ELECTROMAGNETIC INTERACTIONS OF HADRONS
VOLUME 1

Edited by A. Donnachie and G. Shaw

Nuclear Physics Monographs
Series Editors: Erich W. Vogt and John W. Negele

Electromagnetic Interactions of Hadrons
Volume 1

Edited by

A. Donnachie
and
G. Shaw
University of Manchester, England

Plenum Press · New York and London

Library of Congress Cataloging in Publication Data

Main entry under title:

Electromagnetic interactions of hadrons.

(Nuclear physics monographs)
Bibliography: p.
Includes index.
1. Hadrons. 2. Electromagnetic interactions. I. Donnachie, A. II. Shaw, G. III. Series.
QC793.5.H328E44 539.7'5 77-17811
ISBN 0-306-31052-X

A Division of Plenum Publishing Corporation
227 West 17th Street, New York, N.Y. 10011

Printed in the United States of America

Contributors

A. Donnachie, Department of Theoretical Physics, University of Manchester, Manchester, England

A. J. G. Hey, Department of Physics, University of Southampton, Southampton, England

D. W. G. S. Leith, Stanford Linear Accelerator Center, Stanford University, Stanford, California

D. H. Lyth, Department of Physics, University of Lancaster, Lancaster, England

R. G. Moorhouse, Department of Natural Philosophy, University of Glasgow, Glasgow, Scotland

G. Shaw, Department of Theoretical Physics, University of Manchester, Manchester, England

J. K. Storrow, Department of Theoretical Physics, University of Manchester, Manchester, England

Preface

While electromagnetic interactions were first used to probe the structure of elementary particles more than 20 years ago, their importance has only become fully evident in the last 10 years. In the resonance region, photoproduction experiments have provided clear evidence for simple quark model ideas, and confirmed the Melosh-transformed $SU(6)_W$ as a relevant symmetry classification. At higher energies, their most striking feature is their similarity to hadron-induced reactions, and they have provided fresh insight into the ideas developed to explain strong-interaction physics. New dimensions are added by taking the photon off mass shell, both in the spacelike region, where the development of high-energy electron and muon beams has led to the discovery and study of scaling, and the introduction of "partons," and even more dramatically in the timelike region, where the development of high-energy electron–positron storage rings has led to the exciting discoveries of the last four years.

In view of the immense interest stimulated by these developments, an extensive review of our present state of knowledge is both timely and useful. Because of the very wide range of the subject, a cooperative venture presents itself as the most suitable format and is the one we have adopted here. The emphasis throughout is primarily, but not entirely, on phenomenology, concentrating on describing the main features of the experimental data and on the theoretical ideas used directly in their interpretation. As such we hope that it will be of interest and of use to all practicing physicists in the field of elementary particles, including graduate students.

The work is in two volumes. This volume deals with photoproduction and electroproduction in the resonance region and at medium energies, treating mainly two-body and quasi-two-body final states. The companion volume first considers multiparticle production and inclusive reactions, and then goes on to tackle deep inelastic scattering and electron–positron annihilation.

We are deeply indebted to the many authors who have contributed to this work. Their adherence to the proposed guidelines greatly eased the problems of editing, and contributed significantly towards achieving a balanced presentation.

We would like to thank Mrs. S. A. Lowndes of Daresbury Laboratory, for her invaluable assistance in the technical editing of the articles in both this and the companion volume.

Manchester, 1978

A. Donnachie
G. Shaw

Contents of Volume 2

Contents

Chapter 1

Quarks and Symmetries

A. J. G. Hey

Chapter 2

Electromagnetic Excitation and Decay of Hadron Resonances

R. G. Moorhouse

Chapter 3

Low-Energy Pion Photoproduction

A. Donnachie and G. Shaw

Chapter 4

Exclusive Electroproduction Processes

D. H. Lyth

Chapter 5

Form Factors and Electroproduction

A. Donnachie, G. Shaw, and D. H. Lyth

Chapter 6

High-Energy Photoproduction: Nondiffractive Processes

J. K. Storrow

1

Quarks and Symmetries

A. J. G. Hey

1. Introduction

Quarks were first invoked by Gell-Mann and Zweig over ten years ago (Gell-Mann, 1964; Zweig, 1964). They provided some sort of "explanation" for the success of the Eightfold Way based on $SU(3)$ symmetry. One of the novel features about $SU(3)$ symmetry was that the smallest representation, the basic triplet, did not seem to be realized by nature. Gell-Mann and Zweig proposed that in fact it was, but perhaps only in the sense of a building block, q, with the baryons composed of three quarks ($3q$) and mesons of a quark and antiquark ($q\bar{q}$). However, there are several peculiar features about such a scheme. To overcome the difficulty of the earlier Sakata model (Sakata, 1956) the quarks were assigned nonintegral charge and baryon number. Such curious properties have stimulated a painstaking search for quarks as physical particles in their own right, but so far, at least, with no success. One possibility is, of course, that free quarks do not exist and the nonintegral quantum numbers are a flag to tell us that quarks only exist in bound states. Theorists have been trying to embody this "unobservability" in a field-theoretic framework, and one hopeful avenue stems from another curious feature of the quarks. To build up hadrons of integral and half-integral spin, the most economical scheme is to endow the quarks with spin $\frac{1}{2}$. Thus one expects them to be fermions—but if so, they appear to obey "funny" statistics. For this and for other theoretical reasons, one popular hypothesis is that the three quarks come

A. J. G. Hey • Department of Physics, University of Southampton, Highfield, Southampton SO9 5NH, England

in three colors and are triplets of an $SU(3)$ color group. Nonobservability of quarks then finds a simple expression in the statement that all observed hadrons must be color singlets. Non-Abelian gauge theories based on this $SU(3)$ color local gauge group may provide quark confinement.

For the present article it will not be necessary to commit oneself to one or other particular scheme. Instead, the purpose will be to demonstrate that the quark hypothesis has definite phenomenological validity in that there is "something more" than $SU(3)$ present in nature. However, there are many approaches to quarks and quark models, and the one we shall adopt in this article attempts to avoid detailed dynamical assumptions and concentrates on symmetries and algebraic structures. We begin by describing a simple model for the spectrum of excited baryon and meson states based on an $SU(6)\otimes O(3)$ symmetry. Since this is a classification group, these quarks are dubbed "constituent" or "classification" quarks. After introducing the mechanics of $SU(6)$, we discuss the extension of such symmetries to decay processes. The problems here lead us rapidly to the algebraic quark predictions of current algebra, whose postulates may be phrased in terms of "current" quark fields. A discussion of current algebra sum rules and infinite momentum leads naturally to an $SU(6)$ algebra of the so-called good null-plane charges. Instead of discussing some of the other possible approaches to quarks and quark predictions [for comprehensive reviews see Lipkin (1973) and Rosner (1974)], we describe recent attempts by Melosh and others (Melosh 1973; Bucella *et al.* 1970) to knit together the "constituent" and "current" quark approaches to $SU(6)$. Although there are many dynamical questions yet to be answered, the use of the Melosh transformation leads to a unifying and extremely useful framework for algebraic $SU(6)$ phenomenology. We review and contrast the success and scope of this scheme with the predictions of some less formal, more intuitive quark models.

Finally, in the last section we arrive at full circle. In 1964, $SU(3)$ and quarks were introduced to bring order and simplicity to the excited hadron spectrum. The spectrum of new particles discovered at Brookhaven and SLAC in 1974 may upset our preconceived notions about quarks and perhaps generate some real progress in quark spectroscopy!

2. Constituent Quarks

2.1. Introduction to SU(3) and SU(6)

The search for regularities in the hadron resonance spectrum is the starting point of many symmetry schemes. The baryons and mesons are observed to fall into very approximately degenerate $SU(3)$ multiplets. The

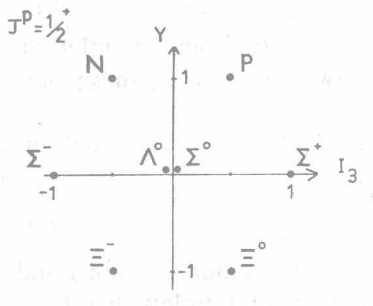

Fig. 1. Nucleon octet.

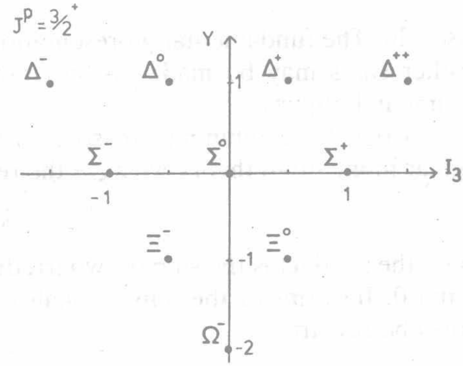

Fig. 2. Delta decuplet.

observed representations may be summarized by the rule

Baryons: **10, 8, 1**

Mesons: **8, 1**

Resonances not falling into these representations are called "exotic," and so far no exotic multiplets have definitely been observed. Figures 1–4 show the lowest-lying baryon and meson $SU(3)$ multiplets—namely, the nucleon octet, delta decuplet, and the pion and rho meson "nonets"—octets and singlets. The multiplets are plotted on an $SU(3)$ weight diagram with I_3 versus Y ($Y = B + S$, the hypercharge). Clearly $SU(3)$ is a much more approximate symmetry than isospin, since the mass splitting within an $SU(3)$ multiplet is much greater.

How can one explain that only **1**'s, **8**'s and **10**'s of $SU(3)$ have been observed? The constituent quark model provides a mnemonic that accounts for this in a very simple way.

Before we discuss combining $SU(3)$ quarks, it is helpful to obtain some insight from the more familiar $SU(2)$ case of angular momentum or

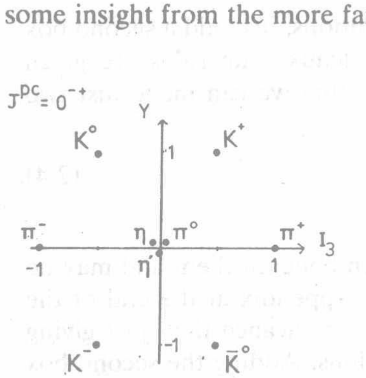

Fig. 3. Pseudoscalar meson nonet.

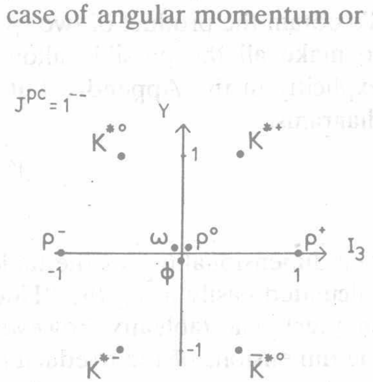

Fig. 4. Vector meson nonet.

isospin. The fundamental representation of $SU(2)$ is the spin-$\frac{1}{2}$ doublet: all other spins may be made up by combining two or more of these fundamental objects.

Consider combining two spin-$\frac{1}{2}$ representations. From the usual angular momentum theory we have the result

$$\tfrac{1}{2}\otimes\tfrac{1}{2} = 1\oplus 0 \tag{2.1}$$

i.e., the product is the sum of two irreducible representations of spin 1 and spin 0. In terms of the dimensionality of the $SU(2)$ multiplets, Eq. (2.1) may be rewritten

$$2\otimes 2 = 3\oplus 1 \tag{2.2}$$

since there are three spin-1 states (spin projections ± 1 and 0) and one spin 0. Furthermore, we know something about the symmetry of the wave functions. Labeling the two spin-$\frac{1}{2}$ representations 1 and 2, the wave functions are explicitly

$$S=1:\quad \begin{cases} \uparrow_1\uparrow_2 \\ 2^{-1/2}(\uparrow_1\downarrow_2+\downarrow_1\uparrow_2) \\ \downarrow_1\downarrow_2 \end{cases} \tag{2.3}$$

$$S=0:\quad 2^{-1/2}(\uparrow_1\downarrow_2-\downarrow_1\uparrow_2)$$

i.e., the spin-1 wave functions are symmetric under the exchange $1\leftrightarrow 2$ and the spin-0 wave function is antisymmetric.

All these properties may be elegantly summarized using a technique invented by an English clergyman named Young (Young, 1901). The fundamental spin-$\frac{1}{2}$ representation corresponds to the "Young diagram" (box) shown below:

$$\tfrac{1}{2}:\quad \square \quad \text{dimensionality } 2$$

To obtain the product of two spin-$\frac{1}{2}$ representations, just add a second box to make all the possible allowed Young tableaux. The rules are given explicitly in the Appendix, but it is plausible that we can make just two diagrams:

$$\underset{2}{\square}\otimes\underset{2}{\square} = \underset{3}{\square\square}\oplus\underset{1}{\begin{array}{c}\square\\ \square\end{array}} \tag{2.4}$$

The dimensionalities of the tableaux are given beneath them and may be calculated easily using the "Hook" rule (see Appendix at the end of the chapter). The tableaux, however, have more significance than just giving the dimensions of the irreducible representations. Adding the second box on the same row corresponds to symmetrizing 1 and 2, while adding in a second column corresponds to antisymmetrizing. Thus the diagrams correspond exactly to the explicit wave functions we wrote down. For $SU(2)$, a column of two boxes is the maximum length of column allowed and corresponds to the singlet or spin-0 representation. Now add a third spin-$\frac{1}{2}$ particle. We know that the answer is

$$1 \otimes \tfrac{1}{2} = \tfrac{3}{2} \oplus \tfrac{1}{2}$$

and (2.5)

$$0 \otimes \tfrac{1}{2} = \tfrac{1}{2}$$

Thus

$$\tfrac{1}{2} \otimes \tfrac{1}{2} \otimes \tfrac{1}{2} = \tfrac{3}{2} \oplus \tfrac{1}{2} \oplus \tfrac{1}{2}$$

or (2.6)

$$2 \otimes 2 \otimes 2 = 4 \oplus 2 \oplus 2$$

What about the symmetries of these wave functions? Clearly the spin-$\frac{3}{2}$ wave function is "symmetric" between spin 1 and spin $\frac{1}{2}$ (the "stretched" state is symmetric under $j_1 \leftrightarrow j_2$). Since the spin-1 state is symmetric between 1 and 2, the spin-$\frac{3}{2}$ state is in fact totally symmetric between 1, 2, and 3. For example:

$$\begin{aligned} |\tfrac{3}{2}\ \tfrac{3}{2}\rangle &= \uparrow_1 \uparrow_2 \uparrow_3 \\ |\tfrac{3}{2}\ \tfrac{1}{2}\rangle &= 3^{-1/2}(|11\rangle \downarrow_3 + 2^{1/2}|10\rangle \uparrow_3) \qquad (2.7) \\ &= 3^{-1/2}(\uparrow_1 \uparrow_2 \downarrow_3 + \uparrow_1 \downarrow_2 \uparrow_3 + \downarrow_1 \uparrow_2 \uparrow_3) \end{aligned}$$

The spin-$\frac{1}{2}$ state obtained from the spin 1 and $\frac{1}{2}$ is "antisymmetric" between spin 1 and spin $\frac{1}{2}$ ($j_1 \leftrightarrow j_2$) and we have

$$\begin{aligned} |\tfrac{1}{2}\ \tfrac{1}{2}\rangle &= 3^{-1/2}(2^{1/2}|11\rangle \downarrow_3 - |10\rangle \uparrow_3) \\ &= 6^{-1/2}(2\uparrow_1 \uparrow_2 \downarrow_3 - \uparrow_1 \downarrow_2 \uparrow_3 - \downarrow_1 \uparrow_2 \uparrow_3) \qquad (2.8) \end{aligned}$$

i.e., symmetric between 1 and 2 but not totally symmetric between 1, 2, and 3. This is known as a state of mixed symmetry. From spin 0 with spin $\frac{1}{2}$ we obtain

$$\begin{aligned} |\tfrac{1}{2}\ \tfrac{1}{2}\rangle &= |00\rangle \uparrow_3 \\ &= 2^{-1/2}(\uparrow_1 \downarrow_2 \uparrow_3 - \downarrow_1 \uparrow_2 \uparrow_3) \qquad (2.9) \end{aligned}$$

i.e., a spin-$\frac{1}{2}$ state of different mixed symmetry—antisymmetric between 1 and 2. All this is again evident from the corresponding Young tableaux

$$1 \otimes \tfrac{1}{2} = \tfrac{3}{2} \oplus \tfrac{1}{2}$$

$$\underset{3}{\square\square} \otimes \underset{2}{\square} = \underset{4}{\square\square\square} \oplus \begin{array}{l}\square\square \\ \square_{2}\end{array} \qquad (2.10a)$$

and

$$0 \otimes \tfrac{1}{2} = \tfrac{1}{2}$$

$$\begin{smallmatrix}\square\\\square\end{smallmatrix} \otimes \square = \begin{smallmatrix}\square\square\\\square\end{smallmatrix} \qquad (2.10b)$$

$$\underset{1}{} \quad \underset{2}{} \quad \underset{2}{}$$

[For $SU(n)$, n boxes in a column corresponds to the singlet and the column can be effectively removed. No more than n boxes can appear in a column.]

Thus for our three spin-$\frac{1}{2}$ decomposition we have in $SU(2)$

$$\underset{2}{\square} \otimes \underset{2}{\square} \otimes \underset{2}{\square} = \underset{4}{\square\square\square} \oplus \underset{2}{\begin{smallmatrix}\square\square\\\square\end{smallmatrix}} \oplus \underset{2}{\begin{smallmatrix}\square\square\\\square\end{smallmatrix}} \qquad (2.11)$$

where the two spin-$\frac{1}{2}$ combinations clearly have different symmetry properties with respect to the three basic objects. They are mixed symmetry states and the spin-$\frac{3}{2}$ state is totally symmetric.

Now let us imitate this with $SU(3)$ using the basic triplet representation (which we may as well call a quark) as a building block (see Fig. 5). Using the standard graphical or tensor methods, one can show that the two-quark product decomposes into the sum of two irreducible representations [this may most easily be proved by a generalization of the usual raising and lowering technique that one uses in $SU(2)$]:

$$\mathbf{3} \otimes \mathbf{3} = \mathbf{6} \oplus \bar{\mathbf{3}} \qquad (2.12)$$

The members of the $\bar{\mathbf{3}}$ representation are illustrated in Fig. 5. Again the explicit wave functions show that the $\mathbf{6}$ is symmetric between 1 and 2 and the $\bar{\mathbf{3}}$ is antisymmetric. For example,

$$\begin{aligned}
\psi\{\mathbf{6}: I = 1 \;\; I_3 = 1\} &= u_1 u_2 \\
\psi\{\mathbf{6}: I = 1 \;\; I_3 = 0\} &= 2^{-1/2}(u_1 d_2 + d_1 u_2) \qquad (2.13) \\
\psi\{\mathbf{6}: I = \tfrac{1}{2} \;\; I_3 = \tfrac{1}{2}\} &= 2^{-1/2}(u_1 s_2 + s_1 u_2)
\end{aligned}$$

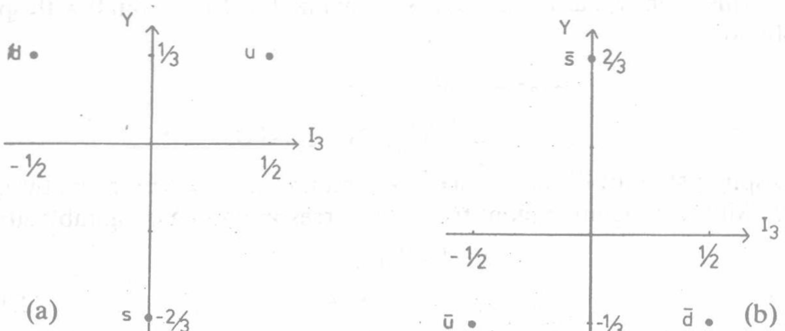

Fig. 5. (a) Quark triplet. (b) Antiquark triplet.