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Editors

Advances in Fractional Calculus

Theoretical Developments and Applications in Physics and Engineering

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Theoretical Developments and Applications in Physics and Engineering

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ADVANCES IN FRACTIONAL CALCULUS

We dedicate this book to the honorable memory of our colleague and friend Professor Peter W. Krempl

Preface

Fractional Calculus is a field of applied mathematics that deals with derivatives and integrals of arbitrary orders (including complex orders), and their applications in science, engineering, mathematics, economics, and other fields. It is also known by several other names such as Generalized Integral and Differential Calculus and Calculus of Arbitrary Order. The name "Fractional Calculus" is holdover from the period when it meant calculus of ration order. The seeds of fractional derivatives were planted over 300 years ago. Since then many great mathematicians (pure and applied) of their times, such as N. H. Abel, M. Caputo, L. Euler, J. Fourier, A. K. Grünwald, J. Hadamard, G. H. Hardy, O. Heaviside, H. J. Holmgren, P. S. Laplace, G. W. Leibniz, A. V. Letnikov, J. Liouville, B. Riemann M. Riesz, and H. Weyl, have contributed to this field. However, most scientists and engineers remain unaware of Fractional Calculus; it is not being taught in schools and colleges; and others remain skeptical of this field. There are several reasons for that: several of the definitions proposed for fractional derivatives were inconsistent, meaning they worked in some cases but not in others. The mathematics involved appeared very different from that of integer order calculus. There were almost no practical applications of this field, and it was considered by many as an abstract area containing only mathematical manipulations of little or no use.

Nearly 30 years ago, the paradigm began to shift from pure mathematical formulations to applications in various fields. During the last decade Fractional Calculus has been applied to almost every field of science, engineering, and mathematics. Some of the areas where Fractional Calculus has made a profound impact include viscoelasticity and rheology, electrical engineering, electrochemistry, biology, biophysics and bioengineering, signal and image processing, mechanics, mechatronics, physics, and control theory. Although some of the mathematical issues remain unsolved, most of the difficulties have been overcome, and most of the documented key mathematical issues in the field have been resolved to a point where many of the mathematical tools for both the integer- and fractional-order calculus are the same. The books and monographs of Oldham and Spanier (1974), Oustaloup (1991, 1994, 1995), Miller and Ross (1993), Samko, Kilbas, and Marichev (1993), Kiryakova (1994), Carpinteri and Mainardi (1997), Podlubny (1999), and Hilfer (2000) have been helpful in introducing the field to engineering, science, economics and finance, pure and applied mathematics communities. The progress in this field continues. Three

recent books in this field are by West, Grigolini, and Bologna (2003), Kilbas, Srivastava, and Trujillo (2005), and Magin (2006).

One of the major advantages of fractional calculus is that it can be considered as a super set of integer-order calculus. Thus, fractional calculus has the potential to accomplish what integer-order calculus cannot. We believe that many of the great future developments will come from the applications of fractional calculus to different fields. For this reason, we are promoting this field. We recently organized five symposia (the first symposium on Fractional Derivatives and Their Applications (FDTAs), ASME-DETC 2003, Chicago, Illinois, USA, September 2003; IFAC first workshop on Fractional Differentiations and its Applications (FDAs), Bordeaux, France, July 2004; Mini symposium on FDTAs, ENOC-2005, Eindhoven, the Netherlands, August 2005; the second symposium on FDTAs, ASME-DETC 2005, Long Beach, California, USA, September 2005; and IFAC second workshop on FDAs, Porto, Portugal, July 2006) and published several special issues which include Signal Processing, Vol. 83, No. 11, 2003 and Vol. 86, No. 10, 2006; Nonlinear dynamics, Vol. 29, No. 1–4, 2002 and Vol. 38, No. 1–4, 2004; and Fractional Differentiations and its Applications, Books on Demand, Germany, 2005. This book is an attempt to further advance the field of fractional derivatives and their applications.

In spite of the progress made in this field, many researchers continue to ask: "What are the applications of this field?" The answer can be found right here in this book. This book contains 37 papers on the applications of Fractional Calculus. These papers have been divided into seven categories based on their themes and applications, namely, analytical and numerical techniques, classical mechanics and particle physics, diffusive systems, viscoelastic and disordered media, electrical systems, modeling, and control. Applications, theories, and algorithms presented in these papers are contemporary, and they advance the state of knowledge in the field. We believe that researchers, new and old, would realize that we cannot remain within the boundaries of integral order calculus, that fractional calculus is indeed a viable mathematical tool that will accomplish far more than what integer calculus promises, and that fractional calculus is the calculus for the future.

Most of the papers in this book are expanded and improved versions of the papers presented at the Mini symposium on FDTAs, ENOC-2005, Eindhoven, The Netherlands, August 2005, and the second symposium on FDTAs, ASME-DETC 2005, Long Beach, California, USA, September 2005. We sincerely thank the ASME for allowing the authors to submit modified versions of their papers for this book. We also thank the authors for submitting their papers for this book and to Springer-Verlag for its

publication. We hope that readers will find this book useful and valuable in the advancement of their knowledge and their field.



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