Wear and Contact Mechanics

Edited by
Luis Rodriguez-Tembleque and Ferri Aliabadi

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Wear and Contact Mechanics

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Luis Rodriguez-Tembleque and Ferri Aliabadi

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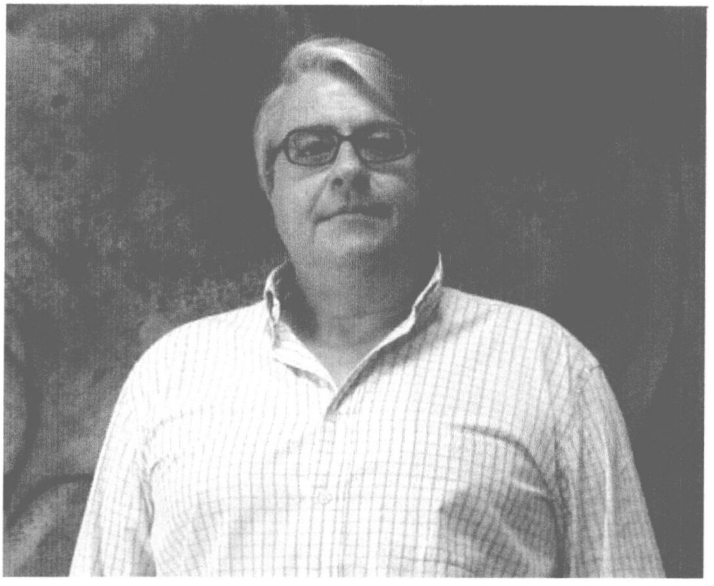
Prologue



Ramón A. Abascal García

(1956 – 2013)

Prof. Ramón Abascal was full professor of Continuum Mechanics and Theory of Structures at the "Escuela Técnica Superior de Ingeniería" (ETSI) of the University of Seville, Spain. He was born in Seville in 1956 and graduated in mechanical engineering at ETSI in 1979. He received his Ph.D. from University of Seville five years later and in 1986 started his teaching career as associated professor at the Department of Continuum Mechanics and Theory of Structures of the ETSI, becoming full professor in 1995.

His research interests were focused on the Boundary Element Method and its applications, with emphasis on elastodynamics (seismic wave propagation and scattering in non-homogenous viscoelastic soils, seismic response of foundations including dynamic soil-structure interaction and nonlinear contact effects due to uplift, guided wave scattering, and ultrasonic waves), fracture mechanics, contact problems (including friction, rolling and wear) and substructure coupling techniques using Lagrange multipliers. During his scientific career, he advised three Ph.D. students and published over forty papers in the most prestigious scientific journals.

But his dedication was not only restricted to research; he was also very concerned with the education of future engineers. He taught with excellence different subjects, such as: Theory of Structures, Advanced Analysis of Structures, Steel Structures and Advanced Finite Elements. Due to his commitment to academic excellence, Ramón was appointed Head of Studies at the ETSI from 2006 to 2010.

First as a Ph.D. student and later as a colleague, I have witnessed the enormous rigor, high demand and passion that Ramón projected on all his work. Everybody in the scientific and educational community, who were fortunate to know him, will remember his enormous generosity, his sense of humor, and, most of all, his significant scientific work. Colleagues and friends will never forget Ramón and will deeply regret losing him.

This work is dedicated to the memory of Prof. Ramón Abascal. We mourn his untimely death as we lose a great engineering educator, researcher and, above all, a good person and friend.

Luis Rodriguez-Tembleque

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The influence of equivalent contact area computation in extended node to surface contact elements

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Abstract. This article aims at extending the node to surface formulation for contact problems with an area regularization as proposed by [1]. For that purpose, two methods are proposed to compute the equivalent contact area attributed to each slave node. The first method, which is based on a geometrical approach through force equivalence, is an original extension of the one proposed in [1] for two-dimensional contact problems, i.e. plane stress and plane strain state, to the axisymmetric modelling context. The second method relies on an energy consistent way obtained through the virtual work principle and the same expression for the equivalent contact area as the one originally cited in [2] is then recovered. First, the node to surface strategy with area regularization is introduced and the aforementioned methods for the equivalent contact area are presented in detail and compared. Afterwards a consistent linearization technique is applied to achieve a quadratic convergence rate in the Newton Raphson iterative procedure used to solve the non-linear equilibrium equations of the underlying finite element model. Finally, two axisymmetric numerical examples are provided in order to compare the aforementioned equivalent contact area evaluations and to demonstrate the performance and the robustness of the consistent approach especially in the neighbourhood the revolution axis.

Introduction

Nowadays, the node to surface formulation [3, 4] is widespread in commercial finite element codes and it is commonly used to solve the contact problems in large deformation which range from the sheet metal forming processes to the impact tests in crashworthiness assessment. Of particular interest in practical engineering applications is the knowledge of the contact pressure and frictional shear stress distribution across the contacting surfaces. However, in the classical node to surface formulation, there are only contact forces directly available at the nodes in contact but some post-processing techniques may be used to transform them into contact pressure and frictional shear stress [5]. From a tribological point of view, micro-macro approaches have recently proposed advanced contact mechanical constitutive models in terms of contact pressure [6, 7, 8] and frictional shear stress [9, 10, 2]. Henceforth, their applicability requires to include an area term in the classical node to surface formulation, if one does not want to rely on more advanced contact formulations, such as mortar-like [11, 12, 13] or segment to segment [14, 15, 16] strategies, where a contact pressure and frictional shear stresses are naturally available in the weak form associated with the contact problem.

Historically, Zavarise et al. [6] introduced for the first time the idea to associate an equivalent contact area to the nodes of the slave surface in a thermomechanical two-dimensional contact finite element formulation due to the presence of complex constitutive models for the thermal contact resistance. These authors decided to compute this area on the slave surface for simplicity's sake, even if the master area is immediately available in the node to surface formulation, because the number of slave nodes in contact with the same master segment should be known in order to determine the area part on the master segment on which each slave node acts.

Following this former work focused on the thermal treatment of the contact formulation, Stupkiewicz [2] proposed an extended node to surface contact element dedicated to more complex frictional laws, depending on the contacting surface area stretching, than the classical Coulomb's law in two-dimensional and axisymmetric modelling context. Since the equivalent contact area term gives rise to a coupling between a slave node and its neighbouring nodes called auxiliary slave nodes, their degrees of freedom should be taken into account to derive the full tangent stiffness matrix associated to this contact element. Even in the frictionless case, the equivalent contact area makes the tangent stiffness matrix unsymmetric and the quadratic convergence rate is reached in the Newton-Raphson iterative scheme, just in case of consistent linearization, as expected.

Recently, Zaravise and De Lorenzis [1] revisited the contact patch test with a frictionless node to surface formulation. If the penalty method is adopted, the non-penetration constraint is regularized without no additional unknowns and a penalty coefficient is introduced to control the amount of penetration between the contacting bodies. Contrary to Lagrange multiplier method, the non-penetration constraint is no longer exactly satisfied but the exact solution to the contact problem is theoretically recovered when the penalty coefficient tends to infinity. Nevertheless, in real-life applications, a finite value of penalty coefficient is set depending on the problem upon consideration, and it is generally bounded to avoid ill-conditioning of the global system of equilibrium equations.

The contact patch test assesses the ability of a contact formulation to transfer a constant contact pressure through two deformable contacting bodies regardless their discretization [17], so that a constant penetration is expected with the penalty method. It is well known that the node to surface formulation is not able to pass the contact patch test. Therefore, local solution errors may occur and won't disappear with mesh refinement. In conjunction with the penalty method, the contact algorithm treatment is much trickier due to the presence of non-zero penetration of the contacting body. To that purpose, the authors proposed successive improvements of the contact algorithm towards a node to surface formulation passing the patch test for linear finite elements. According to them, if all the slave nodes are projected against the same master segment, which is a special case encountered by their algorithm, the presence of an equivalent contact area is a sufficient condition to pass the contact patch test.

In fact, the finite element method converts a pressure distribution applied on a arbitrary two-dimensional surface into uneven nodal forces distribution. In the particular case where the pressure distribution is constant, the nodal force for the nodes at the boundary of the surface is equal to half of the force attributed to each node in the interior of the surface. Since the contact force is linearly related to the penetration by means of a penalty coefficient, considered as a constant value for each slave node, the classical node to surface formulation is not able to reproduce a constant penetration. To solve the problem, the penalty coefficient is weighted by an equivalent contact area, in order to obtain a contact pressure immediately available at the slave node instead of a contact force on one hand, and on the other hand to be able to reproduce a constant contact pressure over the contacting surface, if this area is equal to the sum of half the length of the coincident segments to each slave node.

The motivation of the present paper is to extend the aforementioned work to the axisymmetric modelling context with linear finite elements and to demonstrate that the equivalent contact area should be carefully computed in order to be able to reproduce a constant contact pressure which is the first step towards the development of a node to surface formulation passing the patch test. For that purpose, this paper is organized into two main parts as follows. The first part outlines the node to surface with area regularization as described in [1, 2], and discussed in details the two computational methods used to determine the equivalent contact area in the axisymmetric modelling context, as well as their contribution to the full tangent stiffness matrix through a consistent linearization process. The second part

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deals with the comparison of these methods throughout two computational simulations of academic contact problems. Finally, all numerical examples as well as developments are carried out and implemented respectively in Metafor software [18], which is a in-house object oriented non-linear finite element code for the simulation of solids submitted to large deformations.

Contact formulation

This section briefly introduces the contact strategy adopted to solve the frictionless contact problem between a deformable and a analytically-defined rigid body, both axisymmetric, in the large deformation context. The deformable body is discretized by axisymmetric bilinear finite elements. In axisymmetric modelling, the current position vector $\mathbf{x}$ is written using a cylindrical coordinate system $O\mathbf{e}_r\mathbf{e}_\theta\mathbf{e}_z$ as :

$$\mathbf{x} = \begin{pmatrix} r \\ z \end{pmatrix}, \tag{1}$$

where r is the current radius and z is the current height. For numerical convenience, since the deformable body surface is piecewise continuous, it is considered as the slave whereas the rigid body is the master.

Node to surface formulation with area regularization. In the node to surface formulation, each node of the slave surface is prevented from penetrating into the analytically defined master surface. From a geometrical point of view, this non-penetration constraint is expressed by the normal gap g_n which is the signed distance between a slave point $\mathbf{x}^S$ and its closest projection point $\mathbf{x}^M$ onto the master surface along its outward unit normal $\mathbf{n}^M$ (see Fig. 1):

$$g_n = (\mathbf{x}^S - \mathbf{x}^M(\xi^M)).\mathbf{n}^M(\xi^M), \tag{2}$$

where ξ^M denotes the intrinsic coordinate of this projection point in the master element reference system. The superscripts S and M refer to the slave node and the master surface respectively.

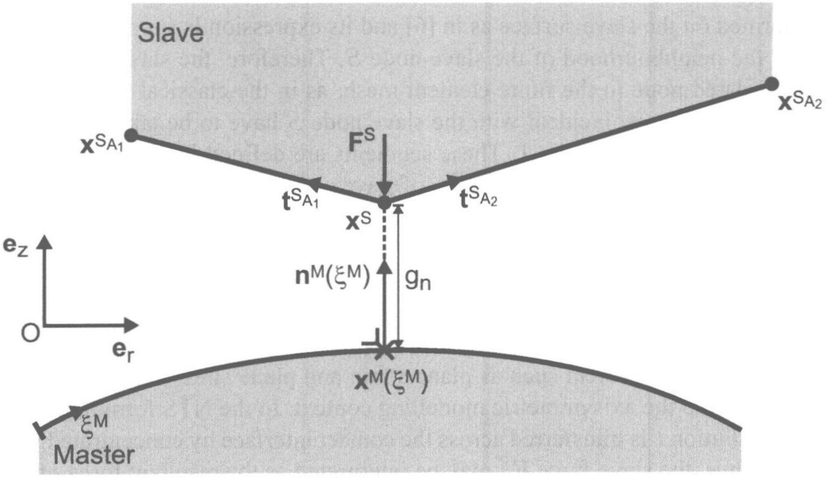


Fig. 1: Axisymmetric node to surface contact element with area regularization.

When the normal gap is negative $g_n \leq 0$, the slave node is considered in contact and a contact force $\mathbf{F}^S$ is applied on it along the normal direction to the master surface $\mathbf{n}^M$

$$\mathbf{F}^S = -t_n A_S \mathbf{n}^M, \tag{3}$$

where t_n is the contact pressure and A_S is the *equivalent contact area* or *slave node area*, introduced and discussed in a further paragraph. Otherwise, if the normal gap is positive $g_n > 0$, there is no contact and, thus no contact force.

If the non-penetration constraint is enforced by the penalty method, a penalty coefficient c_n is introduced and the contact pressure t_n is assumed proportional to the normal gap g_n :

$$t_n = c_n g_n. \tag{4}$$

Finally, the contact contribution $\delta\mathbb{W}_c$ to the virtual work is

$$\delta\mathbb{W}_c = \sum_{I=1}^{N_c} \mathbf{F}^{S_I}.\delta\mathbf{x}^{S_I}, \tag{5}$$

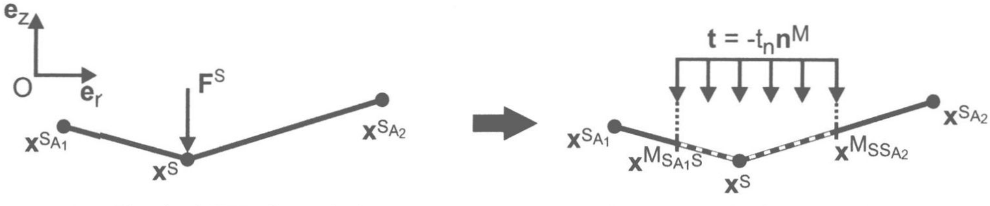
where the summation is performed on all slave node actually in contact and the symbol δ denotes virtual variation.

In the present work, the dimensions of the penalty coefficient c_n depend on the presence of the slave node area A_S as well as the interpretation of the contact variable t_n. In the classical node to surface formulation, the slave node area A_S is hidden in the penalty coefficient c_n expressed in Newton per unit length and the symbol t_n denotes a contact force. Otherwise, the penalty coefficient dimensions are Newton per squared unit length and the symbol t_n stands for a contact pressure. In the literature, the latter approach is termed similarly *node to surface with area regularization* in [1] or *extended node to surface contact element* in [2]. For brevity, the acronyms *NTS* and *NTS-AR* are introduced, that stand for *node to surface* and *node to surface with area regularization* respectively.

Area regularization computational method. In the contact formulation, it remains to determine the expression of the slave node area A_S associated to each slave node. The computation of the slave node area is performed on the slave surface as in [6] and its expression is supposed to depend on the local geometry in the neighbourhood of the slave node S. Therefore, the slave node S is no longer considered as an isolated node in the finite element mesh, as in the classical NTS formulation, and the segments $S_{A_1}S$ and SS_{A_2} coincident with the slave node S have to be taken into account in the NTS-AR formulation, as shown in Fig. 1. These segments are defined by the position vectors $\mathbf{x}^{S_{A_1}}$ and $\mathbf{x}^S$ of slave nodes S_{A_1} and S, and $\mathbf{x}^S$ and $\mathbf{x}^{S_{A_2}}$ of slave nodes S and S_{A_2} respectively. These extra slave nodes are named as *auxiliary slave nodes* and the superscript S_{A_I} refers to the auxiliary slave node I.

Geometrical approach. This approach named *node to surface with geometrical area regularization* formulation (*NTS-AR-GEO*) is a straightforward application of the methodology proposed in [1] for two dimensional contact problem such as plane strain and plane stress state, and it is considered as an original extension to the axisymmetric modelling context. In the NTS formulation, an arbitrary contact traction distribution $\mathbf{t}$ is transferred across the contact interface by concentrated forces applied to the slave nodes. Thus, the slave force $\mathbf{F}^S$ may be interpreted as the resultant force of these contact tractions acting on a surface $\mathbb{S}$ defined in the vicinity of the slave node S (see Fig. 2):

$$\mathbf{F}^S = \int_{\mathbb{S}} \mathbf{t}\, d\mathbb{S}. \tag{6}$$



(a) Classical NTS formulation (b) Geometrical approach

Fig. 2: Node to surface with geometrical area regularization formulation. (a) Classical NTS formulation and (b) geometrical approach.

If the points $M_{S_{A_1}S}$ and $M_{SS_{A_2}}$ correspond to the midpoints of the segments $S_{A_1}S$ and SS_{A_2} respectively, the surface $\mathbb{S}$ is defined as the revolution of the curve which consists of the two straight line segments joining the points $M_{S_{A_1}S}$, S and $M_{SS_{A_2}}$, as depicted in dashed lines in Fig. 2(b). Although there exists an infinity of contact traction distribution corresponding to a given resultant force, a unique solution is found by supposing a constant distribution $\mathbf{t} = -t_n \mathbf{n}^M$ over the surface $\mathbb{S}$.

In case of linear finite elements, the position vector $\mathbf{x}$ of any point within an element (see Fig. 3) is given by

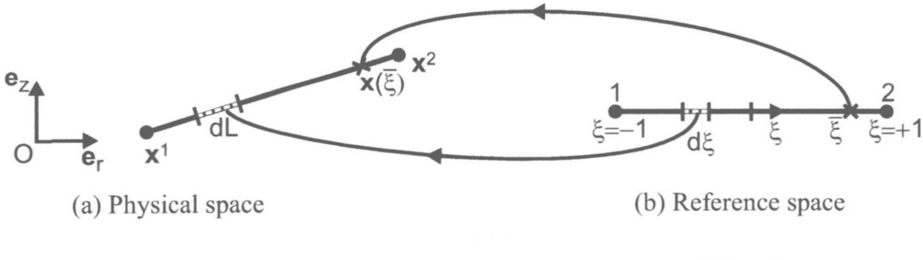
$$\mathbf{x}(\xi) = \sum_{I=1}^{2} N^I(\xi)\mathbf{x}^I \quad \text{with} \quad N^I(\xi) = \frac{1+\xi\xi^I}{2}, \tag{7}$$

where $\mathbf{x}^I$ is the position vector of node I, N^I is the shape function associated to node I, $\xi \in [-1, 1]$ is the reference coordinate in the reference system and $\xi^I = \pm 1$ corresponds to the reference coordinate of the node I according to Fig. 3.

In axisymmetric modelling, the surface area element $d\mathbb{S}$ is given by

$$d\mathbb{S} = r\ d\theta dL = r(\xi)\sqrt{\left(\frac{\partial r}{\partial \xi}\right)^2 + \left(\frac{\partial z}{\partial \xi}\right)^2}\ d\xi d\theta, \tag{8}$$

where the current radius $r(\xi)$ appears due to the integration in the circumferential direction $\mathbf{e}_\theta$ and the second equality holds using the isoparametric mapping $\mathbf{x}(\xi)$ as shown in Fig. 3.



(a) Physical space (b) Reference space

Fig. 3: Axisymmetric linear finite element. (a) Physical space and (b) reference space.

If Eqs. 7 and 8 are combined and then inserted into Eq. 6, one has

$$\mathbf{F}^S = -t_n\mathbf{n}^M \left(\underbrace{\int_0^{2\pi}\int_0^1 \left(N^1(\xi)r_{S_{A_1}} + N^2(\xi)r_S\right)\sqrt{\left(\frac{r_S - r_{S_{A_1}}}{2}\right)^2 + \left(\frac{z_S - z_{S_{A_1}}}{2}\right)^2}\, d\xi d\theta}_{\text{Segment } S_{A_1}S} \right.$$
$$\left. + \underbrace{\int_0^{2\pi}\int_{-1}^0 \left(N^1(\xi)r_S + N^2(\xi)r_{S_{A_2}}\right)\sqrt{\left(\frac{r_{S_{A_2}} - r_S}{2}\right)^2 + \left(\frac{z_{S_{A_2}} - z_S}{2}\right)^2}\, d\xi d\theta}_{\text{Segment } SS_{A_2}} \right), \tag{9}$$

where the subscripts S and S_{A_I} refer to the slave node and auxiliary slave node I respectively.

After integration, this yields

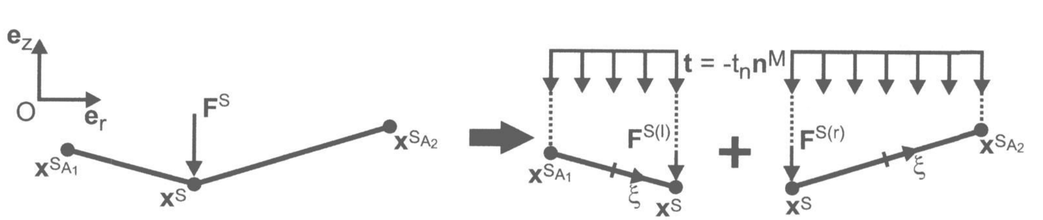
$$\mathbf{F}^S = -t_n\mathbf{n}^M \underbrace{\sum_{I=1}^{2} 2\pi \frac{L^{SS_{A_I}}}{2}\left(\frac{1}{4}r_{S_{A_I}} + \frac{3}{4}r_S\right)}_{A_S^{\text{GEO}}}, \tag{10}$$

where $L^{SS_{A_I}} = \left\|\mathbf{x}^S - \mathbf{x}^{S_{A_I}}\right\|$ denotes the length of the segment $S_{A_I}S$.

Energy consistent approach.

The NTS formulation is a collocation method where the non-penetration constraint is enforced pointwise at the collocation points, which correspond to the nodes of the finite element mesh. It is well known that the classical NTS formulation is recovered if a two-point Newton-Cotes quadrature rule is selected in a segment to segment contact strategy, where the non-penetration constraint is integrated over the contact segment in a weak sense [14]. Thus, the contact force $\mathbf{F}^S$ at the slave node S results into the sum of the consistent nodal contact force contributions coming from all adjacent contact segments as below (see Fig. 4) :

$$\mathbf{F}^S = \underbrace{\mathbf{F}^{S(l)}}_{\text{contact segment } S_{A_1}S} + \underbrace{\mathbf{F}^{S(r)}}_{\text{contact segment } SS_{A_2}}. \tag{11}$$



(a) Classical NTS formulation (b) Consistent approach

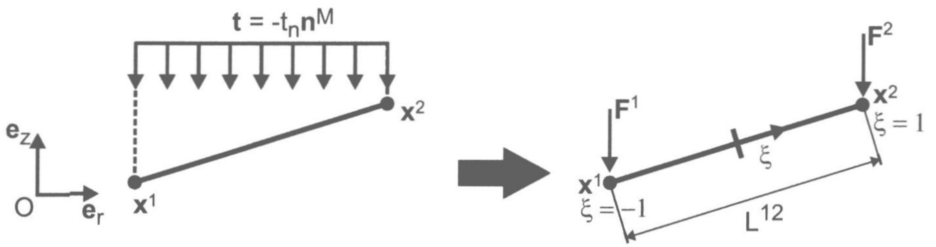
Fig. 4: Node to surface with consistent area regularization formulation. (a) Classical NTS formulation and (b) consistent approach.

Consider the generic segment 12 depicted in Fig. 5 submitted to an arbitrary contact traction distribution $\mathbf{t}$. This segment is defined by the position vectors $\mathbf{x}^1$ and $\mathbf{x}^2$ of nodes 1 and 2 respectively. The virtual work $\delta\mathbb{W}$ transforms these contact tractions into concentrated forces applied at the nodes in an energy consistent way :

$$\delta\mathbb{W} = \int_{\mathbb{S}} \mathbf{t}.\delta\mathbf{x}\ d\mathbb{S} = \sum_{I=1}^{2} \mathbf{F}^I.\delta\mathbf{x}^I, \tag{12}$$

where the consistent nodal forces are

$$\mathbf{F}^I = \int_{\mathbb{S}} N^I(\xi)\mathbf{t}\ d\mathbb{S}. \tag{13}$$



(a) Continuum scheme (b) Finite element approximation

Fig. 5: Axisymmetric contact segment submitted to a constant contact traction distribution. (a) Continuum scheme and (b) finite element approximation.

If it is assumed again that the contact traction distribution is constant over the contact area $\mathbf{t} = -t_n\mathbf{n}^M$ and if the integration is performed in the same way as in the geometrical approach, it comes

$$\mathbf{F}^I = -t_n\mathbf{n}^M \int_0^{2\pi}\int_{-1}^{1} N^I(\xi)r(\xi)\sqrt{\left(\frac{\partial r}{\partial \xi}\right)^2 + \left(\frac{\partial z}{\partial \xi}\right)^2}\ d\xi d\theta. \tag{14}$$

By integrating, one has in compact form :

$$\mathbf{F}^I = -t_n\mathbf{n}^M 2\pi\frac{L^{12}}{2}\left(\frac{3-\xi^I}{6}r_1 + \frac{3+\xi^I}{6}r_2\right), \tag{15}$$

where $L^{12} = \|\mathbf{x}^2 - \mathbf{x}^1\|$ is the length of the segment 12, r_1 and r_2 denote the radii of the node 1 and node 2 respectively.

Finally, if we evaluate Eq. 15 with $\xi^I = 1$ for the contact segment $S_{A_1}S$ and with $\xi^I = -1$ for the contact segment SS_{A_2}, Eq. 11 yields

$$\mathbf{F}^S = -t_n\mathbf{n}^M \underbrace{\sum_{I=1}^{2} 2\pi\frac{L^{SS_{A_I}}}{2}\left(\frac{1}{3}r_{S_{A_I}} + \frac{2}{3}r_S\right)}_{A_S^{\mathrm{CONS}}}. \tag{16}$$

This approach is named *node to surface with consistent area regularization formulation* (*NTS-AR-CONS*) and the expression of the slave node area is the same as originally proposed in [2].

Remark 1 In the two dimensional context such as plane strain and plane stress state, the proposed computational methods will lead to the same equivalent contact area for each slave node, if the continuum is discretized by linear finite elements. If higher order finite elements are used, [19] may be consulted to get further information.

Remark 2 At both ends of the contact boundary, the equivalent contact area must be reduced due to the fact that there is only one segment contribution. Therefore, the sum in Eqs. 10 and 16 reduces to one term and the equivalent contact area takes the following expressions with respect to the considered approach :

- geometrical approach

$$A_S^{\mathrm{GEO}} = 2\pi \frac{L^{SS_{A_I}}}{2} \left(\frac{1}{4} r_{S_{A_I}} + \frac{3}{4} r_S \right), \; I = 1, 2. \tag{17}$$

- consistent approach

$$A_S^{\mathrm{CONS}} = 2\pi \frac{L^{SS_{A_I}}}{2} \left(\frac{1}{3} r_{S_{A_I}} + \frac{2}{3} r_S \right), \; I = 1, 2. \tag{18}$$

If this remark is not taken into account in the NTS-AR formulation, it will lead to problems close to the revolution axis, as shown in the two numerical examples proposed in this paper.

Remark 3 The fundamental hypothesis that the contact pressure distribution is constant over the contact area seems to be very restrictive, but it may be considered as a minimal requirement. In fact, as the mesh is further refined, it may be expected that the contact pressure distribution tends to a constant value across a contact segment. In a former contribution [20], the authors presented a segment to segment contact approach with a perturbed Lagrangian method to enforce the contact constraints, where the contact pressure is supposed to be piecewise constant over the whole contact interface.

Recently, others authors [21] claimed that a contact pressure linearly distributed is the highest approximation level reachable for linear finite elements in their NTS-AR formulation passing the linear contact patch in two-dimensional problems, which may be closely related to the segment to segment contact approach proposed in [16], where a linear distribution of the contact pressure over the contact segment is also assumed. Henceforth, it may be supposed that our hypothesis is accepted for simplicity's sake.

In general, the contact force is a non linear expression and the Newton Raphson iterative scheme is used to solve the global system of equilibrium equations corresponding to the finite element model. If the stretching of the slave node area is considered during the Newton Raphson iterations, the dimension of the contact element residual vector $\mathbf{R}^c$ corresponding to the classical NTS formulation has to be increased, since the degrees of freedom of the auxiliary slave nodes should be inserted in the contact element position vector $\mathbf{x}^c$:

$$\mathbf{R}^c = \begin{pmatrix} \mathbf{F}^S \\ \mathbf{0} \\ \mathbf{0} \end{pmatrix} \quad \text{and} \quad \mathbf{x}^c = \begin{pmatrix} \mathbf{x}^S \\ \mathbf{x}^{S_{A_1}} \\ \mathbf{x}^{S_{A_2}} \end{pmatrix}, \tag{19}$$

where zero entries are added for the residual vector components related to the auxiliary slave nodes.

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