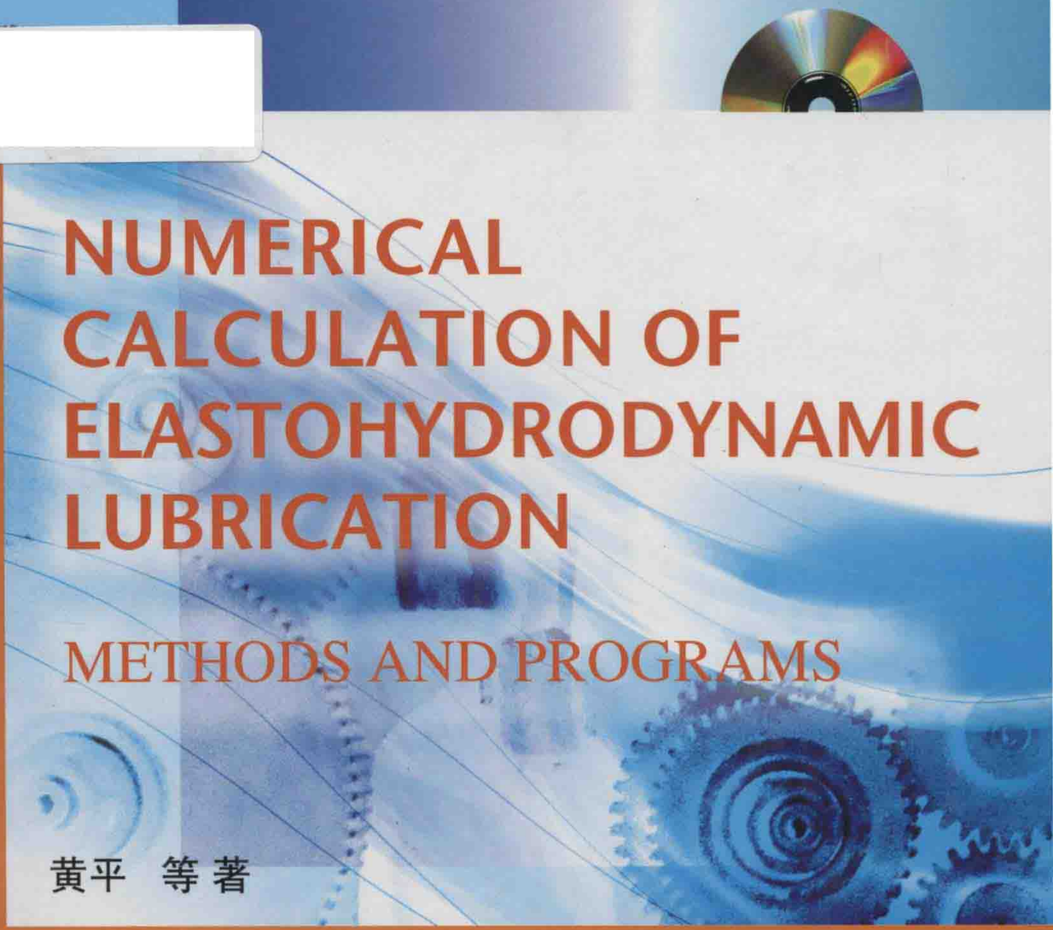

NUMERICAL CALCULATION OF ELASTOHYDRODYNAMIC LUBRICATION

METHODS AND PROGRAMS

黄平　等著

弹性流体动压润滑

——方法与程序

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内 容 简 介

本书介绍了弹性流体动压润滑（弹流润滑）的数值计算方法和程序。全书共分15章，第1~3章分别是：弹性流体动压润滑基本方程、弹性变形数值计算方法和能量方程数值计算方法，这3章的内容是后面各章的基础；第4~10章包括线接触、点接触、椭圆接触弹流问题的等温解和热解的内容，其中第9章分析了速度方向与主轴方向不一致时的椭圆接触弹流问题，这些章节是弹流润滑计算中最基本、最重要的内容；第11~15章分别介绍了脂润滑、双电层效应、非稳态、粗糙度影响、微极流体的弹流润滑计算，这些计算内容是在对前面程序的简单修改的基础上实现的。

本书可以作为机械设计与理论专业研究生及相关专业师生的教材或教学参考书，也可供从事弹流润滑计算分析与研究的工程技术人员参考。

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Preface

The book is a companion book of *Numerical Calculation of Elastohydrodynamic Lubrication: Methods and Programs* [2] written by the author and his graduate students. Since its Chinese publication, it has been widely approved, so we felt necessary to write a book similar to that in elastohydrodynamic lubrication (EHL).

The present book is based on numerical methods and programs of EHL and it describes the calculation of isothermal and thermal EHL problems in line contact, point contact, and elliptical contact in detail. In addition, numerical calculations of some special problems of EHL, such as velocities in different directions, grease lubrication, electric double layer effect, unsteady state, rough surface, and micro-polar fluid are introduced as well.

EHL calculation has a very close relationship with practical engineering. For example, EHL exists in rolling contact bearings and gears. However, compared with calculation of hydrodynamic lubrication, EHL calculation is more difficult. This is because the Reynolds equation, elastic deformation equation, viscosity pressure equation, and energy equation should be solved simultaneously. As EHL solutions are always obtained through iterations, if the iteration method is inappropriately chosen, it often causes the calculation process to be divergent. In this book, we have chosen the more reliable iteration methods and have checked and confirmed that all the programs and examples provided give satisfactory results.

However, it must be noted that because of the complexities of EHL problems, an EHL program is commonly not universal, that is, when the working conditions change, the program may not be able to get a converged or satisfied solution when used directly. In such cases, the reader should note the following conditions.

1. **Divergence for given accuracy:** There always exist some numerical errors in numerical calculation. If the accuracy given in the program cannot be met for some EHL problems, for example, double precision is not used, the calculation process may repeated at a lower level of accuracy. However, it is not really divergent. In such cases, although the accuracy does not meet the requirement, the result can be still considered as a converged solution so that the solution is useful.
2. **No convergence due to limitation of iteration number:** In any iteration program, in addition to level of accuracy, in order not to calculate infinitely, a number must be set to limit the iteration loops. Therefore, although the iteration may have begun to converge, but the limitation of the iteration number has been reached, the calculation will be stopped. In this case, although the result is not convergent, the solution is available. If readers need a more accurate solution, he or she can increase the limitation of number of iterations.
3. **Divergence for small calculation region:** The calculation region of EHL is generally set to be six times of the half width of the contact zone, where the inlet zone is about four times the half-width of the contact zone and the exit zone is about 1.5–2 times. Such a width generally meets the requirement to solve typical EHL problems. However, for some EHL problems, such as light-load, high-speed, high-viscosity, or non-Newtonian fluids, the length may be not large enough to bring the convergent result. For such a situation, please increase the calculation region so as to get a converged solution. However, this may usually bring loss of accuracy and the loss cannot be compensated by the approach to increase the node number.
4. **Divergence for unsuitable problems:** The maximum contact pressure of EHL commonly is between 0.2 and 0.8 GPa in the line or point contacts. If the maximum pressure is very low or very high, the lubrication state may tend to be a hydrodynamic lubrication or the film thickness may be too thin. In such situations, the present EHL calculation programs are not suitable anymore. In such cases, if the reader cannot obtain a convergent solution, he or she must find an appropriate program suitable for his own problem.

Readers must note that most EHL calculation programs are not universal. It is necessary for a user to be familiar with the EHL properties, and modify the corresponding statements in the program accordingly to satisfy the calculation requirements. However, this does not affect the role of the book as it shares the same purpose of the book *Numerical Calculation of Elastohydrodynamic Lubrication: Methods and Programs*. That is, the authors hope to establish an EHL calculation platform so as to avoid repeated EHL calculations and help users to complete EHL calculations and obtain solutions more quickly.

This book is also written by Lai Tianmao (Section 2.3, Chapters 8 and 10), Yang Qianqian (Chapter 4), Wang Yazhen (Chapters 6 and 9), Chen Yingjun (Chapter 11), Bai Shaoxian and Zuo Qiyang (Chapter 12) and Zhang Wei (Chapter 13). Ping Huang is responsible for the remaining chapters and has edited the whole book. Due to time and scope limitations, errors and problems cannot be avoided entirely in the book. We hope that readers' invaluable comments will help to further improve the subsequent editions of this book.

Huang Ping

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of Technology
31 May,2015

Nomenclature

a	is the half-width in the point or ellipse contact region in the x direction (the velocity direction). For the point contact, $a=b=(3wR_x/2E)^{1/3}$
a	is the characteristic length in the x direction
b	is the half-width in the line contact region, $b=\sqrt{8wR/\pi E}$
b	is the radius of the elliptical contact region in the y direction (cross the velocity direction)
b	is the characteristic length in the y direction
c_p	is the specific heat capacity of lubricant
c	is the pressure viscosity coefficient of Cameron equation, which is approximate to $\alpha/15$
c_1 and c_2	are the specific heats of the materials on the up and down surfaces
D	is the density–temperature coefficient, $D=-0.00065\ \mathrm{K}^{-1}$
D_{ij}^{kl}	are the two-dimensional elastic deformation stiffnesses
e_k	is the ellipse rate, $e_k=R_x/R_y$
E	is the equivalent elastic modulus of the two contact surface materials, $1/E=1/2\left(1-\nu_1^2/E_1+1-\nu_2^2/E_2\right)$
E_1 and E_2	are the elastic modulus corresponding to the up and down surfaces of the materials
G^*	is a material parameter, $G^*=\alpha E$
h	is the lubricant film thickness
h_1	is the minimum lubricant thickness film

h_0	is the central film thickness to be determined based on the load balancing condition
H	is the nondimensional film thickness, for the line contact $H = hR/b^2$, for the point or elliptical contact $H = hR_x/a^2$
H_0	is the nondimensional central film thickness
J	is the mechanical equivalent of heat
J	is the inertia factor of micropolar fluid
k	is the thermal conductivity of lubricant
k_1 and k_2	are the thermal conductivities of the up and down surfaces
K_{ij}	is the one-dimensional elastic deformation stiffness
l	is the characteristic length of micropolar fluid
L	is the nondimensional characteristic length of micro-polar fluid
m	is the number of nodes in the y direction
n	is the number of nodes in the x direction
n	is the rheological index, $\leqslant 1$
N	is the coupling coefficient of micropolar fluid
p	is the pressure
p_0	is the pressure–viscosity coefficient, $p_0 = 1.96 \times 10^8$
p_{H}	is Hertz contact pressure, for the line contact $p_{\mathrm{H}} = 2w/\pi b$, for the point contact $p_{\mathrm{H}} = 3w/2\pi a^2$, and for the elliptic contact $p_{\mathrm{H}} = 3w/2\pi ab$
P	is the nondimensional pressure, $P = p/p_{\mathrm{H}}$
P_{tr}	the deformation coefficient of elliptical contact,$P_{\mathrm{tr}} = 3wR_x/\pi^2 a^2 bE$
R	is the equivalent radius of curvature, $1/R = 1/R_1 \pm 1/R_2$; for the outer contact, take + and for the inner contact, take −
R_1 and R_2	are the integrated radii of curvature of the up and down surfaces in the line or point contact
R_x and R_y	are the equivalent radii of curvature in the x and y direction of both surfaces
s	is the sliding–rolling ratio, $s = (u_1 - u_2)/u_s$
t	is the time
T	is the temperature
T	is one of the nondimensional time, $T = Ut/b$
T_0	is the initial temperature, here $T_0 = 303$ K
T^*	is the nondimensional temperature, $T^* = T/T_0$
$u_{\mathrm{s}} = u_1 + u_2/2$	is the average velocity of the up and down surfaces along the x direction
u	is the fluid velocity component in the x direction
u_1 and u_2	are the tangential velocities respectively to the up and down surface in the x direction
U^*	is the velocity parameters, for the line contact $U^* = \eta_0 u_{\mathrm{s}}/ER$, for the point or ellipse contact $U^* = \eta_0 u_{\mathrm{s}}/ER_x$
v	is the fluid velocity component in the y direction
v_{s}	is the average speed in y direction along the two surfaces, $v_{\mathrm{s}} = (v_1 + v_2)/2$

v_1 and v_2	are the tangential velocities respectively to the up and down surface in the y direction
$v(x)$	is the displacement of the elastic deformation generated by pressure
V^*	is the velocity parameter, $V^* = \eta_0 v_s / ER_x$
w	is the load, for line contact w is the load per unit length of the contact, and for the point or ellipse contact w is the total load
w	is the fluid velocity component in the z direction
W	is the nondimensional load, for the line contact $W = \pi/2$, for the point contact $W = 2\pi/3$, and for the elliptical contact $W = 2\pi b/3a$
W^*	is the load parameters, for the line contact $W^* = w/ER$, for the point or elliptical contact $W^* = w/ER_x^2$
x	is the coordinate which is in the same direction of the main speed
x_0	is the inlet coordinate
x_e	is the outlet coordinate
X	is nondimensional coordinate of x, for the line contact $X = x/b$, for the point contact $X = x/a$
X_0 and X_e	are the nondimensional coordinates of the inlet and outlet
y	is the coordinate which is vertical to the main speed
Y	is the nondimensional coordinate in the y direction, $Y = y/a$
z	is the coordinate in the film thickness direction
z	is the coefficient of viscosity pressure formula, for a mineral oil it is generally $z = 0.68$
Z	is the nondimensional coordinate of z, for the line contact $Z = zR/b^2$, for the point or ellipse contact $Z = zR_x/a^2$
α	is the pressure coefficient of oil or base oil of grease in Barus viscosity–pressure formula
α	is the length proportional factor of the contact ellipse, $\alpha = a/b$
α_T	is the coefficient in the density–temperature equation, its unit is $°\mathrm{C}^{-1}$
β	is the viscosity–temperature coefficient in the Barus formula, for oil it usually is $0.03°\mathrm{C}^{-1}$
ΔX	is the equally divided nondimensional increment between the nodes of the mesh, $\Delta X = X_i - X_{i-1}$
ε	is the Reynolds coefficient, for oil $\varepsilon = \rho^* H^3/\eta^* \lambda$, for grease $\varepsilon = \lambda\left(H^{(2+1/n)}/\phi^{*1/n}\right)$ $\varepsilon_0 = \varepsilon_{i-1/2,j} + \varepsilon_{i+1/2,j} + \varepsilon_{i,j-1/2} + \varepsilon_{i,j+1/2}$ $\varepsilon_{i\pm 1/2} = \frac{1}{2}(\varepsilon_i + \varepsilon_{i\pm 1})$ $\varepsilon_{i\pm 1/2,j} = \frac{1}{2}\left(\varepsilon_{i,j} + \varepsilon_{i\pm 1,j}\right)$
ϕ	is the plastic viscosity

ϕ_0	is the plastic viscosity of grease at the normal pressure and at the room temperature
ϕ^*	is the nondimensional viscosity, $\phi^* = \phi/\phi_0$
γ	is the material constant of micropolar fluid
λ	is the parameter of the coefficients ε, for the line contact $\lambda = 12\eta_0 u_s R^2/b^3 p_H$, for the point or elliptical contact $\lambda = 12\eta_0 u_s R_x^2/a^3 p_H$, for grease $\lambda = {p_H}^{1/n} b^{2+1/n}/2u_s(2+1/n)R^{(1+1/n)}2^{1/n}{\phi_0}^{1/n}$
η	is the viscosity of the lubricant
η_0	is the viscosity of the lubricant at $p = 0$ and $T = T_0$
η^*	is the nondimensional viscosity of the lubricant, $\eta^* = \eta/\eta_0$
μ	is the viscosity of Newtonian fluid of micropolar fluid
ρ	is the lubricant density
ρ_0	is the density of lubricant at $p = 0$ and $T = T_0$
ρ^*	is the nondimensional density of lubricant, $\rho^* = \rho/\rho_0$
ρ_1 and ρ_2	are the material densities of the up and down surfaces
χ	is the rotary viscosity of micropolar fluid
ω_1, ω_2, and ω_3	are the rotational angular velocities of micropolar fluid respectively in the x, y, and z directions
ν_1 and ν_2	are the Poisson ratios of the two surface materials

Note: If the above symbols are stated in the text otherwise, the content described here is no longer valid.

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