

INTERMEDIATE ALGEBRA

Third Edition



Weltman  Perez



INTERMEDIATE ALGEBRA

Third Edition

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To our parents,
who have always encouraged
and supported us

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To the Instructor

Purpose

This text is designed for the student who has completed a course in introductory algebra. The purpose of *Intermediate Algebra, Third Edition* is not only to prepare the student for college algebra, but to do it in a non-threatening and organized way. This edition maintains the proven strengths of the successful second edition—the text is mathematically sound, yet written so that students feel comfortable using it. The new features of the third edition support this purpose.

New to the Third Edition

New Features

1. **Applied Algebra** At the beginning of each chapter an application from everyday life is discussed. These applications incorporate some of the mathematics that are covered in the chapter. At the end of each chapter the problem is solved

10

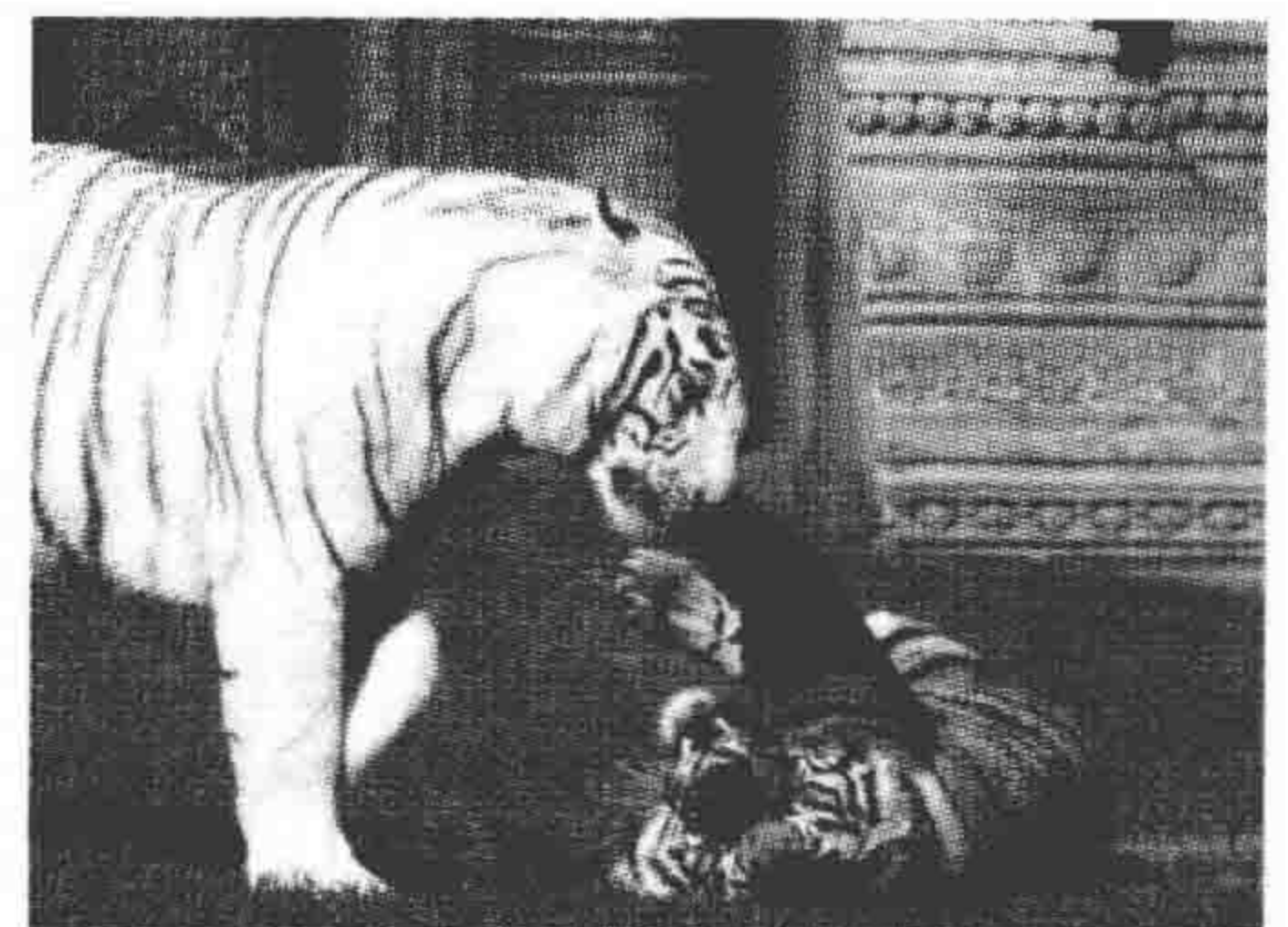
EXPONENTIAL AND LOGARITHMIC FUNCTIONS

The population of most animals in captivity is small in comparison to the animal population in the wild. This concerns zookeepers for two reasons. The inbreeding among a particular species may cause the loss of genetic variation and may introduce undesirable genetic traits. To understand this problem, scientists study the change in heterozygosity (genetic variation) from one generation to the next. A formula that predicts this change is

$$H_t = H_0 \left(1 - \frac{1}{2N}\right)^t$$

where H_0 is the initial heterozygosity, H_t is the heterozygosity after t generations, and N is the number of animals.

This equation is an example of an *exponential equation*. After we have studied exponential and logarithmic equations in this chapter, we will be able to answer questions such as these:



1. What will the heterozygosity be for New Guinea singing dogs after 50 years?
2. When will the heterozygosity for New Guinea singing dogs drop to 50% of its initial heterozygosity?

10.1 Inverse Functions


Given a function, many times we are interested in a function or relation that behaves in an opposite or inverse manner. In this chapter we will study the exponential function and its inverse, the logarithmic function. First we must develop a working knowledge of what we mean by the inverse of a function.


Let us start with a simple example. The equation $y = 2x$ defines a function that takes a value of x and doubles it to get the corresponding value of y . The diagram in Figure 10.1.1 illustrates how values of x in the domain are doubled to get values for y in the range. These ordered pairs are listed in the chart accompanying Figure 10.1.1:

Write Algebra

49. Describe the types of nonlinear systems that are more easily solved using the substitution method.
50. Given a system containing a linear and a quadratic function, why can the solution set contain at most two ordered pairs?

and similar problems are given as exercises. These real-world applications drive home the usefulness of mathematics.

2. **Write Algebra** At the end of most exercise sets “Write Algebra” problems are included. These writing problems require the students to reflect on what they have learned in the section. We feel that the process of putting ideas and concepts into one’s own words can be very beneficial. These problems are at the end of each exercise set, and therefore they can be omitted without difficulty if desired. Since students are asked to use their own thoughts and wording, most of these problems do not have answers in the back of the book. Answers are included when one answer is appropriate.
3. **Graphing Calculator** In several sections you will find examples and exercises that are appropriate to work on a graphing calculator, clearly marked by a graphing calculator screen . Their purpose is to introduce the power of technology at an appropriate level. The examples and exercises center heavily on pattern recognition. Appendix D goes into greater detail on the basics of using the TI-81. These problems are optional and therefore easily skipped without losing continuity.

 The following exercises require the use of a graphing calculator. Use the Trace and Zoom keys of a graphing calculator to approximate the solution (the point of intersection) of the following systems.

51. $3x + 2y = 4$
 $y = x^2$

53. $y = x^2 + 3x - 5$
 $y = -0.5x^2 - 2x + 6$

52. $x^2 + y^2 = 4$
 $x + y = 2$

54. $x^2 + y^2 = 45$
 $y = 0.8x^2 + 3x - 6.4$

4. **Design** Color is used pedagogically throughout, including full-color chapter opening photos linked to the *Applied Algebra* problems.



New Coverage and Organization

1. A section reviewing the techniques of factoring was added to Chapter 3.
2. A set of exercises requiring students to distinguish between equations involving rational expressions and operations involving rational expressions has been added to Section 4.8.
3. Section 6.2 on Relations and Functions from the previous edition has been divided into two sections, resulting in a new Section 6.3 on Function Notation and Combinations of Functions.
4. The Midpoint Formula is new to Section 6.6.
5. Appendix C on Permutations, Combinations, and Probability is also a new feature.

Established Strengths of the Weltman/Perez Series

Features

Our motivation for writing this text was the traditionally high attrition rate in college algebra classes. We have attempted to write a book that will help a student with a weak algebra background bridge the gap between intermediate and college algebra. To do this, we have included the following features:

- 1. Conversational Writing Style** We have attempted to address the student in a conversational style to make the material less formal and distant without, however, sacrificing mathematical integrity. We also have injected some humor to make the subject less dry to a student who typically views math with a combination of fear and boredom.
- 2. Examples with Detailed Step-by-Step Solutions** Examples are thoroughly worked out, including every step that a student should need to follow the solutions.
- 3. Incorrect/Correct Boxes** Boxes containing common student errors are provided to warn the student of mistakes to avoid.
- 4. Exercises** Each exercise set contains a large number of exercises for practice for a total of over 6,500 exercises. Each exercise set starts with problems that test basic skills. The degree of difficulty gradually increases through the set. Similar exercises are typically paired so that a student has both an even and an odd problem that are similar. This encourages students to check their answers yet gives them practice doing problems without the added security of the answer. Calculator exercises are marked appropriately—scientific  and graphing .
- 5. Applications** Applications, while primarily presented in four sections devoted entirely to word problems, are included wherever appropriate throughout the text (see, for example, the exercises in Sections 3.1, 6.2, 7.2, and 10.3). In addition, applications are used at times to motivate the introduction of a new topic (see, for example, the beginning of Section 8.3).
- 6. Reviews** All the important definitions, properties, and formulas are repeated in the chapter reviews to help the student remember them. In addition, the review exercises contain an ample number of problems, including all the different types of problems covered in the chapter. A timed chapter test also helps the student determine if he or she has mastered the concepts of that chapter (see, for example, the Chapter 4 review). Chapters 2 through 11 end with cumulative exercises, helping the student to continually review topics covered in previous chapters.
- 7. Advanced Topics** Some more advanced topics, such as functions and higher-degree equations, are included, but at an introductory level.

Organization

In deciding in what order to present the topics in our book, we were aware of varying needs in intermediate algebra classrooms. Our reasons for choosing the present order of topics include the following:

1. Much of the material in the first three or four chapters will be review; however, we have found that the student can benefit from reviewing some fundamental concepts. While you may choose not to spend class time on some or all of this material, having it at the beginning of the book helps the student easily find a point he or she may want to back up to, in order to review.
2. **The Properties of Exponents** are presented in three chapters—3, 4, and 5. The student is given the first treatment of the product laws of exponents in Chapter 3 and then has ample opportunity to work with them before the next discussion in Chapter 4. Here the student receives the remainder of the definitions and laws governing integer exponents, and he or she is given opportunity to work with them in several sections. Finally, the presentation of rational exponents and radicals in Chapter 5 reinforces the laws of exponents further.
3. **Solving Quadratic Equations by Factoring** was placed in Chapter 3 so that equations leading to quadratic equations could be included in Sections 4.8 (rational equations), 4.9 (applications of rational equations), and 5.6 (radical equations).
4. **Graphing Functions and Equations in Two Variables** are introduced early, in Chapter 6, so that these important concepts for college algebra can be reinforced several times in Chapters 7, 8, 9, and 10.

Supplements

A comprehensive package of supplements is available to facilitate teaching and learning.

For the Instructor

1. **Instructor's Edition** includes answers to all of the exercises in the text including over 800 figures.
2. **Test Bank** contains alternative test forms, including:
 - seven tests per chapter—four free-response and three multiple choice
 - three midterms—two free-response and two multiple choice
 - three final examinations—two free-response and two multiple choice
3. **Computerized Test Generators** are available to adopters of the text in the following formats: **EXP Test** for IBM PC and compatibles and **Exambuilder** for the Macintosh.

For the Student

1. **Student Partial Solutions Manual/Study Guide** consists of worked-out step-by-step solutions to all of the odd-numbered end-of-section exercises, chapter

review exercises, test questions, and Test Your Memory questions. A self-test for each chapter with solutions is also included. This was written by Richard Spangler, Tacoma Community College.

2. **Tutorial Software** is based on the interactive AUTHORWARE[™] program. This text-specific, intuitive tutorial runs on Microsoft[™] WINDOWS and Macintosh platforms. Students are given questions to answer. If they give an incorrect answer, the program will respond with hints; then the student has another opportunity to answer the question. If the student answers incorrectly a second time, the program responds with the correct answer and a step-by-step solution. An on-screen, button-operated calculator can be pulled up for additional help. Password-protected teacher utilities are provided to access their students records. Individual student scorekeeping capabilities allows students to monitor their progress and score in each section of the exercises.
3. **Videotape Series** reviews key topics in the text and features professional math instructors. The videotapes are free to qualified adoptors and students may visit your math laboratory to view the tapes of their choice. These tutorial tapes, produced by Educational Video Resources, can significantly improve students' comprehension and performance in algebra.

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Dennis Weltman
Gilbert Perez

To the Student

Have you ever watched a tennis match on television and marveled at how the pros made it look so easy? You might have noticed that your math instructor makes algebra look easy. And yet in both cases, when you try to play tennis, or work some algebra problems, you find it is not so easy. What's the reason? A tennis pro practices for hours each day. He or she played tennis for years before becoming good enough to play professionally. Your math instructor has been working with math for *years*, spending many hours a day in preparation for class. This is not to imply that you should strive for, or expect to achieve, the level of proficiency of your math instructor. However, if you want to do well in this math course you have to practice. The more you practice, the better you should be. If you don't want to waste your time and money registering for the same algebra course semester after semester, here are some suggestions:

1. *Attend class regularly and ask questions.* Almost no student can learn algebra without going to class. You need to see and hear the instructor's explanations. While you can't expect to understand everything, you should have an idea of how to work the problems. If not, *ask!*
2. *Do your homework soon after class.* You will forget most of what was discussed in class within a few hours unless you do your homework during this time. Waiting until the last minute before the next class meeting to do your homework is the worst thing you can do. Work all problems that are assigned. *Do not* skip over sections or groups of problems that you think are "easy." They may very well be easy for you but you need to practice in order to reduce your chances of making a careless mistake. Remember the tennis pro! If you have time, do problems that were not assigned. Use your notes and text as a reference while doing your homework. However, make it a habit to work some problems during each session without any notes or the textbook. This practice pays off at test time.
3. *Read the book carefully.*
 - It is best to read each section before the material is discussed in class. Try to follow the examples and work some of the exercises. Write questions you have in the book itself. Make notes to yourself in the book. Underline sentences that explain key concepts.

- After class, reread the section carefully. Read the statement of the problem in each example and try to solve the problem before reading the solution given in the example.
 - Exercises that may require a **calculator** are identified by a small calculator that looks like this: scientific  and graphing .
 - We have boxed some **common mistakes**, labelled Incorrect/Correct, that many students make. Make a special note of these when you work the exercises.
 - The **answers** to the odd-numbered exercises are in the back of the book. You should not look at the answers until you have finished working each problem. Remember, on a quiz or test you don't have the answers to refer to. When working "Write Algebra" problems, you will find most will not have answers in the back. Try to work these problems by expressing your own thoughts in your own words. This can be a big help in becoming familiar with concepts.
 - When you have completed a chapter, use the **Chapter Reviews** to go back over the important properties, formulas, and so on, that were introduced in that chapter. Exercises from each section are provided for you to get more practice. When you think you have reviewed the chapter sufficiently, take the timed **Chapter Test** provided. The **Test Your Memory** exercises at the end of each chapter will help to keep "old" topics fresh in your mind.
4. *Try to make the class fun.* Get into a friendly competition with someone from your class. Study with other people in a small group. Explain concepts to each other when someone in the group does not understand something. You will find that this benefits the tutor as well as the tutee.
 5. If you need additional help, a *Student Solutions Manual/Study Guide* by Richard Spangler, Tacoma Community College is also available. Ask your instructor if the videotapes and computer software supplements are available at your campus.

INTERMEDIATE ALGEBRA



1

FUNDAMENTAL CONCEPTS

The National Weather Service warns that if the following conditions are expected to prevail for 3 hr or longer, then a blizzard warning must be issued.

1. Sustained wind speeds of 35 mph or greater or frequent gusts to 35 mph or greater, and
2. Considerable falling and/or blowing snow, frequently reducing the visibility to less than $\frac{1}{4}$ mi.

Although there is no set temperature requirement for a blizzard warning, when the temperature falls below 20°F , forecasters should highlight the life-threatening nature of the cold temperature in combination with other hazardous conditions such as the wind. The chilling effect of the wind during cold temperatures is called the **wind-chill factor**. Although the thermometer may indicate the temperature is 20°F , a 35-mph wind will make it seem as if the temperature is much colder. The formula to determine wind chill is an example of an algebraic expression. At the end of this chapter we will be able to compute what temperature we would perceive under the conditions just described.

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