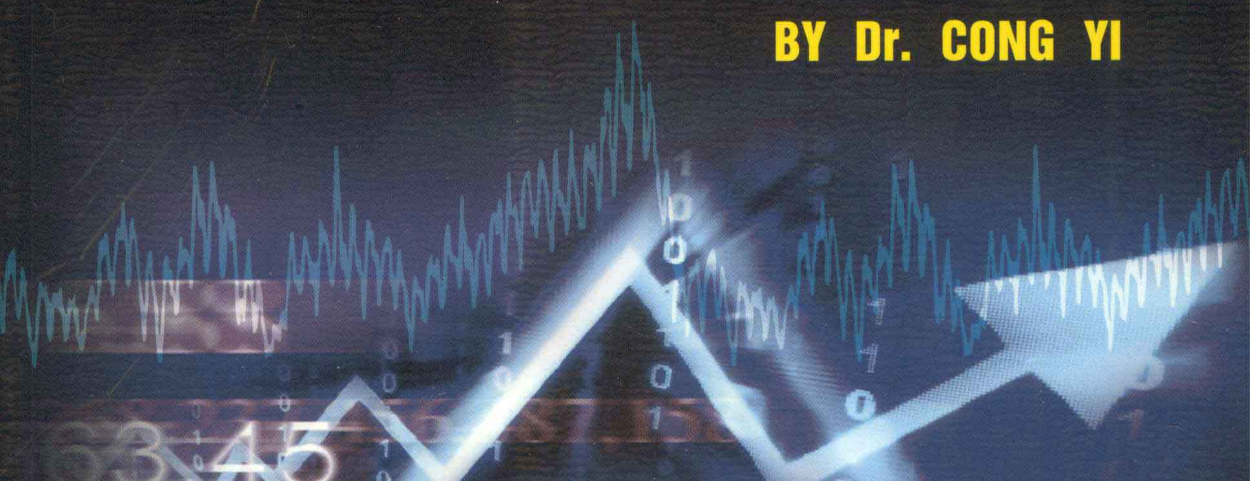
TOPICS IN VOLATILITY MODELS

BY Dr. CONG YI



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To My Parents: Yingsen and Wentao

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Abstract

In this thesis I will present my PhD research work, focusing mainly on financial modelling of asset' s volatility and the pricing of contingent claims (financial derivatives), which consists of four topics:

1. Several changing volatility models are introduced and the pricing of European options is derived under these models;
2. A general local stochastic volatility model with stochastic interest rates (IR) is studied in the modelling of foreign exchange (FX) rates. The pricing of FX options under this model is examined through the use of an asymptotic expansion method, based on Watanabe – Yoshida theory. The perfect/partial hedging issues of FX options in the presence of local stochastic volatility and stochastic IRs are also considered. Finally, the impact of stochastic volatility on the pricing of FX – IR structured products (PRDCs) is examined;
3. A new method of non – biased Monte Carlo simulation for a stochastic volatility model (Heston Model) is proposed;
4. The LIBOR/swap market model with stochastic volatility and jump processes is studied, as well as the pricing of interest rate options under that model.

In conclusion, some future research topics are suggested.

Key words: Changing Volatility Models, Stochastic Volatility Models, Local Stochastic Volatility Models, Hedging Greeks, Jump Diffusion Models, Implied Volatility, Fourier Transform, Asymptotic Expansion, LIBOR Market Model, Monte Carlo Simulation, Saddle Point Approximation.

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Contents

1. General Introduction, Changing Volatility Models and European Options Pricing

1.1 General Introduction

This thesis of my Ph. D work in financial mathematics mainly focuses on financial modeling with non-constant volatility and the pricing of financial derivatives. Financial modelling with non-constant volatility has been a widely studied topic for more than 20 years (cf. [2], [128], [129], etc.) and has become more so in this volatile market environment we are experiencing, since the financial meltdown in late 2007. How to accurately and effectively price and risk manage the derivative products has posted an ever challenging task for academics and practitioners alike.

In this thesis, my research begins with the introduction of *changing volatility models*, which are special cases of *local volatility models* introduced by Dupire (cf. [130]), Derman and Kani (cf. [131]). The introduction of these models has got its own economic meanings, and subsequently, analytic formulae for European option prices under the model settings are obtained. These specific models and the pricing of European options have not been studied before, to the author's best knowledge, and the model implementation can be quickly put into practice. At the end of the chapter, a simple addition of incomplete information on the volatility term extends the model setting to another category of non-constant volatility models: *stochastic volatility models* and more general, *local-stochastic volatility models*, in which the the level of asset volatility not only depends on the level of asset price but also is driven by its own stochastic process.

Local-stochastic volatility models, while their set-up has been discussed in [14], the analytical (or semi-analytical) formulae for European option prices have only been obtained for several specific forms, e. g. SABR model ([30]), Zhou Model ([28]), among others. Chapter 2 mainly discusses the model set-up of stochastic volatility and local-stochastic volatility models, from the general framework (follows Romano and Touzi's setting) to the summary of recent works of different models. At the end of the chapter the author derives the adjustments to the greeks' calculation in the setting of non-constant implied volatility. These adjustments are crucial for the implementation of local-stochastic volatility models and management of volatility risk.

My major work in this thesis begins from chapter 3, where the first part introduces a general form of foreign-exchange (FX) rate modeling with local-stochastic volatility and two stochastic bond price processes. This local-stochastic volatility process is general that it encompasses all the local-stochastic volatility models summarized in chapter 2, such that the specific form of the model is up to the user's preference. Then the pricing of FX vanilla options under this general setting is examined through the use of an asymptotic expansion method, based on Watanabe-Yoshida theory ([95]). This semi-analytical formula is accurate for the pricing of short and medium expiry options and easy to implement, as shown in the appendix.

Later in chapter 2 another model with stochastic interest rates, stochastic volatility and jump process is proposed and the pricing of FX vanilla option can be derived through Fourier Transform approach. This modelling approach is also general that it includes several forms of stochastic volatility models, jump process models and stochastic interest rate models. The pricing formula is accurate even for long maturity options. Then the model calibration and implementation are applied on a complex structured product on FX rate, namely the PRDC.

The last part of chapter 2 discusses the perfect/partial hedging issues of FX options in the presence of local stochastic volatility and stochastic interest rates, as well as the hedging error analysis in the partial hedging process. Also, the impact of stochastic volatility on the pricing of FX-IR structured products (PRDCs) is examined. These two general models are new at the time of writing, and the model calibration, implementation as well as the extensive discussion on hedging process of FX options are first time seen in literature.

In the financial practice, Monte-Carlo simulation has gained more and more importance in the valuation and risk-management of derivatives, especially the complex structured products. How to effectively simulate asset price process with stochastic volatility has been a widely studied area in financial engineering. In chapter 3 a new method of non-biased Monte Carlo simulation for a popular stochastic volatility model (Heston Model) is proposed by the author, by the use of the powerful Saddle point method borrowed from statistics.

The last chapter studies the LIBOR/swap market model with stochastic volatility and jump process, as well as the pricing of interest rate options under the model. Here a new model of bond price is proposed, with stochastic volatility and a general jump process (marked point process), and subsequently the LIBOR forward rate model and swap rate model are derived, by the use of change of measure and various approximation techniques. Finally the pricing formulae of vanilla options in forward rate markets and swap rate markets are derived through Fourier Transform and approximation methods.

1.2 Introduction to Changing Volatility Models

In the financial markets, the volatility of financial assets is changing rather than keeping constant. A simple and realistic case is that volatility depends on the state of the asset and/or its movement path, which will produce incomplete information for the pricing and hedging of contingent claims (cf. [17]). A few special cases will be studied in this chapter including the changing volatility models of the log-normal and Ornstein-Uhlenbeck types. The pricing and hedging results for European options will be provided.

1.3 Model Completeness and European Option Pricing

Consider a probability space $(\Omega, \mathcal{P}, \mathcal{F}_T)$ over a finite time interval $[0, T]$. The market model consists of a risk-free asset $B(t)$ and a risky asset $S(t)$ for time $t \geqslant 0$.

We consider a model given by

$$dS(t) = \mu S(t)dt + \sigma(t)S(t)dW(t) \tag{1.1}$$

$$dB(t) = rB(t)dt \tag{1.2}$$

under the real world measure $\mathcal{P}$.

If we define the volatility process $t \longrightarrow \sigma(t) \in \mathcal{L}^2$ by

$$\sigma(t) = \sigma_1 1_{0 \leqslant t < T_0} + \{\sigma_2 1_{T_0 \leqslant t < T} I_E + \sigma_1 1_{T_0 \leqslant t < T} I_{\Omega/E}\} \tag{1.3}$$

Where σ_1, σ_2 are constants and known at time 0, T_0 is fixed and $T_0 \in \mathcal{F}_0$, the probability set $\mathbf{E}$ is also pre-specified.

We consider the case in which $\mathbf{E} := \{\omega \in \Omega : S_{T_0}(\omega) \geqslant B\} \in \mathcal{F}$ with *barrier* $B \in \mathcal{F}_0$.