Nonlinear Systems Tracking

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Part I

Preface

Systems, control, and computers

0.1 Dynamical systems

Is there a real (dynamical) physical system that is not nonlinear? Probably not. The boundedness of energy and matter sources available for the systems work cause the limitation nonlinearity. There are often geometric and kinematic limitations. The path and its length of every displacement are limited. The maximal speed and acceleration of every technical object are bounded. Some physical processes are inherently nonlinear (e.g., friction). Sophisticated control algorithms are mainly nonlinear. Linear control algorithms cannot ensure a finite reachability time. The study of nonlinear dynamical systems has been attracting more and more research efforts. Their study is challenging for their complexity and for their theoretical and practical importance.

Note 1 ***The three classes of the dynamical systems treated herein***
The classes of the dynamical systems treated herein are:

- *the input-output (IO)* ***systems*** *described by the m-th order time-varying nonlinear vector differential equation (3.62) in the output vector* $\mathbf{Y}$,
- *the input-state-output (ISO)* ***systems*** *described by the first-order time-varying nonlinear vector differential equation (3.65) in the state vector* $\mathbf{X}$, *and by the time-varying nonlinear vector algebraic output equation (3.66) that defines the dependence of the output vector* $\mathbf{Y}$ *on the state vector* $\mathbf{X}$ *and on the input vector* $\mathbf{I}$,
- *the input-internal dynamics-output (IIDO)* ***systems*** *described by the α-th-order time-varying nonlinear vector differential equation [the first equation (3.12)] in the internal dynamics vector* $\mathbf{R}$, *and by the time-varying nonlinear vector algebraic output equation, [the second equation (3.12)], which determines the dependence of the output vector* $\mathbf{Y}$ *on the internal dynamics vector* $\mathbf{R}$ *and on the input vector* $\mathbf{I}$.

Note 2 ***The unifying class of the systems***
We prove how the IIDO system (3.12) reduces to:

- *The IO systems (3.62),*
- *The ISO systems (3.65), (3.66).*

The IO systems (3.62) and the ISO systems (3.65), (3.66) represent special subclasses of the IIDO systems (3.12). The study of the (stability, tracking, trackability) properties of the IIDO systems (3.12) incorporates the study of the same properties of the IO systems (3.62) and the ISO systems (3.65), (3.66). The results (concepts, definitions, criteria, conditions, control algorithms) obtained for the IIDO systems (3.12) simultaneously hold for, and are applicable to, the IO systems (3.62) and the ISO systems (3.65), (3.66).

The IIDO systems (3.12) are the unifying systems. We call them simply ***the systems (3.12)***.

0.2 Dynamical systems and computers

The computer techniques and technology have been very successfully developing. Problems that need the computer's big memory, fast operations, and small computer space are now solvable by computer engineers, computer designers, and by software specialists. The computers enable the development of artificial intelligence. The digital computer now succeeds in carrying out simulations of dynamical processes practically equally well as the analog computer.

The nonlinear systems can abruptly change the character of their dynamical behavior due to a very small, sometimes infinitesimal, variation of initial conditions and/or of the external actions. Their qualitative properties (e.g., controllability, observability, optimality, stability and trackability [279]) can be valid only for initial conditions belonging to a connected bounded infinite set (i.e., can be only local). Such largest set is *the domain* $\mathcal{D}_{(.)}$ *of the corresponding qualitative dynamical property*. It can be open, or closed, but need not be either. The problem of how to determine the shape, the size, and the boundary of the property domain can be very difficult and complex. The computer simulations are very illustrative, but they cannot resolve the problem of the stability domain determination [287]. In order to cope effectively, the corresponding theory should be well developed. It serves as the basis for computer simulations, for engineering design, for its applications, and for further research.

0.3 Dynamical systems and control

A (technical, economical, biological, or social) *dynamical physical system* that cannot realize by itself either its demanded behavior or a real behavior sufficiently close to its requested behavior in the real environment, under arbitrary initial conditions and under (usually unknown, unpredictable) external influences, is a **plant** (also called an **object**). It can be, for example, a submarine, ship, car, train, tool machine, industrial robot, mobile robot, production line, turbine, motor, hydraulic process, thermal process, chemical process, power plant, aircraft, missile, space vehicle, electrical network, factory, human organ, market, or society.

0.4 Control goal

The very control goal, the primary control purpose, is for control to force a plant to behave exactly as demanded (which is the ideal case), *or at least sufficiently closely to the demanded behavior over some (bounded or unbounded) time interval.* The object desired (internal and/or output) behavior expresses its demanded (internal and/or output) dynamical behavior, respectively. Its desired output dynamical behavior is mathematically described by the desired *time* evolution $\mathbf{Y}_d(t)$ of the plant output vector $\mathbf{Y}$. This means that the plant real output response $\mathbf{Y}(t)$ should **follow/track** its desired output response $\mathbf{Y}_d(t)$ sufficiently closely; i.e., control is to force the object to realize/to exhibit an appropriate kind of ***tracking***. Such a control is ***tracking control***. It is clear that **tracking** and **tracking control synthesis** are the fundamental control issues.

Tracking studies essentially started as a *servomechanism* or *servosystem* theory in the broad sense. The pioneering contributions are due to L. A. MacColl, 1945 [402]; H. Lauer, R. Lesnick, and L. E. Matson, 1947 [375]; G. S. Brown and D. P. Campbell,1948 [61]; J. C. West, 1953 [553]; H. Chestnut and R. W. Mayer, 1955 [82]; I. Flügge-Lotz and C. F. Taylor, 1956 [132]; and J. C. Lozier, 1956 [396]. A. I. Talkin [535] constructed (1961) the term "servo tracking". The name *servomechanism* or *servosystem* means the controller that should force the plant output, or forces the controlled plant output, *to follow*, i.e., *to track*, its *time*-varying desired output rather than to track only a constant desired output. The latter is the purpose of the feedback controller called classically the *regulator*.

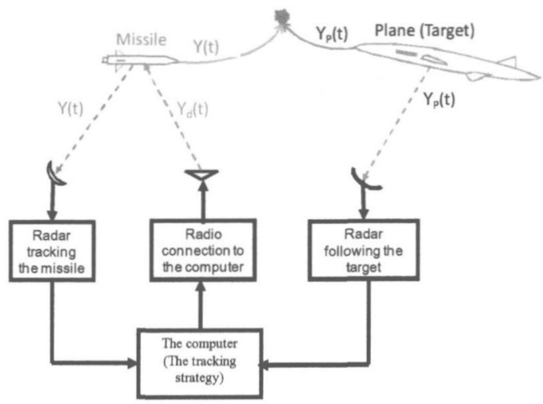

Figure 1: Target (plane) tracking.

0.5 Tracking control tasks

The work by Y. Bar-Shalom [26] was probably the pioneering one that opened the research on the target tracking, Figure 1. Further developments of target tracking are multitarget tracking and the related estimation and data acquisition theories due to Y. Bar-Shalom and T. E. Fortmann [28]; Y. Bar-Shalom and X. R. Li [29], [30]; Y. Bar-Shalom, X. R. Li, and T. Kirubarajan [31]; Y. Bar-Shalom, P. K. Willet, and X. Tian [32]; S. S. Blackman [52]; F. E. Daum [96]-[98]; R. P. S. Mahler [404]; D. B. Reid [496]; and L. D. Stone, C. A. Barlow, and T. L. Corwin [528].

Task 3 ***Estimation and data acquisition to determine the plant desired output behavior***

One tracking control task is to estimate and to gather data about the target motion [the enemy plane flight $\mathbf{Y}_P(t)$] in order to define the desired output behavior $\mathbf{Y}_d(t)$ of the plant (the missile).

The preceding task occupied (multi)target tracking research. What follows does not deal with this tracking control task 3. This means that we assume the following condition holds:

Condition 4 ***Knowledge of the plant desired output behavior***

The plant desired output behavior $\mathbf{Y}_d(t)$ is well defined and known.

The purpose, the goal, and the aim of the plant determine its desired output behavior $\mathbf{Y}_d(t)$. In the case of the missile its goal is to reach and destroy the enemy plane.

Task 5 ***Control synthesis for tracking quality***

Another control task is that control $\mathbf{U}(t)$ acts on the plant so that its real output behavior $\mathbf{Y}(.)$ tracks (follows) sufficiently closely, with a requested tracking quality, its well-defined desired output behavior $\mathbf{Y}_d(.)$, and that $\mathbf{Y}(t)$ reaches $\mathbf{Y}_d(t)$ at a (possibly given, predetermined) finite instant t_R after the initial instant t_0, $t_R > t_0$, to stay equal because t_R until a final instant t_F, $t_R < t_F \leq \infty$,

$$\mathbf{Y}(t) = \mathbf{Y}_d(t), \forall t \in [t_R, t_F),\ t_0 < t_R < t_F \leq \infty, \quad (1)$$

or at least that $\mathbf{Y}(t)$ converges asymptotically to $\mathbf{Y}_d(t)$ as time t goes to infinity:

$$t \longrightarrow \infty \Longrightarrow \mathbf{Y}(t) \longrightarrow \mathbf{Y}_d(t). \quad (2)$$

On the book

0.6 Goals of the book

Various purposes of different plants and the great variety of the requirements on the quality of the plant behavior lead to various tracking demands and characterizations. In order to establish their systematic study we meet the following.

Question 6 *Can we establish the tracking theory in the general framework of time-varying nonlinear dynamical systems (plants and their control systems)?*

By relying on:

- The pioneering works [245], [246], [487], [488], which deal with the absolute tracking of Lurie systems, on the comparative or joint stability and tracking studies in [163], [210], [221], [277], which explain the differences among stability and tracking phenomena and concepts and how they both can be achieved simultaneously
- On the works [150]-[154], [173], [177], [180], [181], [183]-[188], [194]-[197], [210], [212]-[215], [218], [220], [225], [232]-[235], [237], [238], [241], [242], [253], [263], [267], [277]-[283], which contain studies of Lyapunov type tracking concept and properties
- The contributions [183], [185], [191], [196], [212], [234], [236], [240], [239], [243], [269], [284], [286], [467], which established the finite-*time* tracking concept and tracking control synthesis, by referring to the concept of the high-gain tracking [445]-[449]
- On the papers [190], [192], [219], [220], [270], which deal with the guaranteed performance index tracking, by following [194], [195], which introduced and developed the tracking domains concept
- The concept of the practical tracking established in [183], [184], [196], [197], [212], [214], by recalling natural tracking control applications [449]-[453]
- The generalization of both the high-quality tracking theory of linear systems tracking control [279] and the theory of the stability domains of *time*-invariant and *time*-varying nonlinear systems [155]-[164], [170]-[172], [175], [174], [182], [199]-[205], [207]-[209], [217], [216], [287]

we state:

Reply 7 *Yes, we can establish the tracking theory in the general framework of time-varying nonlinear dynamical systems.*

This imposes the following goal.

Goal 8 ***The tracking theory***

The first goal of the book is to establish the tracking theory for time-varying nonlinear plants and their control systems.

The theory should comprise the characteristic tracking concepts, the definitions of the main tracking properties involved in the corresponding tracking concept, the corresponding conditions for the controlled plant to exhibit the tracking property, and various algorithms for tracking control synthesis.

The tracking theory will not only extend stability theory but it will introduce novel qualitative dynamical systems and control concepts that will incorporate various new system properties. It will effectively cope with more complex dynamical problems resulting from the real systems' environmental conditions and from the demands for increasing the quality of the systems and of their controls. It will discover new problems, novel systems properties, and will establish new systems and control methods. It will be applicable to the real (time-varying, hence also to time-invariant, nonlinear, and linear) dynamical and control systems. Its main tools are physics, mathematics, the theory of dynamical and control systems. The engineering is the world of its vivid applications.

Studying the tracking phenomena, concepts, and properties, and attacking the tracking control synthesis problems we meet the following fundamental control problem [279].

Problem 9 ***The fundamental control problem***

a) Do the properties of the plant enable the existence of tracking control for all initial conditions from a neighborhood $\mathfrak{N}(t_0)$ *of* $\mathbf{Y}_{d0}$*, [i.e., of* $\mathbf{Y}_d(t)$ *at the initial moment* $t = t_0$*], for all permitted disturbances* $\mathbf{D}(.)$ *belonging to a functional family* $\mathfrak{D}$ *and for every object desired output behavior* $\mathbf{Y}_d(.)$ *belonging to another functional family* $\mathfrak{Y}_d$*?*

b) If, and only if, they do then the plant is ***trackable*** *over* $\mathfrak{D}\times\mathfrak{Y}_d$*. What are the conditions on the plant to be trackable?*

c) What are the size, the shape, and the boundary of the trackability domain?

Goal 10 ***The trackability theory***

The second goal of the book is to establish the trackability theory for time-varying nonlinear plants and their control systems.

The theory should comprise the perfect and imperfect trackability properties.

Once the first and the second goals of the book are achieved then we meet the core control goal.

Goal 11 ***The tracking control synthesis***

The core control goal is the effective synthesis of the tracking control, under the action of which, the plant behavior satisfies all the imposed tracking requirements resulting from its purpose.

The book should present the crucial tracking control concepts that comprise effective tracking control algorithms.

We can now summarize that the book is on a part of the qualitative theory of the nonlinear control systems; i.e., it is on *the tracking theory*, on *the trackability theory,* and on *the tracking control synthesis for time-varying nonlinear plants* as parts of control theory.

The general aim of this book is to treat and effectively solve the problem of achieving the tracking control task whatever the physical (biological, chemical, economical, electrical, electromechanical, mechanical, social) nature of the plant by using appropriate mathematical data and description of the plant.

The aim of the book is to contribute to the creation and the establishment of the tracking and trackability theories of nonlinear plants and of their control systems, and to contribute to the development of the synthesis of tracking control of nonlinear plants.

This, it is hoped, should enrich essentially the corresponding university control systems courses, should open new directions for research in control theory and should enable fruitful new various control engineering applications.

What follows represents a further tracking control aimed development:

- of the nonlinear systems stability theory [6]-[12], [20]-[25], [37], [38], [45] -[48], [56], [66], [67], [71], [78], [83]-[86], [92]-[94], [99], [107], [113], [123], [125], [149], [155]-[164], [160]-[164], [170]-[172], [175], [176], [178], [182], [198]-[211], [217], [216], [222]-[224], [227], [231], [266], [280], [281], [287], [292]-[297], [300], [304]-[306], [311], [316], [322], [346], [356], [358], [359], [364], [363], [367]-[374], [383], [384], [400], [401], [406], [405], [410]-[417], [419]-[421], [427]-[433], [440], [454], [455], [463], [473], [479], [497], [504], [506], [511], [512], [531]-[534], [542]-[548], [562]-[565], [578],

- and of the nonlinear control systems theory including works on tracking [1]-[5], [10], [13], [14], [16], [19], [33]-[35], [39], [43], [51], [53]-[56], [58]-[65], [68]-[70], [72]-[77], [79]-[82], [87]-[91], [95], [100]-[106], [108]-[110], [112], [114]-[124], [126]-[128], [130], [132], [134]-[138], [141]-[147], [150]-[154], [165]-[169], [181]-[180], [183]-[197], [212]-[215], [218]-[221], [225], [226], [232]-[247], [260], [263]-[265], [267]-[270], [277]-[279], [281]-[286], [288]-[291], [298], [307]-[310], [312]-[315], [317]-[321], [323]-[344], [345]-[355], [360]-[362], [366], [376]-[382], [385]-[393], [394]-[399], [402], [403], [407]-[409], [418], [423], [424], [437]-[439], [441]-[453], [456]-[457], [465]-[474], [475]-[478], [483]-[492], [493], [495], [498], [499], [500]-[505], [507]-[513], [515]-[521], [524]-[530], [535]-[541], [549]-[561], [566]-[577]

as well as the development of the trackability theory of *time*-invariant linear or nonlinear plants [165]-[169], [232]. [233], [235]-[241], [279], [282]-[286] to *time*-varying nonlinear plants.

There is not in this book an available space for examples, simulations, or applications. They are left open to everybody for future works.

0.7 The book structure and composition

There are in the sequel eleven parts, eight of which compose the main body of the book. The parts mainly contain chapters that incorporate sections. Some sections contain subsections. The systems defined by the α-th-order nonlinear vector differential equation that describes the system state behavior and by the nonlinear algebraic vector equation that determines the system output dependence on the system internal dynamics, on the control action, and on the external disturbances are **the input-internal dynamics-output** ($IIDO$) **systems**. Their special classes are **the input-state-output** (ISO) **systems** and **the input-output** (IO) **systems**. The first-order nonlinear vector differential equation defines the internal dynamics, i.e., the state dynamics, and the nonlinear algebraic vector equation defines the output behavior of the ISO systems. The mathematical model of the IO systems is the m-th-order nonlinear vector differential equation that shows directly how the system output behavior depends on the external (control and disturbance) actions on the system. The book treats the $IIDO$ systems so that all the results are directly applicable to both IO systems and ISO systems. For the sake of simplicity the $IIDO$ systems are simply called *the systems*, if not stated otherwise.

The *time*-varying and nonlinear nature of the systems ensures the generality of the concepts, definitions, conditions, criteria, and control algorithms.

The main body begins with the essential topics for dynamical systems and their control. The next parts start with new concepts that incorporate definitions of new systems and control properties and end with the criteria or with the conditions on the system.

The fact that the book contributes new concepts, definitions, and system and control features, the author suggests to the reader to pass through the text continuously, without omitting any part, chapter, section, or subsection.

In order to describe the systems, to define their various dynamical and control properties, and to cope effectively with the dynamical and control problems caused by the systems complexity, the book uses the new simple vector and matrix notation analogous to the scalar notation, [252], [279].

0.8 In gratitude

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