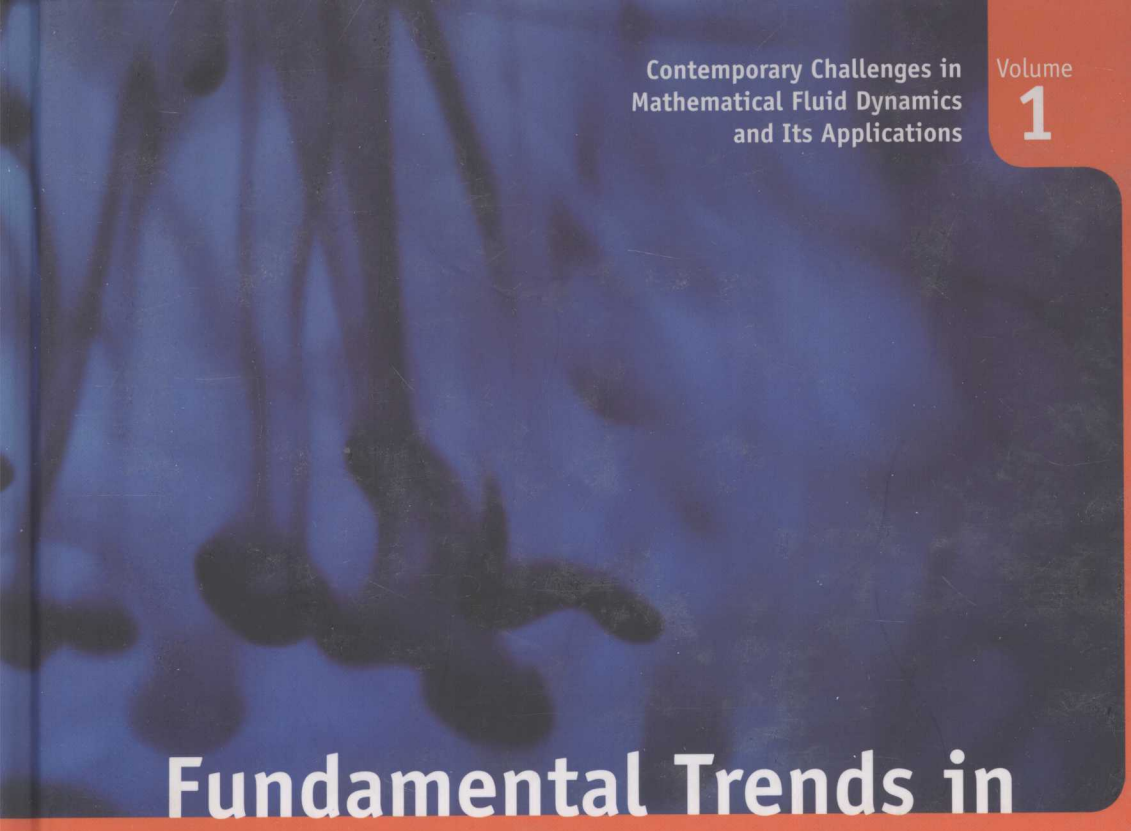

Contemporary Challenges in Mathematical Fluid Dynamics and Its Applications

Volume 1

Fundamental Trends in Fluid-Structure Interaction

Giovanni P Galdi
Rolf Rannacher
Editors

World Scientific

Contemporary Challenges in Mathematical Fluid Dynamics and Its Applications

Volume 1

Fundamental Trends in Fluid-Structure Interaction

Editors

Giovanni P Galdi
University of Pittsburgh, USA

Rolf Rannacher
Heidelberg University, Germany

World Scientific

NEW JERSEY · LONDON · SINGAPORE · BEIJING · SHANGHAI · HONG KONG · TAIPEI · CHENNAI

Published by

World Scientific Publishing Co. Pte. Ltd.

5 Toh Tuck Link, Singapore 596224

USA office: 27 Warren Street, Suite 401-402, Hackensack, NJ 07601

UK office: 57 Shelton Street, Covent Garden, London WC2H 9HE

British Library Cataloguing-in-Publication Data

A catalogue record for this book is available from the British Library.

FUNDAMENTAL TRENDS IN FLUID-STRUCTURE INTERACTION

Contemporary Challenges in Mathematical Fluid Dynamics and Its Applications — Vol. 1

ISBN-13 978-981-4299-32-9

ISBN-10 981-4299-32-4

Typeset by Stallion Press

Email: enquiries@stallionpress.com

Printed by FuIsland Offset Printing (S) Pte Ltd. Singapore

Fundamental Trends in Fluid-Structure Interaction

Series on Contemporary Challenges in Mathematical Fluid Dynamics and Its Applications

Founder and Editor: Giovanni P. Galdi

Vol. 1 Fundamental Trends in Fluid-Structure Interaction
Editors: G. P. Galdi and R. Rannacher

PREFACE

Interaction of a fluid with a solid body is a widespread phenomenon in Nature. It occurs at different scales and in different applied disciplines: swimming of fish, flight of an airplane, transport of material through water pipelines and blood flow in human arteries are just few significant examples.

Interestingly enough, even though the mathematical theory of the motion of bodies in a liquid is one of the oldest and most classical problems in fluid mechanics, owed to the seminal contributions of Stokes, Kirchhoff, and Thomson (Lord Kelvin), only very recently have mathematicians become interested in a systematic study of the basic problems related to fluid-structure interaction, from both analytical and numerical viewpoints.

In fact, contributions to the subject are nowadays growing at such a fast pace that it is highly desirable to have an updated information on the state of the art.

This book is a unique collection of fundamental papers written by world renowned experts aimed at furnishing the highest level of development in several significant areas of fluid-structure interaction.

Specifically, the contribution of Th. Dunne *et al.* is devoted to a numerical analysis of the problem of a viscous fluid interacting with a deformable elastic body. In particular, it reviews the pros and cons of whether it is more appropriate to use a Lagrangean or Eulerian formulation.

The article by V. Heuveline and P. Wittwer provides a detailed survey on the progress over the recent years made on the problem of the interaction of an exterior Navier–Stokes flow with a rigid structure at low Reynolds number.

The paper of M. Razzaq *et al.* centers around the use of the Arbitrary Lagrangean Eulerian formulation in the numerical resolution of the problem of fluid-solid interaction. As an application, the influence of endovascular stent implantation onto cerebral aneurysm hydrodynamics is investigated.

J. San Martin and M. Tucsnak consider the coupled problem of the interaction of a fluid with a number of rigid bodies. Their contribution surveys the fundamental mathematical analysis that is at the basis of the problem.

Finally, the article of A. Quarteroni presents some of the basic models that are used to describe blood flow dynamics in local arterial environments and to predict the vessel wall deformation in compliant arteries.

We hope that the diversity of the topics along with the different approaches will allow the reader to have a global and updated view on the latest results on the subject and on the relevant open questions.

Editors

Giovanni P. Galdi and Rolf Rannacher

CONTENTS

CHAPTER 1

NUMERICAL SIMULATION OF FLUID-STRUCTURE INTERACTION BASED ON MONOLITHIC VARIATIONAL FORMULATIONS

Th. Dunne, R. Rannacher* and Th. Richter

Institute of Applied Mathematics
University of Heidelberg, INF 293/294
D-69120 Heidelberg, Germany
**rannacher@iwr.uni-heidelberg.de*

The dilemma in modeling the coupled dynamics of fluid-structure interaction (FSI) is that the fluid model is normally based on an Eulerian perspective in contrast to the usual Lagrangian formulation of the solid model. This makes the setup of a common variational description difficult. However, such a variational formulation of FSI is needed as the basis of a consistent Galerkin discretization with *a posteriori* error control and mesh adaptation, as well as the solution of optimal control problems based on the Euler–Lagrange approach. This article surveys recent developments in the numerical approximation of FSI problems based on "monolithic" variational formulations. The modeling is based either on an arbitrary Lagrangian–Eulerian (ALE) or a fully Eulerian–Eulerian (Eulerian) description of the (incompressible) fluid and the (elastic) structure dynamics. These global one-field formulations constitute a strongly implicit coupling of the dynamics of fluid and structure which, in contrast to the commonly used weakly coupled two-field formulations, provides the basis for a robust and efficient solution process. In this context a fully consistent treatment of mesh adaptation (DWR method) and optimal control ("all-at-once" approach) becomes possible within a Galerkin finite element discretization.

1. Introduction

Computational fluid dynamics and computational structure mechanics are two major areas of numerical simulation of physical systems. With the introduction of high performance computing it has become possible to tackle

systems with a coupling of fluid and structure dynamics. General examples of such fluid-structure interaction (FSI) problems are flow transporting elastic particles (particulate flow), flow around elastic structures (airplanes, submarines) and flow in elastic structures (haemodynamics, transport of fluids in closed containers). In all these settings the dilemma in modeling the coupled dynamics is that the fluid model is normally based on an Eulerian perspective in contrast to the usual Lagrangian approach for the solid model. This makes the setup of a common variational description difficult. However, such a variational formulation of FSI is needed as the basis of a consistent approach to residual-based *a posteriori* error estimation and mesh adaptation as well as to the solution of optimal control problems by the Euler–Lagrange method. This is the subject of the present paper, which is largely based on the doctoral dissertation of the first author Dunne[22] and the survey article Dunne and Rannacher.[24]

Combining the Eulerian and the Lagrangian setting for describing FSI involves conceptional difficulties. On one hand the fluid domain itself is time-dependent and depends on the deformation of the structure domain. On the other hand, for the structure the fluid boundary values (velocity and the normal stress) are needed. In both cases values from one problem are used for the other, which is costly and can lead to a drastic loss of accuracy. A common approach to dealing with this problem is to separate the two models, solve each separately, and so converge iteratively to a solution which satisfies both together with the interface conditions (Fig. 1). Solving the separated problems serially multiple times is referred to as a "partitioned (or segregated) approach".

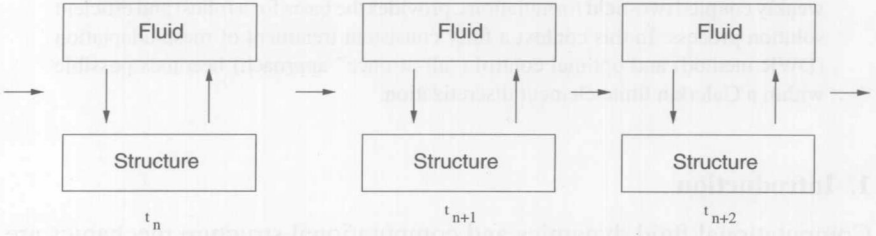

Fig. 1. Partitioned approach, Lagrangian and Eulerian frameworks coupled.

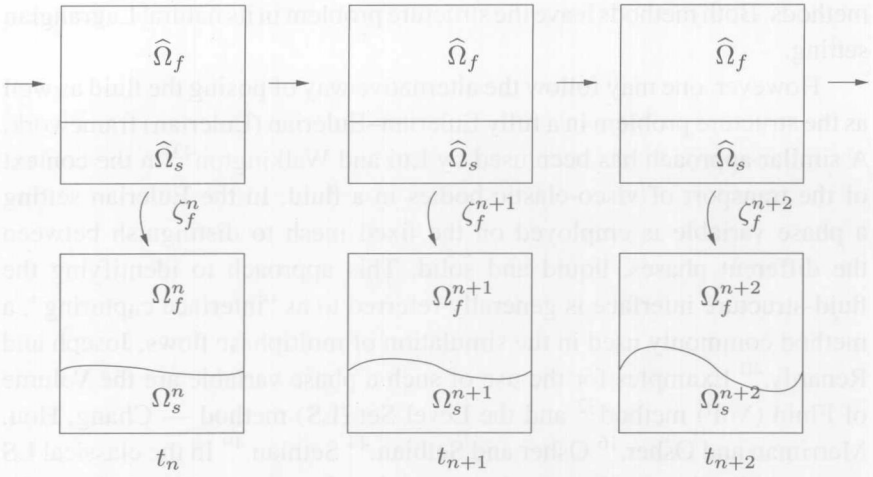

Fig. 2. Transformation approach, both frameworks Lagrangian.

A partitioned approach does not contain a variational equation for the fluid-structure interface. To achieve this, usually an auxiliary unknown coordinate transformation function T_f is introduced for the fluid domain. With its help the fluid problem is rewritten as one on the transformed domain which is fixed in time. Then, all computations are done on the fixed reference domain and as part of the computation the auxiliary transformation function T_f has to be determined at each time step. Figure 2 illustrates this approach for the driven cavity problem considered in Sec. 8. Such so-called "arbitrary Lagrangian–Eulerian" (ALE) methods are used in Huerta and Liu,[35] Wall,[58] Hron and Turek,[33] and corresponding transformed space-time finite element formulations in Tezduyar, Behr and Liou[51,52] and Tezduyar, Sathe, Stein and Aureli.[53] For other ways of dealing with implicit coupling in FSI models, we refer to Vierendeels[56] and Wall, Gerstenberger, Gamnitzer, Forster and Ramm.[59] Computational comparisons of partitioned and monolithic approaches have recently been made in Heil, Hazel and Boyle.[30]

Both, the partitioned and the transformation approach to overcome the Euler–Lagrange discrepancy explicitly track the fluid-structure interface by mesh adjustment and are generally referred to as "interface tracking"

methods. Both methods leave the structure problem in its natural Lagrangian setting.

However, one may follow the alternative way of posing the fluid as well as the structure problem in a fully Eulerian–Eulerian (Eulerian) framework. A similar approach has been used by Lui and Walkington[43] in the context of the transport of visco-elastic bodies in a fluid. In the Eulerian setting a phase variable is employed on the fixed mesh to distinguish between the different phases, liquid and solid. This approach to identifying the fluid-structure interface is generally referred to as "interface capturing", a method commonly used in the simulation of multiphase flows, Joseph and Renardy.[40] Examples for the use of such a phase variable are the Volume of Fluid (VoF) method[32] and the Level Set (LS) method — Chang, Hou, Merriman and Osher,[16] Osher and Sethian,[44] Sethian.[49] In the classical LS approach the distance function has to continually be reinitialized, due to the smearing effect by the convection velocity in the fluid domain. This makes the use of the LS method delicate for modeling FSI problems particularly in the presence of cornered structures. To cope with this difficulty, in Dunne[21,22] a variant of the LS method, the Initial Position (IP) method, has been proposed that makes reinitialization unnecessary and which easily copes with cornered structures. This approach does not depend on the specific structure model.

The key variable in structure dynamics is the deformation, and since this depends on the deflection, it is understandable why structure dynamics is preferably described in the Lagrangian frame. The set of "initial positions" (IP set) of all structure points enables us to describe the deformations in the Eulerian frame. This set is then transported with the structure velocity in each time step. Based on this concept the displacement is now available in an Eulerian sense. Also its gradient has to be rewritten appropriately, which will be explained in Sec. 4.2. Since the fluid-structure interface will be crossing through cells, we will have to also transport the IP set in the fluid domain.

If we were to use the fluid velocity for the advection of the IP set, this would lead to entanglement of the respective displacements, which would "wreak havoc" on the interface cells. This is a known problem with LS approaches. A common way for fixing this problem has been to occasionally fix the LS field between the time steps. The problem with this approach is

that the variational formulation is no longer consistent. As an alternative, we harmonically continue the structure velocity into the fluid domain. In the fluid domain, we then use this velocity for advecting the IP set. Since an IP set is available in both domains, we can always at each point determine if it belongs to the fluid or solid part of the model.

Again this approach is similar to the LS approach. Actually, it is possible to also develop a model for FSI using the level set approach, Legay, Chessa and Belytschko.[41] But when developing a complete variational formulation the two key characteristics of the LS approach also become the main cause of concern: reinitialization and the signed distance function. Although the problem of reinitialization here can also be avoided by using an harmonically extended velocity, the trouble concerning corner approximation persists. In contrast to this, by using an initial position set, we are deforming a virtual mesh of the structure which is extended into the whole domain.

The equations we use are based on the momentum and mass conservation equations for the flow of an incompressible Newtonian fluid and the deformation of a compressible St. Venant–Kirchhoff or likewise incompressible neo-Hookean solid. The spatial discretization is by a second-order finite element method with conforming equal-order (bilinear) trial functions using "local projection stabilization", Becker and Braack.[4,5] The time discretization uses the second-order "Fractional-Step-θ" scheme originally proposed by Bristeau, Glowinski and Periaux.[13] This method has the same complexity as the Crank–Nicolson scheme but better stability properties, see Rannacher.[46]

Based on the Eulerian variational formulation of the FSI system, we use the "dual weighted residual" (DWR) method, described in Becker and Rannacher,[7–9] Becker, Heuveline and Rannacher,[6] Bangerth and Rannacher,[2] Braack and Richter,[10] to derive "goal-oriented" *a posteriori* error estimates. The evaluation of these error estimates requires the approximate solution of a linear dual variational problem. The resulting *a posteriori* error indicators are then used for automatic local mesh adaptation. The full application of the DWR method to FSI problems requires a Galerkin discretization in space as well as in time. Due to the use of a difference scheme in time, in this paper we are limited to "goal-oriented" mesh adaptation in computing steady states or (somewhat heuristically) to quasi-steady states within the

time stepping process. The incorporation of automatic time-step control will be the subject of forthcoming work.

The method for computing FSI described in this paper is validated at a stationary model problem that is a lid-driven cavity involving the interaction of an incompressible Stokes fluid with a linearized incompressible neo-Hookean solid. Then, as a more challenging test the self-induced oscillation of a thin elastic bar immersed in an incompressible fluid is treated (FLUSTRUK-A benchmark, see Hron and Turek.[34]) For this test problem, our method is also compared against a standard "arbitrary Lagrange Eulerian" (ALE) approach. The possible potential of the fully Eulerian formulation of the FSI problems is indicated by its good behavior for large structure deformations. All computations and visualizations are done using the flow-solver package GASCOIGNE[26] and the graphics package VISUSIMPLE.[57] The details on the software implementation can be found in Dunne.[21–23]

The outline of this paper is as follows. Section 2 ("Notation") introduces the basic notation for the ALE as well as the Eulerian formulation of the FSI problem and Sec. 3 ("Reference frameworks") discusses the reference frameworks, *Lagrangian* and *Eulerian*, which will be used throughout this paper. The corresponding variational formulations are developed in detail, first separately for the structure and fluid parts in Sec. 4 ("Variational formulations of fluid and structure problems") and then for the coupled FSI problem in Sec. 5 ("Variational formulations of the FSI problem"). Section 6 ("Discretization") describes the discretization in space and time as well as the techniques for solving the algebraic systems and for evaluating directional derivatives. The derivation of *a posteriori* error estimates and strategies for mesh adaptation is explained in Sec. 7 ("Mesh adaptation"). In Sec. 8 ("Numerical test 1: elastic flow cavity") the newly proposed Eulerian method is validated at a stationary test problem "elastic flow cavity". Then, Sec. 9 ("Numerical test 2: FSI benchmark FLUSTRUK-A") contains the results obtained by the two approaches, ALE and Eulerian, for the solution of the benchmark problem FLUSTRUK-A (oscillations of a thin elastic bar) for various combinations of material models and flow conditions. The paper is closed by Sec. 10 ("Summary and future development") which gives a summary and points at some directions of ongoing and future research on the basis of the approaches described in this paper.

2. Notation

We begin with introducing some notation which will be used throughout this paper. By $\Omega \subset \mathbb{R}^d$ ($d = 2$ or $d = 3$), we denote the domain of definition of the FSI problem. The domain Ω is supposed to be *time independent* but to consist of two possibly time-dependent subdomains, the fluid domain $\Omega_f(t)$ and the structure domain $\Omega_s(t)$. Unless needed, the explicit time dependency will be skipped in this notation. The boundaries of Ω, Ω_f, and Ω_s are denote by $\partial\Omega$, $\partial\Omega_f$, and $\partial\Omega_s$, respectively. The common interface between Ω_f and Ω_s is $\Gamma_i(t)$, or simply Γ_i.

The initial structure domain is denoted by $\widehat{\Omega}_s$. Spaces, domains, coordinates, values (such as pressure, displacement, velocity) and operators associated to $\widehat{\Omega}_s$ (or $\widehat{\Omega}_f$) will likewise be indicated by a "hat".

Partial derivatives of a function f with respect to the i-th coordinate are denoted by $\partial_i f$, and the total time-derivative by $d_t f$. The divergence of a vector and tensor is written as $\operatorname{div} f = \sum_i \partial_i f_i$ and $(\operatorname{div} F)_i = \sum_j \partial_j F_{ij}$. The gradient of a vector valued function v is the tensor with components $(\nabla v)_{ij} = \partial_j v_i$.

By $[f]$, we denote the jump of a (possibly discontinuous) function f across an interior boundary, where n is always the unit vector n at points on that boundary.

For a set X, we denote by $L^2(X)$ the Lebesque space of square-integrable functions on X equipped with the usual inner product and norm

$$(f, g)_X := \int_X fg\, dx, \quad \|f\|_X^2 = (f, f)_X,$$

respectively, and correspondingly for vector- and matrix-valued functions. Mostly the domain X will be Ω, in which case we will skip the domain index in products and norms. For Ω_f and Ω_s, we similarly indicate the associated spaces, products, and norms by a corresponding index "f" or "s".

We will generally use roman letters, V, for denoting spaces of functions depending only on spatial variables and calligraphic letters, $\mathcal{V}$, for spaces of functions depending additionally on time. Let $L_X := L^2(X)$ and $L_X^0 := L^2(X)/\mathbb{R}$. The functions in L_X (with $X = \Omega$, $X = \Omega_f(t)$, or $X = \Omega_s(t)$) with first-order distributional derivatives in L_X make up the Sobolev space $H^1(X)$. Further, $H_0^1(X) = \{v \in H^1(X) : v_{|\partial X_D} = 0\}$, where ∂X_D is that part of the boundary ∂X at which Dirichlet boundary conditions are

imposed. Further, we will use the function spaces $V_X := H^1(X)^d$, $V_X^0 := H_0^1(X)^d$, and for time-dependent functions

$$\mathcal{L}_X := \mathcal{L}^2[0,T;L_X], \quad \mathcal{V}_X := \mathcal{L}^2[0,T;V_X] \cap \mathcal{H}^1[0,T;V_X^*],$$
$$\mathcal{L}_X^0 := \mathcal{L}^2[0,T;L_X^0], \quad \mathcal{V}_X^0 := \mathcal{L}^2[0,T;V_X^0] \cap \mathcal{H}^1[0,T;V_X^*],$$

where V_X^* is the dual of V_X^0, and $\mathcal{L}^2$ and $\mathcal{H}^1$ indicate the corresponding properties in time. Again, the X-index will be skipped in the case of $X = \Omega$, and for $X = \Omega_f$ and $X = \Omega_s$ a corresponding index "f" or "s" will be used.

3. Reference Frameworks

In modeling the variation of a spatial continuum in time two approaches are commonly used. The *Lagrangian* or *material* framework and the *Eulerian* or *spatial* framework. Both approaches have the simple goal of describing how a certain scalar quantity of interest $f : \mathbb{R}^d \times I \to \mathbb{R}$ changes in space and with time. The choice of the "reference point" of the value f is what distinguishes the two frameworks. We denote by $x \in \mathbb{R}^d$ and $t \in I$ the spatial and temporal coordinates, respectively. The function f is assumed to be sufficiently smooth with respect to space and time.

3.1. *Lagrangian framework*

In the Lagrangian framework one observes the value at a preselected point that is moving (and possibly accelerating) steadily through space. The initial position of the point at the initial time t_0, we define as $\hat{x}$. Thus, the position of the point is a function of the initial position $\hat{x}$ and time t,

$$x = x(\hat{x}, t).$$

We define the velocity v of this point as the total time derivative of its position x,

$$v(x,t) := d_t x(\hat{x},t) = \partial_t x + \hat{\nabla} x \, d_t \hat{x}. \tag{3.1}$$

Since $\hat{x}$ is the position of the point at an initial time it follows that it does not change in time, i.e., $d_t \hat{x} = 0$ and $v = \partial_t x$.

To be more precise, in the Lagrangian framework we should refer to $f(x,t)$ as $\hat{f}(\hat{x},t) := f(x(\hat{x},t),t)$. Visually one can imagine that we are

observing the value at a *material point* that was initially at the position $\hat{x}$ and is moving through space with velocity v. The total time derivative of $\hat{f}$ in the Lagrangian framework can thus be written:

$$d_t \hat{f}(\hat{x}, t) = \partial_t \hat{f}(\hat{x}, t) + \hat{\nabla} \hat{f}(\hat{x}, t)\, d_t \hat{x} = \partial_t \hat{f}(\hat{x}, t). \tag{3.2}$$

Since the Lagrangian approach describes the movement and deformation of individual particles and volumes it follows that this framework is the natural approach for modeling structure dynamics.

3.2. *Eulerian framework*

In the Eulerian framework one observes the value at a fixed point x in space. Hence this framework is also referred to as a *spatial framework*. Looking back at the Lagrangian framework one can imagine that at the point x at different times there will continuously be different material points moving through. Each such material points will have a respective initial position $\hat{x}$. Thus, the velocity v at this space-time position (x, t) is still to be understood as the velocity of the material point with the initial position $\hat{x}$, i.e., $v(x, t) := d_t x(\hat{x}, t) = \partial_t x$.

In an Eulerian framework the quantity of interest is written as $f(x, t)$ with x and t being anywhere within the permitted space-time continuum. Taking the total time derivative of f leads to

$$d_t f(x, t) = \partial_t f(x, t) + \nabla f(x, t)\, d_t x = \partial_t f(x, t) + v \cdot \nabla f(x, t). \tag{3.3}$$

The second term is referred to as the "transport" or "convection term". This term is a characteristic difference between the Eulerian and Lagrangian frameworks. In the Lagrangian framework, when the total time derivative is expanded into all its partial derivatives, there is no convective term due to the spatial parameter being constant in time. In contrast, in Eulerian frameworks convection can generally be expected in the expanded total time derivative.

The Eulerian framework presents itself as the natural approach for modeling fluid flow. This follows as a consequence that one is less interested in the individual behavior of particles and more interested in flow properties at certain spatial points in the flow domain. In viscous fluids with behavior similar to soft materials, a Lagrangian approach would be plausible.