Design and Analysis of Mechanisms

A Planar Approach

Michael J. Rider

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Preface

The intent of this book is to provide a teaching tool that features a straightforward presentation of basic principles while having the rigor to serve as basis for more advanced work. This text is meant to be used in a single-semester course, which introduces the basics of planar mechanisms. Advanced topics are not covered in this text because the semester time frame does not allow these advanced topics to be covered. Although the book is intended as a textbook, it has been written so that it can also serve as a reference book for planar mechanism kinematics. This is a topic of fundamental importance to mechanical engineers.

Chapter 1 contains sections on basic kinematics of planar linkages, calculating the degrees of freedom, looking at inversions, and checking the assembling of planar linkages. Chapter 2 looks at position analysis, both graphical and analytical, along with a vector approach, which is the author's preferred method. Chapter 3 looks at graphical design of planar linkages including four-bar linkages, slider–crank mechanisms, and six-bar linkages. Chapter 4 looks at the analytical design of the same planar linkages found in the previous chapter. Chapter 5 deals with velocity analysis of planar linkages including the relative velocity method, the instant center method, and the vector approach. Chapter 6 deals with the acceleration analysis of planar linkages including the relative acceleration method and the vector approach. Chapter 7 deals with the static force analysis of planar linkages including free body diagrams, equations for static equilibrium, and solving a system of linear equations. Chapter 8 deals with the dynamic force analysis based on Newton's law of motion, conservation of energy and conservation of momentum. Adding a flywheel to the mechanism is also investigated in this chapter. Chapter 9 deals with spur gears, contact ratios, interference, basic gear equations, simple gear trains, compound gear trains, and planetary gear trains. Chapter 10 deals with fundamental cam design while looking at different types of followers and different types of follower motion and determining the cam's profile.

There are numerous problems at the end of each chapter to test the student's understanding of the subject matter.

Appendix A discusses the basics of using the Engineering Equation Solver (EES) and how it can be used to solve planar mechanism problems. Appendix B discusses the basics of MATLAB and how it can be used to solve planar mechanism problems.

Contents

1

Introduction to Mechanisms

1.1 Introduction

Engineering involves the design and analysis of machines that deal with the conversion of energy from one source to another using the basic principles of science. Solid mechanics is one of these branches. It contains three major sub-branches: kinematics, statics, and kinetics. Kinematics deals with the study of relative motion. Statics is the study of forces and moments apart from motion. Kinetics deals with the result of forces and moments on bodies. The combination of kinematics and kinetics is referred to as dynamics. However, dynamics deals with the study of motion caused by forces and torques. For mechanism design, the desired motion is known and the task is to determine the type of mechanism along with the required forces and torques to produce the desired motion. This text covers some of the mathematics, kinematics, and kinetics required to perform planar mechanism design and analysis.

A mechanism is a mechanical device that transfers motion and/or force from a source to an output. A linkage consists of links generally considered rigid which are connected by joints such as pins or sliders. A kinematic chain with at least one fixed link becomes a mechanism if at least two other links can move. Since linkages make up simple mechanisms and can be designed to perform complex tasks, they are discussed throughout this book.

A large majority of mechanisms exhibit motion such that all the links moved in parallel planes. This text emphasizes this type of motion, which is called two-dimensional planar motion. Planar rigid body motion consists of rotation about an axis perpendicular to the plane of motion and translation in the plane of motion. For this text, all links are assumed rigid bodies.

Mechanisms are used in a variety of machines and devices. The simplest closed form linkage is a 4-bar, which has three moving links plus one fixed link and four pinned joints. The link that does not move is called the ground link. The link that is connected to the power source is called the input link. The follower link contains a moving pivot point relative to ground and it is typically considered as

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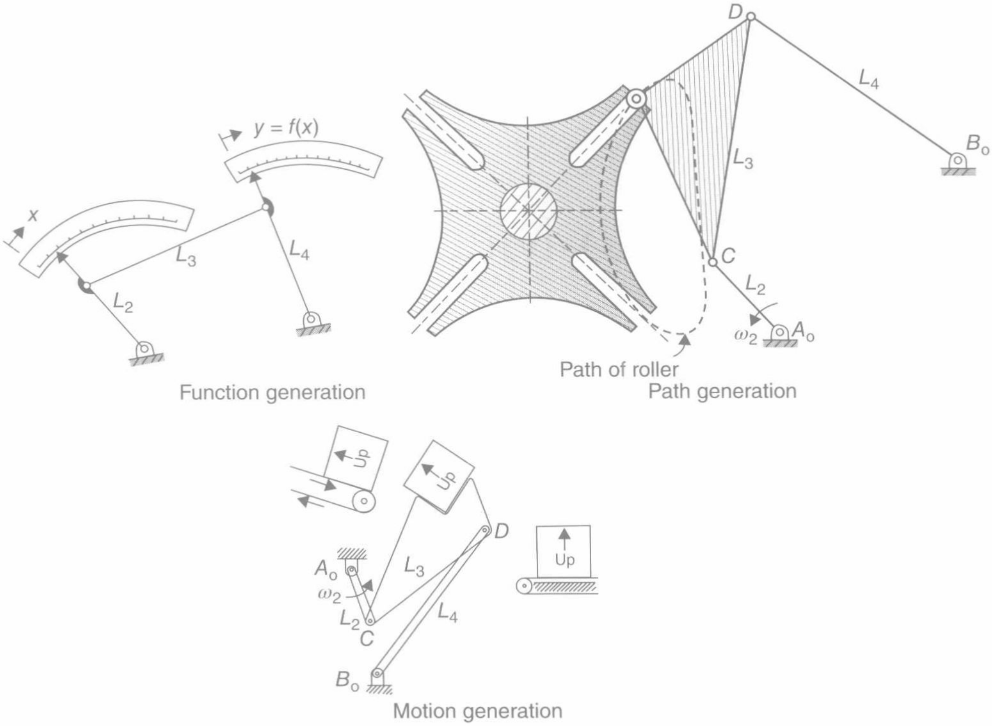

Figure 1.1 4-Bar linkages

the output link. The coupler link consists of two moving pivots, points C and D, thereby coupling the input link to the output link. A point on the coupler link generally traces out a sixth-order algebraic coupler curve. Very different coupler curves can be generated by using a different tracer point on the coupler link. Hrones and Nelson's *Analysis of 4-Bar Linkages* [1] published in 1951 shows many different types of coupler curves and their appropriate 4-bar linkage.

The 4-bar linkage is the most common chain of pin-connected links that allows relative motion between the links (see Figure 1.1). These linkages can be classified into three categories depending on the task that the linkage performs: function generation, path generation, and motion generation. A **function generator** is a linkage in which the relative motion or forces between the links connected to ground is of interest. In function generation, the task does not require a tracer point on the coupler link. In **path generation**, only the path of the tracer point on the coupler link is important and not the rotation of the coupler link. In **motion generation**, the entire motion of the coupler link is important, that is, the path that the tracer point follows and the angular orientation of the coupler link.

1.2 Kinematic Diagrams

The first step in designing or analyzing a mechanical linkage is to draw the kinematic diagram. A kinematic diagram is a "stick-figure" representation of the linkage as shown in Figure 1.2.

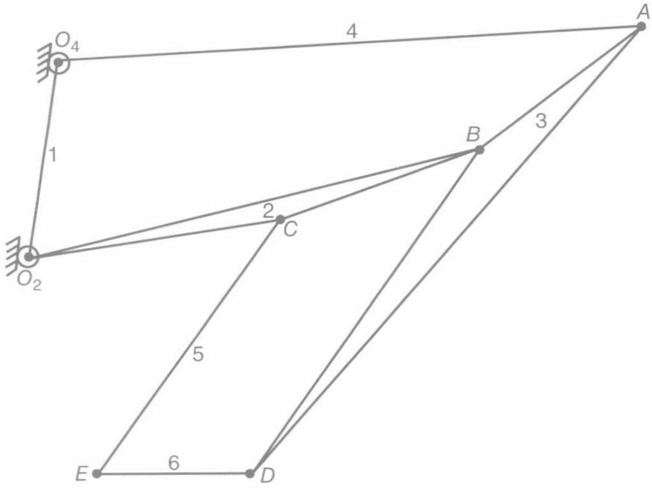

Figure 1.2 Kinematic diagram

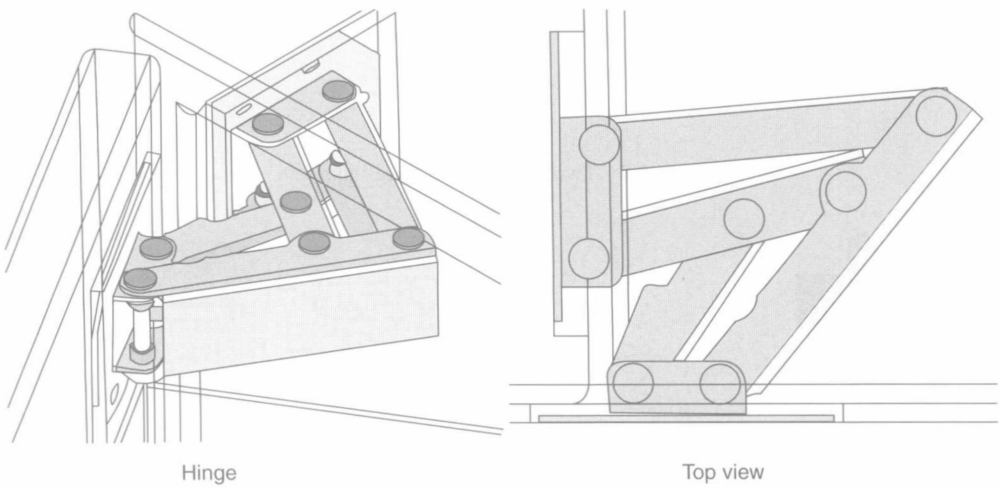

Figure 1.3 Physical system

The kinematic diagram is made up of nodes and straight lines and serves the same purpose as an electrical circuit schematic used for design and analysis purposes. It is a simplified version of the system so you can concentrate on the analysis and design instead of the building of the system. The actual 3D model is shown in Figure 1.3.

For convenience, the links are numbered starting with the ground link as number 1, the input link as number 2, then proceeding through the linkage. The purpose of a kinematic diagram is to show the relative motion between links. For example, a slider depicts translation while a pin joint depicts rotation. The joints are lettered starting with letter A, B, C, etc. On some kinematic

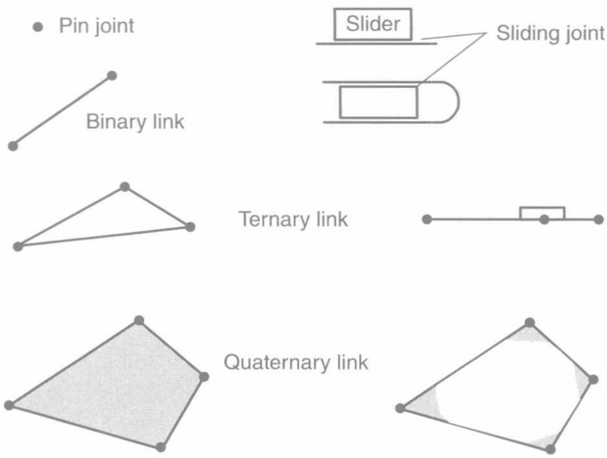

Figure 1.4 Planar links and joints

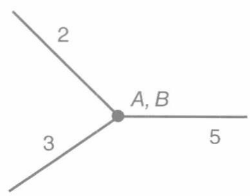

Figure 1.5 Two joints where three links join

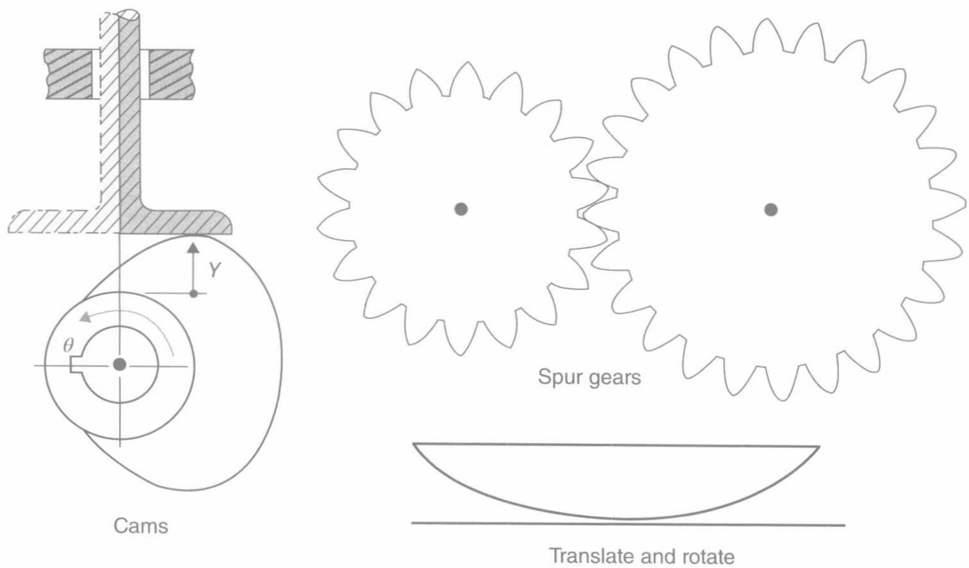

Figure 1.6 Half joints

diagrams, it is preferred to label fixed rotational pin joints using the letter O; thus link 2 connected to ground at a fixed bearing would be labeled O_2 and link 4 connected to ground at a fixed bearing would be O_4. Both notations are used in this book.

A link is a rigid body with at least two nodes. A node is a point on a link that attaches to another link. Connecting two links together forms a joint. The two most common types of nodes are the pin joint and the sliding joint; each has one degree of freedom. Links are categorized by the number of joints present on them. For example, a binary link has two nodes and a ternary link has three nodes (see Figure 1.4).

If three links come together at a point, the point must be considered as two joints since a joint is the connection between two links, not three links (see Figure 1.5).

A full joint has one degree of freedom. A half joint has two degree of freedom. Figure 1.6 shows half joints which can translate and rotate. A system with one degree of freedom requires one input to move all links. A system with two degrees of freedom requires two inputs to move all links. Thus, the degrees of freedom represent the required number of inputs for a given system.

1.3 Degrees of Freedom or Mobility

Kutzbach's criterion for 2D planar linkages calculates the number of degrees of freedom or mobility for a given linkage.

$$M = 3(L-1) - 2J_1 - J_2$$

L = Number of links including ground

J_1 = Number of one degree of freedom joints (full joints)

J_2 = Number of two degrees of freedom joints (half joints)

If we consider only full joints, then the mobility can also be calculated using the following equation which is a modification of Gruebler's equation. Note that the number of ternary links in the mechanism does not affect its mobility.

$$M = B - Q - 2P - 3$$

B = Number of binary links (2 nodes)

Q = Number of quaternary links (4 nodes)

P = Number of pentagonal links (5 nodes)

A 4-bar linkage has one degree of freedom. So does a slider-crank mechanism as seen in Figure 1.7. Each has four binary links and four full joints.

$$M = 3(L-1) - 2J_1 - J_2 = 3(4-1) - 2(4) - 0 = 1$$

or

$$M = B - Q - 2P - 3 = 4 - 0 - 2(0) - 3 = 1$$

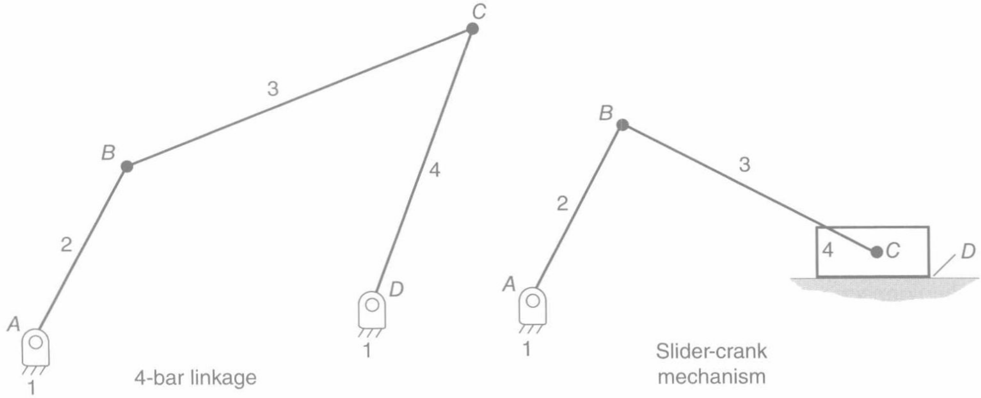

Figure 1.7 Mechanisms with four links

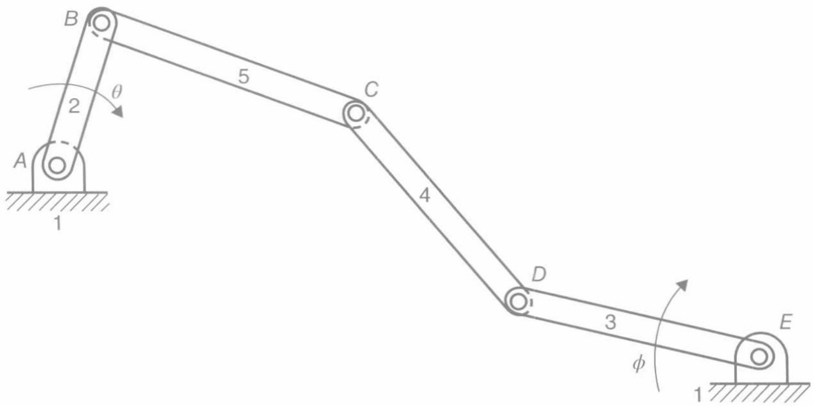

Figure 1.8 Five-bar linkage

A 5-bar linkage has two degrees of freedom. It has five binary links and five full joints. The 5-bar requires two separate inputs as shown in Figure 1.8.

$$M = 3(5-1) - 2(5) - 0 = 2$$

A 6-bar linkage with two ternary links has one degree of freedom. It has six links and seven full joints. A 6-bar linkage can be put together in two different configurations, the Watt 6-bar and the Stephenson 6-bar, as shown in Figure 1.9. What is the difference between these configurations?

The Watt 6-bar linkage has the two ternary links connected together, whereas the Stephenson 6-bar linkage has the two ternary link separated by a binary link.

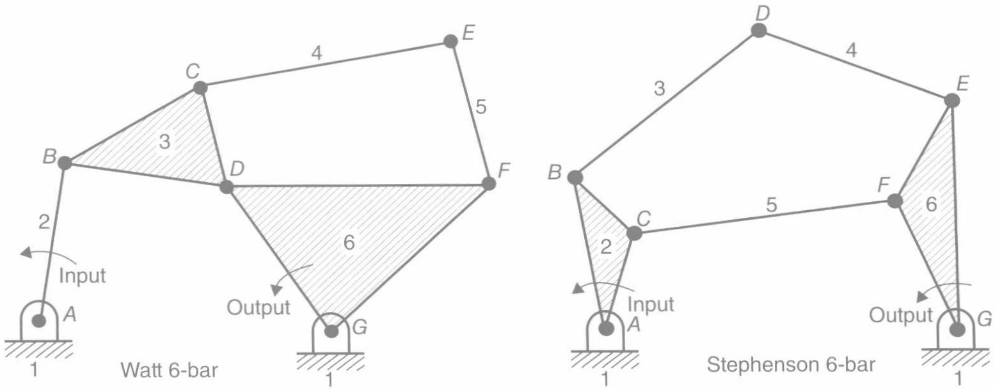

Figure 1.9 6-Bar linkage (2 configurations)

1.4 Grashof's Equation

A mark of a "good design" is simplicity. A design with the fewest moving parts is in general less expensive and more reliable. With this in mind, a 4-bar linkage is best if it works for your application.

Grashof's equation states that at least one link will rotate through 360° if $S+L \leq P+Q$ where

$$S = \text{Shortest link}$$

$$L = \text{Longest link}$$

$$P, Q = \text{Other two links}$$

If Grashof's equation is not true, then no link will rotate through 360°. If $S+L > P+Q$, then the 4-bar linkage is a triple rocker. If $S+L = P+Q$, then all inversions are either double-crank mechanisms or crank-rocker mechanisms with a change-over point where it will move from an open loop configuration to a crossed configuration.

If $S+L \leq P+Q$, then based on where the shortest link is located, the 4-bar linkage is as follows:

a. Crank-crank if S is the ground link
b. Crank-rocker if S is the input link
c. Double-rocker if S is the coupler link (see Figure 1.10)
d. Rocker-crank if S is the output link

1.5 Transmission Angle

In Figure 1.11, the acute angle between the coupler link, L_3, and the follower link, L_4, is called the transmission angle, μ. For equilibrium of link 4, the sum of the torques about point B_o must be zero. Since the coupler link, link 3, is a two-force member, the force that link 3 applies to link 4 is along link 3. Thus, the torque that link 4 experiences about B_o is$\text{Torque}_4 = \text{Force}_{34} L_4 \sin\mu$.

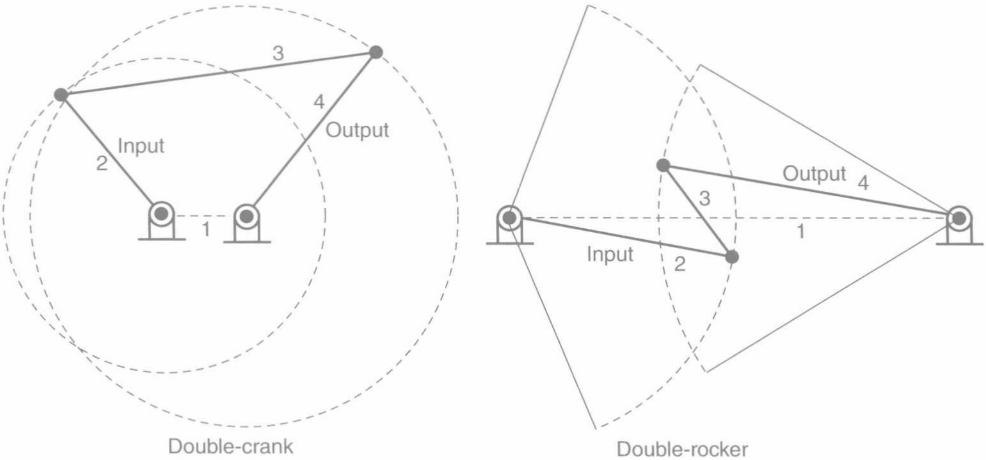

Figure 1.10 Double-crank and double-rocker

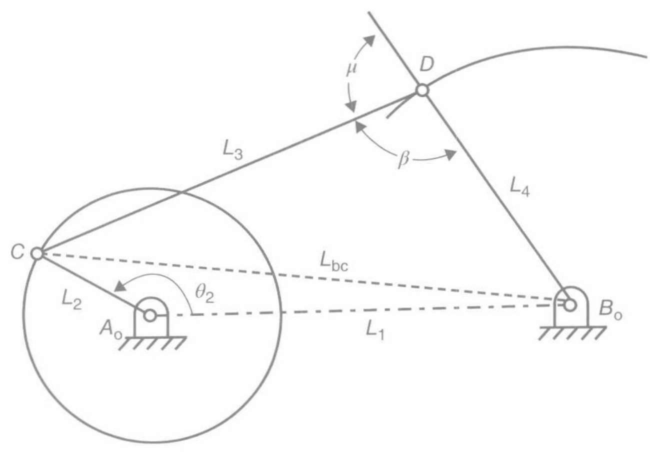

Figure 1.11 Transmission angle

Now looking at the two triangles that have L_{bc} in common, we can use the cosine law and relate θ_2 to β.

$$L_{bc}{}^2 = L_1{}^2 + L_2{}^2 - 2L_1L_2\cos\theta_2$$

and

$$L_{bc}{}^2 = L_3{}^2 + L_4{}^2 - 2L_3L_4\cos\beta$$

If we set the two equations equal to each other and solve for β, we have:

$$\beta = \cos^{-1}\left(\frac{L_3{}^2 + L_4{}^2 - \left(L_1{}^2 + L_2{}^2 - 2L_1L_2\cos\theta_2\right)}{2L_3L_4}\right)$$

Since the transmission angle is always an angle less than 90°, the transmission angle becomes as follows:

$$\mu = \text{Minimum}(\beta, 180^\circ - \beta)$$

A small transmission angle is undesirable for several reasons. As the transmission angle decreases, the output torque on the follower decreases for the same coupler-link force. If the output torque is constant, then the coupler link force must increase as the transmission angle decreases. This could lead to links buckling or connecting pins shearing. Also, as the transmission angle decreases, the position of the follower link becomes more sensitive to linkage lengths and hole tolerances at the connecting pins. To avoid this, the transmission angle should be above 40° at all times. Note that $\sin(40^\circ) = 0.64$, and thus the follower torque will be reduced to approximately two-third of its maximum, which occurs when the transmission angle is at 90°.

For the linkage in Figure 1.12, the extreme values of the transmission angle, μ' and μ'', can be obtained when the input link 2 is aligned with the ground link, link 1.

Using the cosine law again leads to the following:

$$\beta' = \cos^{-1}\left(\frac{L_3^{\,2} + L_4^{\,2} - (L_1 - L_2)^2}{2L_3L_4}\right)$$

$$\mu' = \text{Minimum}(\beta', 180^\circ - \beta')$$

and

$$\beta'' = \cos^{-1}\left(\frac{L_3^{\,2} + L_4^{\,2} - (L_1 + L_2)^2}{2L_3L_4}\right)$$

$$\mu'' = \text{Minimum}(\beta'', 180^\circ - \beta'')$$

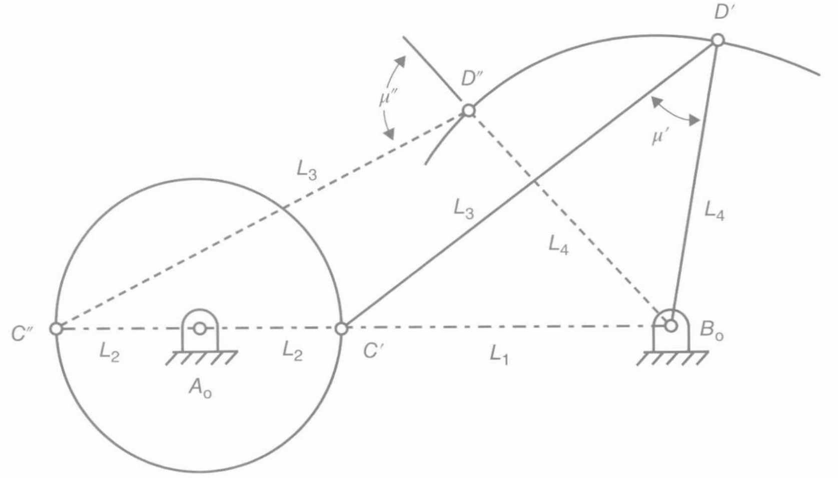

Figure 1.12 Transmission angle extremes

1.6 Geneva Mechanism

A Geneva mechanism converts a constant rotational motion into an intermittent translational motion. The *Geneva Drive* is also called the *Maltese Cross*. The Geneva mechanism was originally invented by a watch maker from Geneva to prevent the spring of a watch from being over-wound. In operation, a drive wheel with a pin enters into one of several slots on the driven wheel and thus advances it by one step. The drive wheel has a raised circular disc that serves to lock the driven wheel in a fixed position between steps. Figure 1.13 shows a five-slot and six-slot Geneva wheel mechanism.

Figure 1.14 shows several shots of the motion of the output wheel as the input wheel rotates.

The following are equations for designing a Geneva wheel mechanism. The first four variables are design parameters chosen by the designer. The rest of the variables are calculated based on these design parameters (see Figure 1.15).

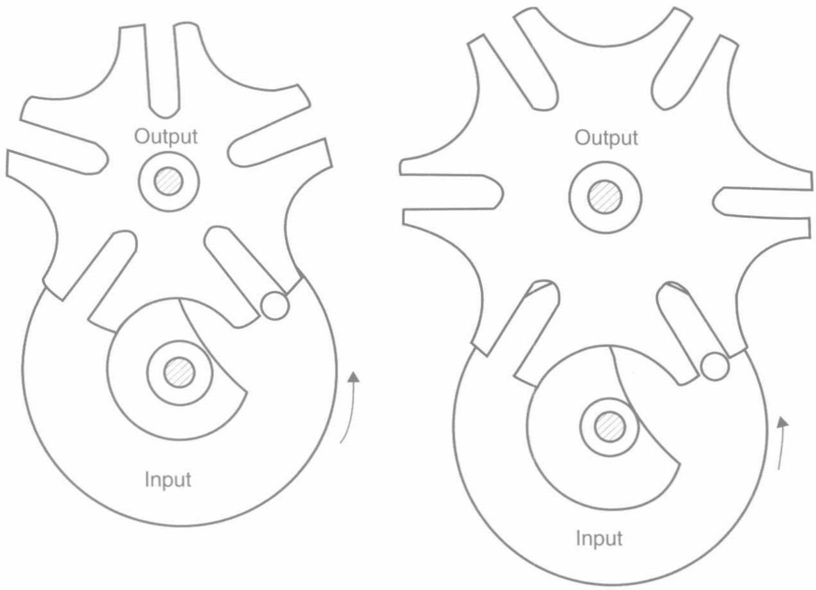

Figure 1.13 Geneva mechanism

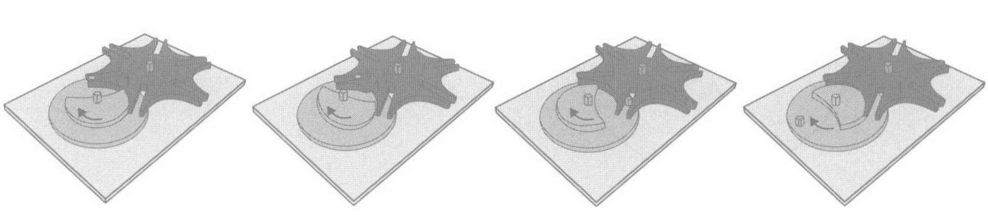

Figure 1.14 Motion of Geneva mechanism