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Phase Transitions and Renormalization Group

相变与重正化群

（影印版）

〔法〕齐恩-朱斯坦（J. Zinn-Justin） 著

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序　言

物理学是研究物质、能量以及它们之间相互作用的科学。她不仅是化学、生命、材料、信息、能源和环境等相关学科的基础，同时还是许多新兴学科和交叉学科的前沿。在科技发展日新月异和国际竞争日趋激烈的今天，物理学不仅囿于基础科学和技术应用研究的范畴，而且在社会发展与人类进步的历史进程中发挥着越来越关键的作用。

我们欣喜地看到，改革开放三十多年来，随着中国政治、经济、教育、文化等领域各项事业的持续稳定发展，我国物理学取得了跨越式的进步，做出了很多为世界瞩目的研究成果。今日的中国物理正在经历一个历史上少有的黄金时代。

在我国物理学科快速发展的背景下，近年来物理学相关书籍也呈现百花齐放的良好态势，在知识传承、学术交流、人才培养等方面发挥着无可替代的作用。从另一方面看，尽管国内各出版社相继推出了一些质量很高的物理教材和图书，但系统总结物理学各门类知识和发展，深入浅出地介绍其与现代科学技术之间的渊源，并针对不同层次的读者提供有价值的教材和研究参考，仍是我国科学传播与出版界面临的一个极富挑战性的课题。

为有力推动我国物理学研究、加快相关学科的建设与发展，特别是展现近年来中国物理学者的研究水平和成果，北京大学出版社在国家出版基金的支持下推出了"中外物理学精品书系"，试图对以上难题进行大胆的尝试和探索。该书系编委会集结了数十位来自内地和香港顶尖高校及科研院所的知名专家学者。他们都是目前该领域十分活跃的专家，确保了整套丛书的权威性和前瞻性。

这套书系内容丰富，涵盖面广，可读性强，其中既有对我国传统物理学发展的梳理和总结，也有对正在蓬勃发展的物理学前沿的全面展示；既引进和介绍了世界物理学研究的发展动态，也面向国际主流领域传播中国物理的优秀专著。可以说，"中外物理学精品书系"力图完整呈现近现代世界和中国物理

科学发展的全貌，是一部目前国内为数不多的兼具学术价值和阅读乐趣的经典物理丛书。

“中外物理学精品书系”另一个突出特点是，在把西方物理的精华要义“请进来”的同时，也将我国近现代物理的优秀成果“送出去”。物理学科在世界范围内的重要性不言而喻，引进和翻译世界物理的经典著作和前沿动态，可以满足当前国内物理教学和科研工作的迫切需求。另一方面，改革开放几十年来，我国的物理学研究取得了长足发展，一大批具有较高学术价值的著作相继问世。这套丛书首次将一些中国物理学者的优秀论著以英文版的形式直接推向国际相关研究的主流领域，使世界对中国物理学的过去和现状有更多的深入了解，不仅充分展示出中国物理学研究和积累的“硬实力”，也向世界主动传播我国科技文化领域不断创新的“软实力”，对全面提升中国科学、教育和文化领域的国际形象起到重要的促进作用。

值得一提的是，“中外物理学精品书系”还对中国近现代物理学科的经典著作进行了全面收录。20 世纪以来，中国物理界诞生了很多经典作品，但当时大都分散出版，如今很多代表性的作品已经淹没在浩瀚的图书海洋中，读者们对这些论著也都是“只闻其声，未见其真”。该书系的编者们在这方面下了很大工夫，对中国物理学科不同时期、不同分支的经典著作进行了系统的整理和收录。这项工作具有非常重要的学术意义和社会价值，不仅可以很好地保护和传承我国物理学的经典文献，充分发挥其应有的传世育人的作用，更能使广大物理学人和青年学子切身体会我国物理学研究的发展脉络和优良传统，真正领悟到老一辈科学家严谨求实、追求卓越、博大精深的治学之美。

温家宝总理在 2006 年中国科学技术大会上指出，“加强基础研究是提升国家创新能力、积累智力资本的重要途径，是我国跻身世界科技强国的必要条件”。中国的发展在于创新，而基础研究正是一切创新的根本和源泉。我相信，这套“中外物理学精品书系”的出版，不仅可以使所有热爱和研究物理学的人们从中获取思维的启迪、智力的挑战和阅读的乐趣，也将进一步推动其他相关基础科学更好更快地发展，为我国今后的科技创新和社会进步做出应有的贡献。

“中外物理学精品书系”编委会　主任
中国科学院院士，北京大学教授
王恩哥
2010 年 5 月于燕园

Phase Transitions and Renormalization Group

Jean Zinn-Justin

Dapnia, CEA/Saclay, France

and

Institut de Mathématiques de Jussieu-Chevaleret,
Université de Paris VII

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Preface

Quantum field theory is at the basis of a notable part of the theoretical developments of twentieth century physics. The model that describes all fundamental interactions, apart from gravitation, at the microscopic scale, is a quantum field theory. Perhaps more surprisingly, quantum field theory has also led to a complete understanding of the singular macroscopic properties of a wide class of phase transitions near the transition point as well as statistical properties of some geometrical models.

However, unlike Newtonian or non-relativistic quantum mechanics, a quantum field theory in its most direct formulation leads to severe conceptual difficulties due to the appearance of infinities in the calculation of physical observables. Eventually, the problem of infinities was solved empirically by a method called *renormalization*. Only later did the method find a satisfactory interpretation, in the framework of the *renormalization group*. The problem of infinities is related to an unexpected phenomenon, the non-decoupling of very different length-scales in some physical situations.

It is within the framework of statistical physics and continuous phase transitions that the discussion of these conceptual problems is the simplest. This work thus tries to provide an elementary introduction to the notions of continuum limit and universality in physical systems with a large number of degrees of freedom. We will emphasize the role of Gaussian distributions and their relations with the mean-field approximation and Landau's theory of critical phenomena. We will show that quasi-Gaussian or mean-field approximations cannot describe correctly phase transitions in two and three space dimensions. We will assign this difficulty to the coupling of very different physical length-scales, even though the systems we will consider have only local, that is, short-range interactions. To analyse the problem, a new concept is required: the renormalization group, whose fixed points allow understanding the universality of physical properties at large distance, beyond the quasi-Gaussian or mean-field approximations.

Renormalization group arguments then lead to the idea that, in critical systems, correlations at large distance near the transition temperature can be described by local statistical field theories, formally quantum field theories in imaginary time.

This work corresponds to a course delivered, in various forms, for three years at the University of Paris 7 and first published in French [1]. It is organized in the following way.

Chapter 1 contains a short, semi-historical, introduction that tries to describe the evolution of ideas from the first works in quantum field theory [2–5] to the application of renormalization group methods to phase transitions.

In Chapter 2, we have collected a number of technical results concerning generating functions, Gaussian measures and the steepest descent method, which are indispensable for the understanding of the work.

Chapter 3 introduces several basic topics of the work: the notions of continuum limit and universality, through the examples of the central limit theorem and the random walk. We show that universality originates from the small probability of large deviations from the expectation value in probability distributions, which translates into an hypothesis of locality in random walks. In both examples, universality is related to the appearance of asymptotic Gaussian distributions. We then show that, beyond a direct calculation, universality can also be understood as resulting from the existence of fixed points of transformations acting on the space of probability distributions. These very simple examples will allow us to introduce immediately the renormalization group terminology. Finally, the existence of continuum limits leads naturally to a description in terms of path integrals.

In Chapter 4, we begin the study of classical statistical systems, the central topic of this work, with the example of one-dimensional models. This enables us to introduce the terminology of statistical physics, like correlation functions, thermodynamic limit, correlation length, and so on. Even if one-dimensional systems with short-range interactions do not exhibit phase transitions, it is nevertheless possible to define a continuum limit near zero temperature. Moreover, in the case of short-range interactions, these systems can be solved exactly by the transfer matrix method, and thus provide interesting pedagogical examples.

The continuum limit of one-dimensional models again leads to path integrals. We describe some of their properties in Chapter 5 (for a more systematic discussion see, for example, Ref. [6]).

In Chapter 6, we define more general statistical systems, in an arbitrary number of space dimensions. For convenience, we use the ferromagnetic language, even though, as a consequence of universality, the results that are derived in this work apply to much more general statistical systems. In addition to complete and connected correlation functions (whose decay properties at large distance, called cluster properties, are recalled), which we have already defined in the preceding chapters, we introduce vertex functions, which are related to the thermodynamic potential. The free energy and thermodynamic potential, like connected correlation functions and vertex functions, are related by a Legendre transformation of which we discuss a few properties.

Chapter 7 is devoted to the concept of phase transition, a concept that is far from being trivial in the sense that a phase transition requires the interaction of an infinite number of degrees of freedom. We first solve exactly a particular model in the limit in which the number of space dimensions becomes infinite. In this limit, the model exhibits a behaviour that the analysis presented in the following chapters will identify as quasi-Gaussian or mean-field like. Then, we discuss, in general terms, the existence of phase transitions as a function of the space dimension. We emphasize the difference between models with discrete and continuous symmetries in dimension two.

In Chapter 8, we examine the universal properties of phase transitions in the quasi-Gaussian or mean-field approximations. We study the singularities of thermodynamic functions at the transition point as well as the large-distance behaviour of the two-point correlation function. We summarize the universal properties in the form of Landau's theory [7]. We stress the peculiarities of models with continuous symmetries at low temperature due to the appearance of Goldstone modes. Finally, we evaluate corrections to the quasi-Gaussian approximation and show that the approximation is only consistent in space dimension larger than 4 (following the lines of Ref. [8]). We mention the possible existence of tricritical points.

In Chapter 9, we introduce the general concept of renormalization group [5] in the spirit of the work [9]. We study the role of fixed points and their stability properties. We exhibit a particular fixed point, the Gaussian fixed point, which is stable in dimension larger than 4. We identify the leading perturbation to the Gaussian fixed point in dimension ≤ 4. We discuss the possible existence of a non-Gaussian fixed point near dimension 4.

In Chapter 10, using the assumptions introduced in Chapter 9, we show that it is indeed possible to find a non-Gaussian fixed point in dimension $d = 4 - \varepsilon$ [10], both in models with reflection and rotation symmetries. We briefly introduce the field theory methods [11, 12] that we will describe more thoroughly in the following chapters. Finally, we present a selection of numerical results concerning critical exponents and some universal amplitude ratios [13–17], obtained by field theory methods using both the Callan–Symanzik formalism in three dimensions and the ε-expansion extrapolated to $\varepsilon = 1$.

Chapter 11 contains a general discussion of renormalization group equations and the properties of the corresponding fixed points, for a whole class of models that possess more general symmetries than the reflection and rotation groups considered so far, generalizing somewhat the results presented in [8, 18]. In particular, the analysis leads to an interesting conjecture, relating decay of correlation functions and stability of fixed points [8, 19].

With Chapter 12, we begin a more systematic presentation of field theory methods. Beyond a simple generalization of the perturbative methods already presented in the preceding chapters, several new concepts are introduced like the loop expansion, dimensional continuation and regularization [20].

With these technical tools, we can then justify, in Chapter 13, asymptotic renormalization group equations obtained by varying the cut-off, as they appear in field theory [4, 21]. General universality properties follow, as well as methods of calculating universal quantities as an expansion in powers of the deviation $\varepsilon = 4 - d$ from dimension 4. We conclude the chapter by a short presentation of the alternative formalism of renormalization group equations in renormalized form [22–25], in particular Callan–Symanzik equations [22] directly relevant to the numerical results reported in Chapter 10.

A class of field theories with an $O(N)$ orthogonal symmetry can be solved in the $N \to \infty$ limit, as we show in Chapter 14. All universal properties derived within the framework of the ε-expansion can also be proved at fixed dimension, within the framework of an expansion in powers of $1/N$ [26–36].

In models with continuous symmetries, phase transitions are dominated, at low temperature and large distance, by the interaction between Goldstone (massless) modes. The interaction can be described by the non-linear σ-model. Its study, using the renormalization group, allows generalizing the scaling properties of the critical theory at the transition to the whole low-temperature phase and studying properties of the phase transition near dimension 2 [37–40].

The renormalization group of quantum field theory has an interpretation as an asymptotic renormalization group when the relevant fixed point is close to the Gaussian fixed point. Quite early, more general formulations of the renormalization group have been proposed, which do not rely on such an assumption [41–42]. They lead to functional renormalization group (FRG) equations that describe the evolution of the effective interaction, but which are much more difficult to handle than the equations arising in field theory. They have been used to prove renormalizability without relying on a direct analysis of Feynman diagrams, unlike more traditional methods [43]. Moreover, more recently, they have inspired a number of new approximation schemes, different from the perturbative scheme of field theory [44]. Thus, for both pedagogical and practical reasons, we have decided to describe then in this work.

Finally, in the appendix, we have collected various technical considerations useful for a better understanding of the material presented in the work, and a few additional results concerning the FRG and functional flow equations based on partial integration over low-momentum (IR) modes [45].

Contents

1 Quantum field theory and the renormalization group

Without a minimal understanding of quantum or statistical field theories (formally related by continuation to imaginary time), the theoretical basis of a notable part of twentieth century physics remains incomprehensible.

Indeed, field theory, in its various incarnations, describes fundamental interactions at the microscopic scale, singular properties of phase transitions (like liquid–vapour, ferromagnetic, superfluid, separation of binary mixture,...) at the transition point, properties of diluted quantum gases beyond the model of Bose–Einstein condensation, statistical properties of long polymeric chains (as well as self-avoiding random walks), or percolation, and so on.

In fact, quantum field theory offers at present the most comprehensive framework to discuss physical systems that are characterized by a large number of strongly interacting local degrees of freedom.

However, at its birth, quantum field theory was confronted with a somewhat unexpected problem, the problem of *infinities*. The calculation of most physical processes led to infinite results. An empirical recipe, *renormalization*, was eventually discovered that allowed extracting from divergent expressions finite predictions. The procedure would hardly have been convincing if the predictions were not confirmed with increasing precision by experiment. A new concept, the *renormalization group* related in some way to the renormalization procedure, but whose meaning was only fully appreciated in the more general framework of the theory of phase transitions, has led, later, to a satisfactory interpretation of the origin and role of renormalizable quantum field theories and of the renormalization process.

This first chapter tries to present a brief history of the origin and the development of quantum field theory, and of the evolution of our interpretation of renormalization and the renormalization group, which has led to our present understanding.

This history has two aspects, one directly related to the theory of fundamental interactions that describes physics at the microscopic scale, and another one related to the theory of phase transitions in macroscopic physics and their universal properties. That two so vastly different domains of physics have required the development of the same theoretical framework, is extremely surprising. It is one of the attractions of theoretical physics that such relations can sometimes be found.

A few useful dates:

1925 Heisenberg proposes a quantum mechanics, under the form of a mechanics of matrices.

1926 Schrödinger publishes his famous equation that bases quantum mechanics on the solution of a non-relativistic wave equation. Since relativity theory was already well established when quantum mechanics was formulated, this may surprise.

In fact, for accidental reasons, the spectrum of the hydrogen atom is better described by a non-relativistic wave equation than by a relativistic equation without spin,* the Klein–Gordon equation (1926).

1928 Dirac introduces a relativistic wave equation that incorporates the spin 1/2 of the electron, which describes much better the spectrum of the hydrogen atom, and opens the way for the construction of a relativistic quantum theory. In the two following years, Heisenberg and Pauli lay out, in a series of articles, the general principles of quantum field theory.

1934 First correct calculation in quantum electrodynamics (Weisskopf) and confirmation of the existence of divergences, called ultraviolet (UV) since they are due, in this calculation, to the short-wavelength photons.

1937 Landau publishes his general theory of phase transitions.

1944 Exact solution of the two-dimensional Ising model by Onsager.

1947 Measurement of the so-called Lamb shift by Lamb and Retherford, which agrees well with the prediction of quantum electrodynamics (QED) after cancellation between infinities.

1947–1949 Construction of an empirical general method to eliminate divergences called *renormalization* (Feynman, Schwinger, Tomonaga, Dyson, *et al*).

1954 Yang and Mills propose a non-Abelian generalization of Maxwell's equations based on non-Abelian gauge symmetries (associated to non-commutative groups).

1954–1956 Discovery of a formal property of quantum field theory characterized by the existence of a *renormalization group* whose deep meaning is not fully appreciated (Peterman–Stückelberg, Gell-Mann–Low, Bogoliubov–Shirkov).

1967–1975 The Standard Model, a renormalizable quantum field theory based on the notions of non-Abelian gauge symmetry and spontaneous symmetry breaking, is proposed, which provides a complete description of all fundamental interactions, but gravitation.

1971–1972 After the initial work of Kadanoff (1966), Wilson, Wegner, *et al*, develop a more general concept of renormalization group, which includes the field theory renormalization group as a limit, and which explains universality properties of continuous phase transitions (liquid–vapour, superfluidity, ferromagnetism) and later of geometrical models like self-avoiding random walks or percolation.

1972–1975 Several groups, in particular Brézin, Le Guillou and Zinn-Justin, develop powerful quantum field theory techniques that allow a proof of universality properties of critical phenomena and calculating universal quantities.

1973 Using renormalization group arguments, Politzer and Gross–Wilczek establish the property of *asymptotic freedom* of a class of non-Abelian gauge theories, which allows explaining the free-particle behaviour of quarks within nucleons.

1975–1976 Additional information about universal properties of phase transitions are derived from the study of the non-linear σ model and the corresponding $d-2$ expansion (Polyakov, Brézin–Zinn-Justin).

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* intrinsic angular momentum of particles, that takes half-integer (fermions) or integer (bosons) values in units of $\hbar$.