

Quantum Theory and Gravitation

Edited by

A.R. Marlow

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*Department of Physics
Loyola University
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PREFACE

This volume contains the results of the second in a series of meetings at Loyola in New Orleans between physicists and mathematicians concerned with the fundamental questions of modern theoretical physics. The first conference in June 1977 centered around foundational problems in quantum theory. Emboldened by the success of that meeting, the conference organizers (C. H. Brans and A. R. Marlow) decided to cast their net in a wider arc and invite a distinguished group of participants to focus their collective talents in May 1979 on the unification of the two theories that define twentieth century physics: quantum theory and general relativity.

The diversity of the participants is reflected in the diversity of the final results presented in this volume: overviews designed to locate and clearly define the problem areas, specific solutions to problems in quantum theory or relativity, frontal attacks on the central question itself. One of the premises from which a conference of this type arises is the belief that mathematicians and physicists talking and working together can accomplish more than either group separately. We believe this premise has been sustained. In particular, the formative influence on the shape of the final results contributed by a distinguished group of stimulators, facilitators, constructive critics, and just plain good listeners (Andrew M. Gleason, R. J. Greechie, Paul R. Halmos, Cecile DeWitt-Morette, Bryce S. DeWitt) should not go unrecognized. Throughout the conference the participation of John A. Wheeler, the one figure in modern physics who has perhaps contributed most definitively to our present understanding of both quantum theory and gravitation, acted as a yeast to make the whole thing rise.

Albert Einstein founded relativity theory and contributed mightily to quantum theory. The success of the present efforts to unify the two theories in the centennial year of his birth must be left to the judgment of the reader. As a possible guide through this volume, one might read first Wheeler's survey of pregeometry as foundation, then Brans' study of the problem areas, then Marlow's construction of a relativistic quantum model from quantum logic as pregeometry, and finally use Blakemore's conference survey (last paper) as a key to the full variety of the contributions.

Needless to say, the contributions of so many persons are needed to make a conference and publication of this type come about that no listing can be adequate. However, we feel that we must attempt to repay in some small way our debt of

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PREGEOMETRY: MOTIVATIONS AND PROSPECTS

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Spacetime is normally treated as a continuum. Likewise for many purposes, an elastic medium is usefully regarded as a continuum or a piece of cloth as continuum. However, an elastic substance reveals at a crack that the concept of "ideal elastic medium" is a fiction. Cloth shows at a selvege that it is not a continuous medium but woven out of thread. Spacetime--with or without "gauge" or "phase" or "internal spin" degrees of freedom--often considered to be the ultimate continuum of physics, evidences nowhere more clearly than at big bang and at collapse that it cannot be a continuum. Obliterated in those events is not only matter, but the space and time that envelop that matter. If the elastic medium is built out of electrons and nuclei and nothing more, if cloth is built out of thread and nothing more, we are led to ask out of what "pregeometry" the geometry of space and spacetime are built. What are the motives for asking about "pregeometry;" and what the clues?

Investigators of an earlier age often asked, "Why does space have dimension three?" Many interesting proposals were

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put forward in response to this question (Kant, 1868; Poincaré, 1913; Ehrenfest, 1917; Reichenbach, 1928; Weyl, 1963). In the end, however, we can believe it is the wrong question for understanding space. Might we not more appropriately ask, "How does the world manage to give the impression that it has dimension three?"

A "homogeneous isotropic elastic substance" manages to give the impression that it has two and only two elastic constants. Nevertheless, a closer look shows that the very concept of elasticity is only an approximation. There is no such thing as "elasticity" in the space between the electron and the nucleus. Moreover, a hundred years of the study of elasticity would never have revealed atoms, molecules and the complicated dependence on distance of the forces that hold them together. Nor would a hundred years of the study of atomic and molecular forces have revealed that they go back for their origin to electrons, nuclei and Schrödinger's equation and nothing more. The direction of understanding went, not from the large to the small, but from the small to the large. If elasticity was the last place to look for a clue to Schrödinger's equation, geometry would seem the last place to look for a clue to pregeometry.

It is possible to dispose of two unproductive ideas of pregeometry--a lattice and Borel set--and yet confess the total absence today of any productive idea of pregeometry.

The word "dozens" is too small to describe the number of papers which treat space as a lattice. They include (1) Heisenberg's (1930) consideration and rejection of a lattice geometry as a way to deal with the self-energy difficulty of

electron theory, (2) Snyder's (1947a, 1947b) proposal and development of relativistic commutation relations between space and time coordinates, (3) Regge's (1961) skeletonization of Riemannian geometry as means both to bring out the geometric content of Einstein's standard geometrodynamics and as algorithm for calculating the evolution of geometry with time, and (4) treating field theory in general, whether electrodynamics or Yang-Mills field or other fields, as taking place on a lattice of points rather than on a continuum, as a means to make tractable the mathematical analysis of some of those field theories (Maxwell, 1877; Kogut, 1978). All of these investigations treat spacetime as a pre-existing continuum; they do not look at it as an approximation to an underlying structure, a pregeometry, a substrate of quite a different kind.

If it does not lead us to pregeometry to accept already in advance a space, or a spacetime continuum of a definite dimensionality, and skeletonize it, what about the opposite approach? Can one take a set of points, a so-called Borel set, that in the beginning has no dimensionality, assemble these points into the most diverse configurations and look for a consideration of probability, by way of Feynman's (Feynman, 1942; Feynman and Hibbs, 1965) sum over histories or otherwise, that will give preference to three dimensions as compared to other dimensions (Wheeler, 1964)? Here also too much geometric structure is presupposed to lead to a believable theory of geometric structure. No one has ever come forward with a way to ascribe a weight or a probability amplitude to the configuration of a Borel set of points

that did not rest on some idea of distance between point and point. But to admit distance at all is to give up on the search for pregeometry.

At least one try has been made at a concept of pregeometry that breaks loose at the start from all mention of geometry and distance, "Pregeometry as the calculus of propositions" (Wheeler, 1971; Misner et al., 1973): "...make a statistical analysis of the calculus of propositions in the limit where the number of propositions is great and most of them are long. Ask if parameters force themselves on one's attention in this analysis (1) analogous in some small measure to the temperature and entropy of statistical mechanics but (2) so much more numerous, and everyday dynamic in character, that they reproduce the continuum of everyday physics." However, a later analysis (Patton, 1975) found nothing in mathematical logic supportive of this proposal. On the contrary, what is most attractive in mathematical logic--the theorem of Gödel (1931, 1934) and related theorems (Cohen, 1966) is the least attractive. The modern revolution in mathematical logic points, not toward some chosen branch of mathematical logic as the natural foundation for pregeometry and physics, but away. The Gödel theorem of undecidability shows up in number theory, in the theory of transfinite members, in set theory and in any formal axiomatic system of more than minimal complexity.

In the end we are led back from mathematics to physics in the search for a clue to pregeometry. The only thing that could be worse than not finding pregeometry automatically contained in mathematics would be finding it automatically

contained in mathematics. How could one believe any account of the foundation for the central structure of physics, spacetime, which proceeded without reference to the quantum, the overarching principle of all physics?

The central lesson of the quantum has been stated in the words, "No elementary phenomenon is a phenomenon until it is an observed (registered) phenomenon" (Wheeler, 1979). Nowhere does this feature of nature show more conspicuously than in so-called "delayed-choice" experiments (Wheeler, 1978). Nowhere is this "question and answer" way of converting conceivabilities into actualities illustrated in a more homely context than in the "surprise version" of the game of twenty questions (Wheeler, 1978). There one sees the one who thought he was an observer pure and simple willy-nilly converted into a participator. Both in the game and in the elementary quantum phenomenon the observer-participator converts conceivability into actuality. If at this elementary level we already have a mechanism for building part of what we call reality, why should we look further for a mechanism to build all of what we call reality--including spacetime itself? Does not the famous "razor" of Duns Scotus and William of Occam, "Essentia non sunt multiplicanda praeter necessitatem" instruct us not to look for two methods of constructing reality when we already have one?

That is the task; what is the vision (Wheeler, 1979)?
(1) Law without law with no before before the big bang and no after after collapse. The universe and the laws that guide it could not have existed from everlasting to everlasting. Law must have come into being (Peirce, 1940).

Moreover, there could have been no message engraved in advance on a tablet of stone to tell them how to come into being. They had to come into being in a higgledy-piggledy way, as the order of genera and species came into being by the blind accidents of billions upon billions of mutations, and as the second law of thermodynamics with all its dependability and precision comes into being out of the blind accidents of motion of molecules who would have laughed at the second law if they had ever heard of it. (2) "Individual events. Events beyond law. Events so numerous and so uncoordinated that flaunting their freedom from formula, they yet fabricate firm form." (3) These events, not of some new kind, but the elementary act of question to nature and a probability-guided answer given by nature, the familiar everyday elementary quantum act of observer-participancy. (4) Billions upon billions of such acts giving rise, via an overpowering statistics, to the regularities of physical law and to the appearance of a continuous spacetime. Far though one is from seeing how to spell out this vision, let alone appraise it, this is one conception of what it would mean to "understand geometry in terms of pregeometry."

In brief, we confront two imperatives and one great issue. First, the gates of time tell us that physics must be built from a foundation that has no physics; or still more briefly: "Must Build." Second, elementary quantum acts of observer-participatorship: "Do Build." Finally, how are billions upon billions of these elementary building acts organized-- if they are--to make up the grand structure that we call "reality"; or, in brief: "How Build?" No more attractive

ffers itself for attacking this great issue than the
formation is processed to make "meaning." On what else
comprehensible universe be built but on the demand for
sibility?

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