization (DRX). However similar stress-strain behavior can also be due to flow instability. Likewise, DRX may occur in cases where the behavior is steady state. It is therefore not advisable to conclude on the deformation mechanism from the shapes of the stress-strain curves alone.

One of the early attempts [6] to evaluate the mechanisms of hot working was to use a kinetic rate equation basically of the type:

$$\dot{\varepsilon} = A\sigma^{n} \exp(-Q/RT) \qquad (1)$$

where $\dot{\varepsilon}$: strain rate, σ: flow stress, Q: activation energy, R: gas constant, T: temperature, A and n: constants. In order to fit the experimental data more closely and over a wider range, several functions of σ like the hyperbolic sine function have been used [6] but apparently these do not have any other significance. For use in finite element models, simple polynomial fits are found to be more convenient mathematically. On the basis of an apparent activation energy, deformation mechanisms may be evaluated and microstructural correlations obtained with a temperature compensated strain rate parameter, Z defined as:

$$Z = \dot{\varepsilon} / A \exp(Q / RT) = \sigma^{n} \qquad (2)$$

The kinetic rate equation (Eq.1) is valid over a narrow range of temperature and strain rate, and when considered over a wide range n and Q become temperature and strain rate dependent. The kinetic analysis is applicable for pure metals and dilute alloys but when extended to commercial alloys with complex microstructures, the apparent activation energy values become too complex to interpret in terms of a single mechanism. Further, the kinetic model does not specifically lead to optimization of intrinsic workability nor can it be applied universally for microstructural control without knowing the specific ranges or domains where it is valid.

Frost and Ashby [7] were the first to represent the materials response in the form of a Deformation Mechanism Map. These are plots of normalized stress vs. homologous temperature showing the area of dominance of each flow mechanism, calculated using fundamental parameters. The emphasis in the Deformation Mechanism Maps has been essentially on the creep mechanisms applicable to lower strain rates and the maps are very useful for alloy design. However, mechanical processing is done at strain rates orders of magnitude higher than those observed during creep deformation and therefore involves different microstructural regimes. Considering strain rate as one of the direct variables and temperature as the other, Raj [8] extended the concept of Ashby's maps to construct a processing map which is shown schematically in Fig. 1.1. The Raj map represents the limiting conditions for two damage mechanisms: (i) cavity formation at hard particles in a soft matrix occurring at lower temperatures and higher strain rates, and (ii) wedge cracking at grain boundary triple junctions occurring at higher temperatures and lower strain rates. At very high strain rates, a regime representing adiabatic heating was identified. In principle, there is always a region which may be termed "safe" for processing where neither of the two damage mechanisms nor adiabatic heating occurs. Using an atomstic approach, processing maps were developed by Raj for pure metals as well as dilute alloys.

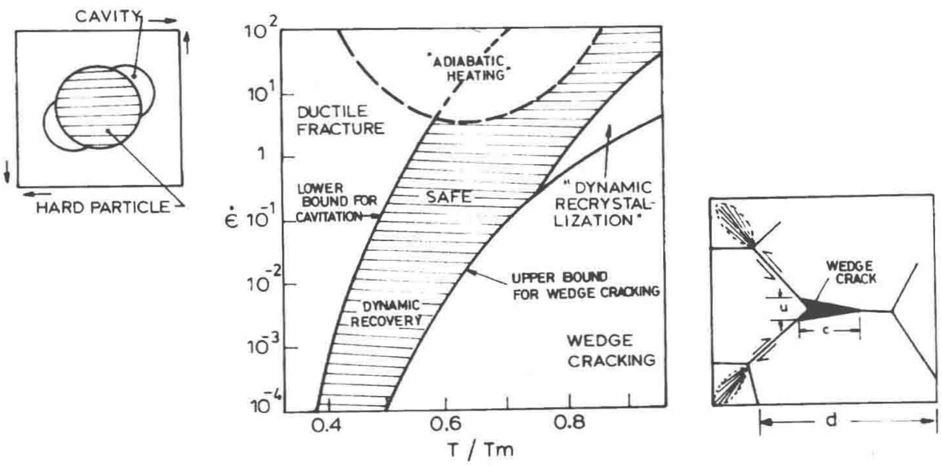


Fig. 1.1 Raj map for aluminum showing limiting conditions for damage nucleation (schematic) [8].

1.2 What is a Processing Map?

A processing map is an explicit representation of the response of a material, in terms of microstructural mechanisms, to the imposed process parameters and consists of a superimposition of a power dissipation and an instability map. These are developed on the basis of the Dynamic Materials Model (DMM) [9] which is essentially a continuum model using the concepts of systems engineering [10], extremum principles of irreversible thermodynamics with application to continuum mechanics of large plastic flow [11] and those describing the stability and self organization of chaotic systems [12]. DMM may be viewed as a bridge between the continuum mechanics of large plastic deformation and the development of dissipative microstructures in the material. The different aspects of processing maps and the principles and applications have been reviewed from the point of view of microstructural control [13-15].

1.2.1 Processing System

The model considers mechanical processing as a system, an example of which is shown in Fig. 1.2 with reference to the forging process. The system consists of a source of power (e.g. a hydraulic power pack), a store of power (tools like anvil, ram and die) and a dissipator of power (the workpiece). Energy is generated by the source, transmitted to the tools to store the power and transferred to the workpiece through an interface (lubricant). The workpiece itself dissipates the energy while it undergoes plastic flow to take the shape imposed by the deformation zone. The response of each of the above system elements depends upon their individual constitutive equations which should be evaluated for modeling their behavior. If the constitutive behavior of the system elements could be modeled accurately, they may be linked together such that suitable process controls may be designed for optimization. In this system, it is important to note that *power* or *energy per second* is to be considered and not energy *per se*, since the response of the system depends on how fast or slow the energy is input, bringing in "time" as an independent variable to make the system "dynamic". While the integration of all the system elements has not been achieved so far, the characteristics of the dissipator element (work-piece) are considered to be most important in designing the control system.

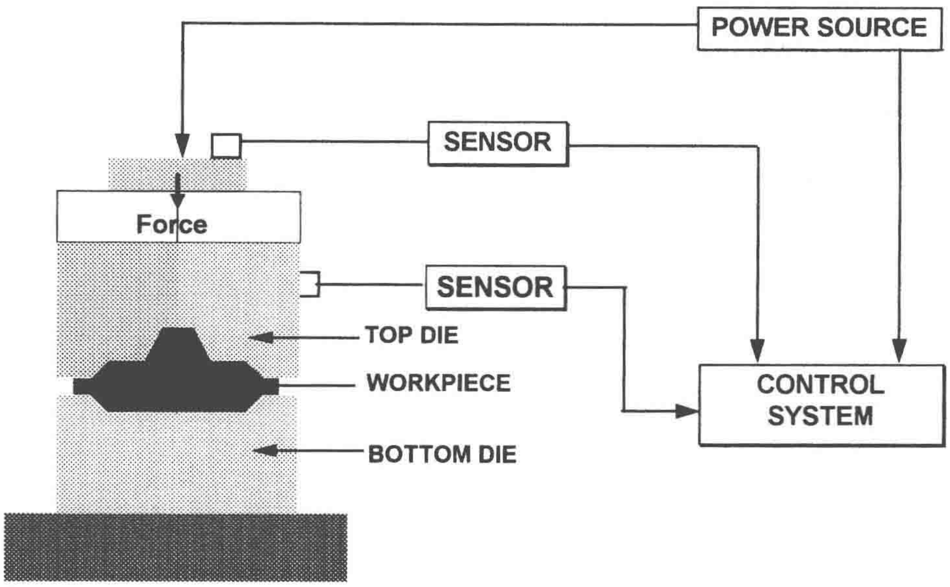


Fig. 1.2 Processing system elements with forging as an example.

The constitutive equation of the workpiece describes the manner in which energy is converted at any instant into two forms - thermal and microstructural, which are not recoverable by the system. The following definitions apply to the dissipator element of the metal processing system [13,14]:

(i) Dissipator: During hot deformation the workpiece dissipates all the power that is input to it since the stress strain curves exhibit either steady-state or flow softening behavior.

(ii) Non-linear: The relation between the flow variable (strain rate) and the effort variable (flow stress) is non-linear when considered over a wide range. The workpiece is therefore a non-linear dissipator of power.

(iii) Dynamic: Time taken to achieve a particular strain depends upon the strain rate and is an independent variable in the constitutive relation between flow stress and strain rate.

(iv) Irreversible: Since the workpiece is subjected to large plastic deformation, the system is irreversible. The principles of irreversible thermodynamics as applied to the continuum mechanics of large plastic flow are relevant in describing the entropy changes occurring within the material.

(v) Away from Equilibrium: Since the strains are not imposed in infinitesimally small increments and the strain rates normally encountered in metal forming are large, the system is considered to be away from equilibrium. It is relevant to mention that the laws of equilibrium are universal while the behavior of the system may be very specific away from equilibrium.

(vi) *Sensitivity to initial conditions:* Small changes in the chemistry, initial microstructure, temperature, strain rate and strain can cause a large change in the response of the system or lead to different mechanisms of deformation. In view of the non-linearity and sensitivity to initial conditions, the material system exhibits deterministic chaos similar to that occurring in other dissipative systems.

1.2.2 Power Dissipation Map

The non-linear dynamics of a dissipative system are generally analyzed using state space variables [3]. For a materials system with a given chemistry (composition) and processing "history", one set of physical state (or phase) space variables for hot working consists of (1) temperature of deformation, (2) strain rate, (3) strain, and (4) the dissipative state of the microstructure. The state-control variables are temperature, ram velocity and the extent of deformation. While these are related to the first three state space variables respectively, there is no simple or direct method of representing the dissipative state of microstructure. For this purpose, the extremum principles of irreversible thermodynamics of the quasi-static processes of large plastic deformation are helpful. Ziegler [11] has shown that the behaviour of such a system follows the principle of maximum rate of entropy production, which is equivalent to the principles of least irreversible force or least velocity corresponding to the velocity and force spaces respectively. At a given temperature in the hot working regime, the rate of dissipation work (power) is directly proportional to the rate of internal entropy production [9] which is always positive since the process is irreversible.

$$P = \bar{\sigma} \cdot \dot{\bar{\varepsilon}} = \theta \cdot \frac{d^{(i)}S}{dt} \geq 0 \qquad (1)$$

where $\bar{\sigma}$ is the effective stress, $\dot{\bar{\varepsilon}}$ is the effective strain rate, θ is the temperature and $\frac{d^{(i)}S}{dt}$ is the rate of internal entropy production. The total rate of entropy production consists of two complementary parts [10]. The first part (generally larger) consists of "conduction entropy" which is due to the conduction of heat from where it is generated (due to plastic flow) to the colder parts of the body. The second part is due to a microstructural dissipation which lowers the flow stress for plastic flow (dislocation movement). Ziegler [11] represented these two in terms of dissipative functions in the velocity and force space and showed that the instantaneously dissipated total power ($\bar{\sigma}.\dot{\bar{\varepsilon}}$) is given by:

$$P = \int_0^{\dot{\bar{\varepsilon}}} \bar{\sigma}.d\dot{\bar{\varepsilon}} + \int_0^{\bar{\sigma}} \dot{\bar{\varepsilon}}.d\bar{\sigma} = G + J \qquad (2)$$

where $\bar{\sigma}$ is the effective stress and $\dot{\bar{\varepsilon}}$ is the effective strain rate. In terms of physical systems terminology [11], the first integral is called G content and the second one a J co-content since it is a complementary part of G content. The constitutive equation decides the relative values of power dissipation through the heat conduction and microstructural dissipation since the origin of viscoplasticity is in the microstructural dissipation. For plastically deforming materials, the power law:

$$\bar{\sigma} = K(T, \bar{\varepsilon}, \dot{\bar{\varepsilon}})\dot{\bar{\varepsilon}}^{m(T,\bar{\varepsilon},\dot{\bar{\varepsilon}})} \qquad (3)$$

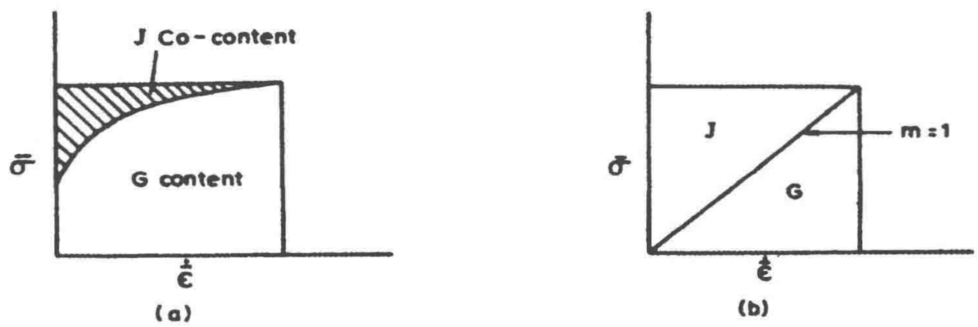


Fig. 1.3 (a) Schematic representation of the constitutive equation in a non-linear power dissipator (b) Ideal linear dissipator.