SWOKOWSKI ▪ Sixth Edition

Fundamentals of College Algebra

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Sixth Edition

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Preface

This book is a major revision of the previous five editions of *Fundamentals of College Algebra.* One of my goals was to maintain the mathematical soundness of earlier editions, while making discussions somewhat less formal by rewriting, placing more emphasis on graphs, and by adding new examples and figures. Another objective was to stress the usefulness of the subject matter through a great variety of applied problems from many different disciplines. Finally, suggestions for improvements from users of previous editions led me to change the order in which certain topics are presented. The comments that follow highlight some of the changes and features of this edition.

CHANGES FOR THE SIXTH EDITION

- Chapter 1 is streamlined. The concepts that formerly appeared in eight sections are now presented in four.
- The Binomial Theorem is discussed in Section 1.5 instead of late in the book.
- Complex numbers are defined in Section 2.4, allowing for a complete treatment of quadratic equations in Chapter 2.
- Greater emphasis is given to graphical interpretations for the domain and range of a function in Section 3.4.
- Section 3.6, *Operations on Functions,* has not appeared in previous editions.
- The discussion of inverse functions in Section 3.7 is simplified and integrated with the concept of a one-to-one function.
- The material on division of polynomials, synthetic division, and zeros of polynomials has been reorganized and moved forward in the text.
- Oblique asymptotes for graphs of rational functions are discussed in Section 4.6.
- Chapter 5 has been completely rewritten, with much more attention given to the natural exponential and logarithmic functions and their applications.
- Logarithmic tables have been deemphasized, since calculators are much more efficient and accurate. Teachers who feel that students should be instructed on the use of tables and the technique of linear interpolation will find suitable material in Appendix I.

- In Chapter 6 greater emphasis is given to finding solutions of systems of linear equations by means of the echelon form of a matrix.
- Partial fractions are introduced in Section 6.4 as an application of systems of linear equations.

FEATURES

Applications Previous editions contained applied problems, but most of them were in the fields of engineering, physics, chemistry, or biology. In this revision other subjects are also considered, such as physiology, medicine, sociology, ecology, oceanography, marine biology, business, and economics.

Examples Each section contains carefully chosen examples to help students understand and assimilate new concepts. Whenever feasible, applications are included to demonstrate the usefulness of the subject matter.

Exercises Exercise sets begin with routine drill problems and gradually progress to more difficult types. As a rule, applied problems appear near the end of the set, to allow students to gain confidence in manipulations and new ideas before attempting questions that require an analysis of practical situations.

There is a review section at the end of each chapter, consisting of a list of important topics and pertinent exercises.

Answers to the odd-numbered exercises are given at the end of the text. Instructors may obtain an answer booklet for the even-numbered exercises from the publisher.

Calculators Calculators are given much more emphasis in this edition. It is possible to work most of the exercises without the aid of a calculator; however, instructors may wish to encourage their use to shorten numerical computations. Some sections contain problems labeled *Calculator Exercises,* for which a calculator should definitely be employed.

Text Design A change in page size has made it possible to place most figures in margins, as close as possible to where they are first mentioned in the text. A new use of a second color for figures and statements of important facts should make it easier to follow discussions and remember major ideas. Graphs are usually labeled and color-coded to clarify complex figures. Many figures have been added to exercise sets to help visualize important aspects of applied problems.

Flexibility Hundreds of syllabi from schools that used previous editions attest to the flexibility of the text. Sections and chapters can be rearranged in many different ways, depending on the objectives and the length of the course.

SUPPLEMENTS

Instructors may obtain a manual containing worked-out solutions for approximately one-third of the exercises, authored by Stephen Rodi of Austin Community College. Test banks that can be used for quizzes and examinations are also available from the publisher. Students who need additional help may purchase, from their bookstore, *A Programmed Study Guide* by Roy Dobyns of Carson-Newman College. This guide is designed to assist self-study by reinforcing the mathematics presented in the lectures and the text.

ACKNOWLEDGMENTS

I wish to thank Michael Cullen of Loyola Marymount University for supplying all the new exercises dealing with applications. This large assortment of problems provides strong motivation for the mathematical concepts introduced in the text. Because of his significant input on exercise sets, Michael should be considered as a coauthor of this edition.

This revision has benefited from the comments of users of previous editions. I wish to thank the following individuals, who reviewed the manuscript and offered many helpful suggestions:

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I also wish to single out the following mathematics educators, who met with me and representatives from my publisher for several days during the summer of 1984: Michael Eames of Keene State College, Carol Edwards of St. Louis Community College, John Hornsby of the University of New Orleans, and William Robinson of Ventura College. Their general comments about pedagogy and their specific recommendations about the content of mathematics courses were extremely helpful in improving the book.

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I am grateful for the excellent cooperation of the staff of Prindle, Weber & Schmidt. Two people in the company deserve special mention. They are Senior Editor David Pallai and my production editor Kathi Townes. The present form of the book was greatly influenced by their efforts, and I owe them both a debt of gratitude. Moreover, their personal friendship has often been a source of comfort during the years we have worked together.

In addition to all of the persons named here, I express my sincere appreciation to the many unnamed students and teachers who have helped shape my views on mathematics education.

EARL W. SWOKOWSKI

Contents

CHAPTER 1

Fundamental Concepts of Algebra

The material in this chapter is basic to the study of algebra. ■ We begin by discussing properties of real numbers. ■ Next we turn our attention to exponents and radicals, and how they may be used to simplify complicated algebraic expressions.

1.1 What Is Algebra?

A good foundation in algebra is essential for advanced courses in mathematics, the natural sciences, and engineering. It is also required for solving problems in business, industry, statistics, and many other fields. Indeed, algebraic methods can be applied to every situation that makes use of numerical processes.

Algebra evolved from the operations and rules of arithmetic. The study of arithmetic begins with addition, multiplication, subtraction, and division of numbers, such as

$$4 + 7, \quad (37)(681), \quad 79 - 22, \quad \text{and} \quad 40 \div 8.$$

In *algebra* we introduce symbols or letters such as a, b, c, d, x, y to denote *arbitrary* numbers and, instead of special cases, we often consider *general* statements, such as

$$a + b, \quad cd, \quad x - y, \quad \text{and} \quad x \div a.$$

This *language of algebra* serves a twofold purpose. First, it may be used as a shorthand to abbreviate and simplify long or complicated statements. Second, it is a convenient means of generalizing many specific statements. To illustrate, at an early age, children learn that

$$2 + 3 = 3 + 2, \qquad 4 + 7 = 7 + 4, \qquad 1 + 8 = 8 + 1,$$

and so on. In words, this property may be phrased "if two numbers are added, then the order of addition is immaterial; that is, the same result is obtained whether the second number is added to the first, or the first number is added to the second." This lengthy description can be shortened and at the same time made easier to understand by means of the algebraic statement

$$a + b = b + a,$$

where a and b denote arbitrary numbers.

Illustrations of the generality of algebra may be found in formulas used in science and industry. For example, if an airplane flies at a constant rate of 300 mph (miles per hour) for two hours, then the distance it travels is

$$(300)(2) \quad \text{or} \quad 600 \text{ miles}.$$

If the airplane's rate is 250 mph and the elapsed time is 3 hours, then the distance traveled is

$$(250)(3) \quad \text{or} \quad 750 \text{ miles}.$$

If we introduce symbols and let r denote the constant rate, t the elapsed time, and d the distance traveled, then the two situations we have described are special cases of the general algebraic formula

$$d = rt.$$

When specific numerical values for r and t are given, the distance d may be found readily by substitution in the formula. The formula may also be used to solve related problems. For example, suppose the distance between two cities is 645 miles, and we wish to find the constant rate that would enable an airplane to cover that distance in 2 hours and 30 minutes. Thus, we are given

$$d = 645 \text{ miles} \quad \text{and} \quad t = 2.5 \text{ hours},$$

and we must find r. Since $d = rt$, it follows that

$$r = \frac{d}{t}$$

and hence, for our special case,

$$r = \frac{645}{2.5} = 258 \text{ mph}.$$

Thus, if an airplane flies at a constant rate of 258 mph, then it will travel 645 miles in 2 hours and 30 minutes. In like manner, if we are given r, we can find the time t required to travel a distance d by means of the formula

$$t = \frac{d}{r}.$$

This indicates how the introduction of an algebraic formula allows us not only to solve special problems conveniently but also to enlarge the scope of our knowledge by suggesting new problems that can be considered.

We have given several elementary illustrations of the value of algebraic methods. There are an unlimited number of situations where a symbolic approach may lead to insights and solutions that would be impossible to obtain using only numerical processes. As you proceed through this text and go on either to more advanced courses in mathematics or to fields that employ mathematics, you will become even more aware of the importance and the power of algebraic techniques.

1.2 Real Numbers

Real numbers are employed in all phases of mathematics, and you are probably well acquainted with symbols that are used to represent them, such as

$$1, \quad 73, \quad -5, \quad \tfrac{49}{12}, \quad \sqrt{2}, \quad 0, \quad \sqrt[3]{-85}, \quad 0.33333\ldots, \quad \text{and} \quad 596.25.$$

The real numbers are said to be **closed** relative to operations of addition (denoted by $+$) and multiplication (denoted by $\cdot$). This means that to every pair a, b of real numbers there corresponds a unique real number $a + b$, called the **sum** of a and b, and a unique real number $a \cdot b$ (also written ab), called the **product** of a and b. If a and b denote the same real number, we write $a = b$ (read "a equals b"). An expression of this type is called an **equality.**

The special numbers 0 and 1, referred to as **zero** and **one,** respectively, have the properties $a + 0 = a$ and $a \cdot 1 = a$ for every real number a. Each real number a has a **negative,** denoted by $-a$, such that $a + (-a) = 0$, and each nonzero real number a has a **reciprocal,** $\frac{1}{a}$, such that $a\left(\frac{1}{a}\right) = 1$. These and other important properties are included in the following list, in which a, b, and c denote arbitrary real numbers.

COMMUTATIVE PROPERTIES	$a + b = b + a,$	$ab = ba$
ASSOCIATIVE PROPERTIES	$a + (b + c) = (a + b) + c,$	$a(bc) = (ab)c$
IDENTITIES	$a + 0 = a = 0 + a,$	$a \cdot 1 = a = 1 \cdot a$
INVERSES	$a + (-a) = 0 = (-a) + a,$	$a\left(\frac{1}{a}\right) = 1 = \left(\frac{1}{a}\right)a \quad \text{if } a \neq 0$
DISTRIBUTIVE PROPERTIES	$a(b + c) = ab + ac,$	$(a + b)c = ac + bc$

The real numbers 0 and 1 are sometimes referred to as the **additive identity** and **multiplicative identity,** respectively. The negative, $-a$, is also called the **additive inverse** of a and, if $a \neq 0$, $\frac{1}{a}$ is called the **multiplicative inverse** of a. The symbol a^{-1} may be used in place of $\frac{1}{a}$, as indicated by the following definition.

DEFINITION OF a^{-1}

$$a^{-1} = \frac{1}{a}; \qquad a \neq 0$$

Since $a + (b + c)$ and $(a + b) + c$ are always equal, we may, without ambiguity, use the symbol $a + b + c$ to denote this real number. Similarly, the notation abc is used for either $a(bc)$ or $(ab)c$. An analogous situation exists if four or more real numbers are added or multiplied. Thus, if a, b, c, and d are real numbers, then we write $a + b + c + d$ for their sum and $abcd$ for their product, regardless of how the numbers are grouped or interchanged.

The Distributive Properties are useful for finding products of many different types of expressions. The next example provides one illustration.

EXAMPLE 1 If a, b, c, and d denote real numbers, show that

$$(a + b)(c + d) = ac + bc + ad + bd.$$

SOLUTION Using the two Distributive Properties,

$$\begin{aligned}(a + b)(c + d) &= (a + b)c + (a + b)d \\ &= (ac + bc) + (ad + bd) \\ &= ac + bc + ad + bd.\end{aligned}$$

■

If $a = b$ and $c = d$, then $a + c = b + d$ and $ac = bd$. This is often called the **substitution principle,** since we may think of replacing a by b and c by d in the expressions $a + c$ and ac. As a special case, using the fact that $c = c$ gives us the following rules.

If $a = b$, then $a + c = b + c$.
If $a = b$, then $ac = bc$.

We sometimes refer to those rules by the statements "Any number c may be added to both sides of an equality" and "Both sides of an equality may be multiplied by the same number c."

The following theorem can be proved. (See Exercises 54–55.)

THEOREM

$a \cdot 0 = 0$ for every real number a.
If $ab = 0$, then either $a = 0$ or $b = 0$.

The preceding theorem implies that $ab = 0$ *if and only if* either $a = 0$ or $b = 0$. The phrase "if and only if," which is used throughout mathematics, always has a twofold character. Here it means that if $ab = 0$, then $a = 0$ or $b = 0$ and, *conversely*, if $a = 0$ or $b = 0$, then $ab = 0$. Consequently, if both $a \neq 0$ and $b \neq 0$, then $ab \neq 0$; that is, *the product of two nonzero real numbers is always nonzero.*

The following properties can also be proved.

PROPERTIES OF NEGATIVES

$$-(-a) = a$$
$$(-a)b = -(ab) = a(-b)$$
$$(-a)(-b) = ab$$
$$(-1)a = -a$$

The operation of **subtraction** (denoted by $-$) is defined as follows:

DEFINITION OF SUBTRACTION

$$a - b = a + (-b)$$

Division (denoted by $\div$) is defined as follows:

DEFINITION OF DIVISION

$$a \div b = a\left(\frac{1}{b}\right) = ab^{-1}; \qquad b \neq 0$$

The symbol a/b or $\dfrac{a}{b}$ is often used in place of $a \div b$, and we refer to it as the **quotient of *a* by *b*** or the **fraction *a* over *b*.** The numbers a and b are called the **numerator** and **denominator,** respectively, of the fraction. It is important to note that since 0 has no multiplicative inverse, a/b is not defined if $b = 0$; that is, *division by zero is not permissible.* Also note that if $b \neq 0$, then

$$1 \div b = \frac{1}{b} = b^{-1}.$$

The following properties of quotients may be established, where all denominators are nonzero real numbers.

PROPERTIES OF QUOTIENTS

$$\frac{a}{b} = \frac{c}{d} \quad \text{if and only if} \quad ad = bc$$

$$\frac{a}{b} = \frac{ad}{bd}, \qquad \frac{a}{-b} = \frac{-a}{b} = -\frac{a}{b}$$

$$\frac{a}{b} + \frac{c}{b} = \frac{a+c}{b}, \qquad \frac{a}{b} + \frac{c}{d} = \frac{ad+bc}{bd}$$

$$\frac{a}{b} \cdot \frac{c}{d} = \frac{ac}{bd}, \qquad \frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c} = \frac{ad}{bc}$$

The **positive integers,** 1, 2, 3, 4, . . . , may be obtained by adding the real number 1 successively to itself. The numbers $-1, -2, -3, -4, \ldots$ are called **negative integers.** The **integers** consist of all positive and negative integers together with the real number 0.

If a, b, and c are integers and $c = ab$, then a and b are called **factors,** or **divisors,** of c. For example, since

$$6 = 2 \cdot 3 = (-2)(-3) = 1 \cdot 6 = (-1)(-6),$$

we see that $1, -1, 2, -2, 3, -3, 6$, and -6 are factors of 6.

A positive integer p different from 1 is **prime** if its only positive factors are 1 and p. The first few primes are 2, 3, 5, 7, 11, 13, 17, and 19. The **Fundamental Theorem of Arithmetic** states that every positive integer different from 1 can be expressed as a product of primes in one and only one way (except for order of factors). Some examples are:

$$12 = 2 \cdot 2 \cdot 3, \qquad 126 = 2 \cdot 3 \cdot 3 \cdot 7, \qquad 540 = 2 \cdot 2 \cdot 3 \cdot 3 \cdot 3 \cdot 5.$$

A **rational number** is a real number of the form a/b, where a and b are integers and $b \neq 0$. Real numbers that are not rational are called **irrational numbers.** The ratio of the circumference of a circle to its diameter is irrational and is denoted by π. It is often approximated by the decimal 3.1416 or by the rational number $\frac{22}{7}$. We use the notation $\pi \approx 3.1416$ to indicate that π *is approximately equal to* 3.1416.

There is no rational number b such that $b^2 = 2$, where b^2 denotes $b \cdot b$. However, there is an *irrational* number, denoted by $\sqrt{2}$, such that $(\sqrt{2})^2 = 2$. In general, we have the following definition.

DEFINITION OF SQUARE ROOT

Let a be a nonnegative real number. The **principal square root of *a*,** denoted by $\sqrt{a}$, is the *nonnegative* real number b such that $b^2 = a$.

We often refer to $\sqrt{a}$ simply as the *square root of a*. Some square roots are rational. For example,

$$\sqrt{25} = 5, \qquad \sqrt{\tfrac{9}{4}} = \tfrac{3}{2}, \qquad \sqrt{16} = 4.$$

Other square roots, such as $\sqrt{3}$, $\sqrt{5}$, and $\sqrt{\frac{7}{2}}$, are irrational.

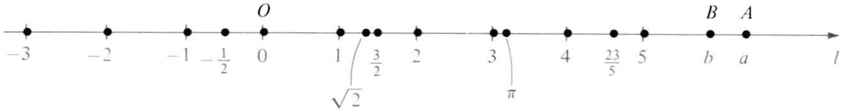
Real numbers may be represented by decimal expressions. Decimal representations for rational numbers either are terminating or are nonterminating and repeating. For example, we can show by division that

$$\tfrac{5}{4} = 1.25 \qquad \text{and} \qquad \tfrac{177}{55} = 3.2181818\ldots,$$

where the digits 1 and 8 repeat indefinitely. Decimal representations for irrational numbers are always nonterminating and nonrepeating.

Real numbers may be represented geometrically by points on a line l in such a way that for each real number a there corresponds one and only one point on l, and conversely, to each point P on l there corresponds precisely one real number. Such an association is called a **one-to-one correspondence.** We first choose an arbitrary point O, called the **origin,** and associate with it the real number 0. Points associated with the integers are then determined by laying off successive line segments of equal length on either side of O as illustrated in Figure 1.1. The points corresponding to rational numbers, such as $\frac{23}{5}$ and $-\frac{1}{2}$, are obtained by subdividing the equal line segments. Points associated with certain irrational numbers,

FIGURE 1.1



such as $\sqrt{2}$, can be found by geometric construction. (See Exercise 47.) To every irrational number there corresponds a unique point on l, and conversely, every point that is not associated with a rational number corresponds to an irrational number.

The number a that is associated with a point A on l is called the **coordinate** of A. An assignment of coordinates to points on l is called a **coordinate system** for l, and l is called a **coordinate line,** or a **real line.** A direction can be assigned to l by taking the **positive direction** along l to the right and the **negative direction** to the left. The positive direction is noted by placing an arrowhead on l as shown in Figure 1.1.

The numbers that correspond to points to the right of O in Figure 1.1 are called **positive real numbers,** whereas numbers that correspond to points to the left of O are **negative real numbers.** *The real number* 0 *is neither positive nor negative.*

If a and b are real numbers and $a - b$ is positive, we say that ***a* is greater than *b*** and we write $a > b$. An equivalent statement is that ***b* is less than *a*,** written $b < a$. The symbols $>$ or $<$ are called **inequality signs** and expressions such as $a > b$ or $b < a$ are called **inequalities.** From the manner in which we constructed the coordinate line l in Figure 1.1, we see that if A and B are points with coordinates a and b, respectively, then $a > b$ (or $b < a$) *if and only if* A *lies to the right of* B. The following definition is stated for reference, where a and b denote real numbers.

DEFINITIONS OF > AND <

$a > b$ means $a - b$ is positive.
$b < a$ means $a - b$ is positive.

The expressions $a > b$ and $b < a$ have exactly the same meaning. As illustrations we may write

$$5 > 3 \quad \text{since} \quad 5 - 3 = 2 \text{ is positive;}$$
$$-6 < -2 \quad \text{since} \quad -2 - (-6) = -2 + 6 = 4 \text{ is positive;}$$
$$-\sqrt{2} < 1 \quad \text{since} \quad 1 - (-\sqrt{2}) = 1 + \sqrt{2} \text{ is positive;}$$
$$2 > 0 \quad \text{since} \quad 2 - 0 = 2 \text{ is positive;}$$
$$-5 < 0 \quad \text{since} \quad 0 - (-5) = 5 \text{ is positive.}$$

The last two illustrations are special cases of the following general properties.

$$a > 0 \quad \text{if and only if } a \text{ is positive.}$$
$$a < 0 \quad \text{if and only if } a \text{ is negative.}$$