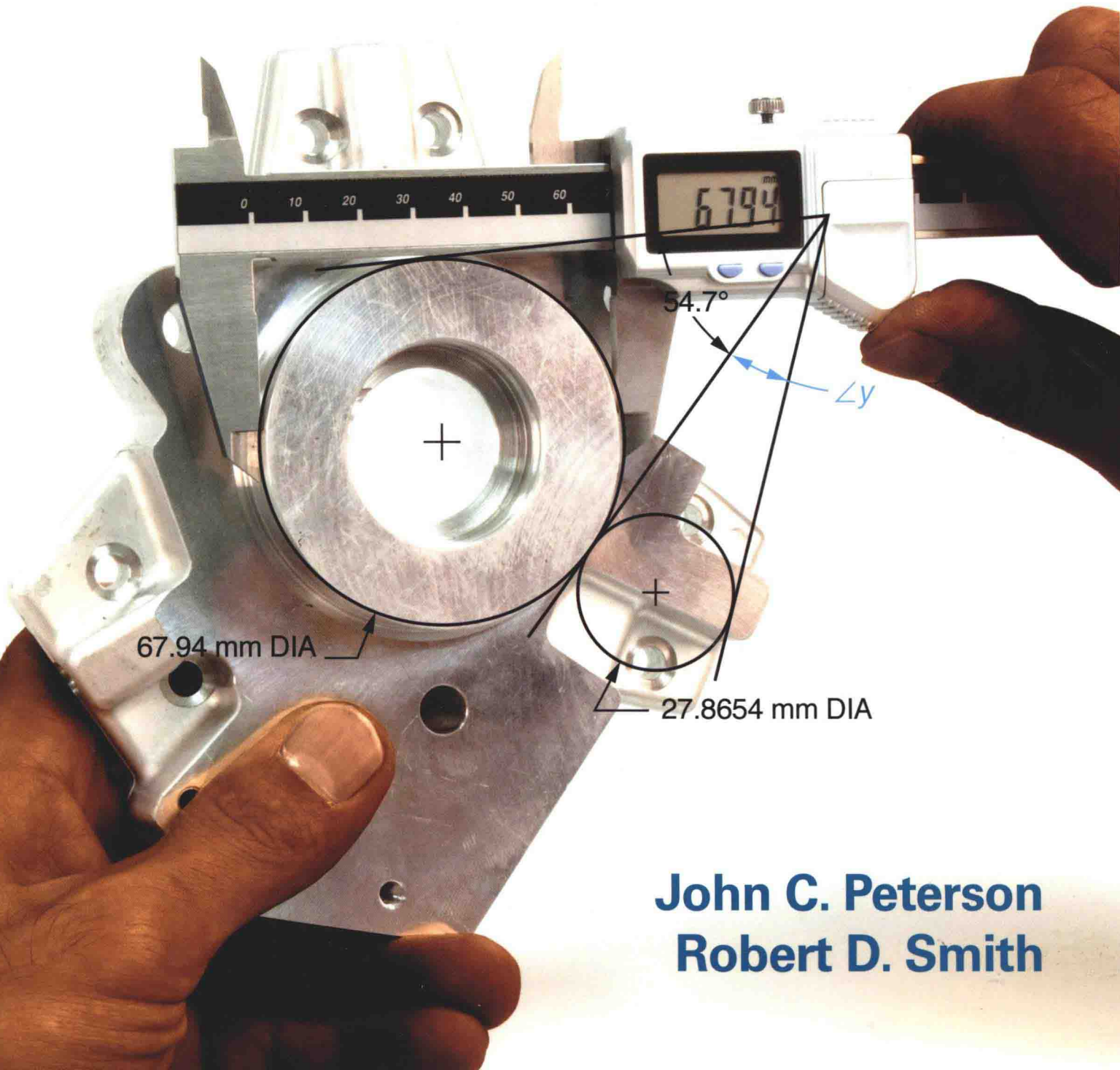


# MATHEMATICS

# Machine Technology

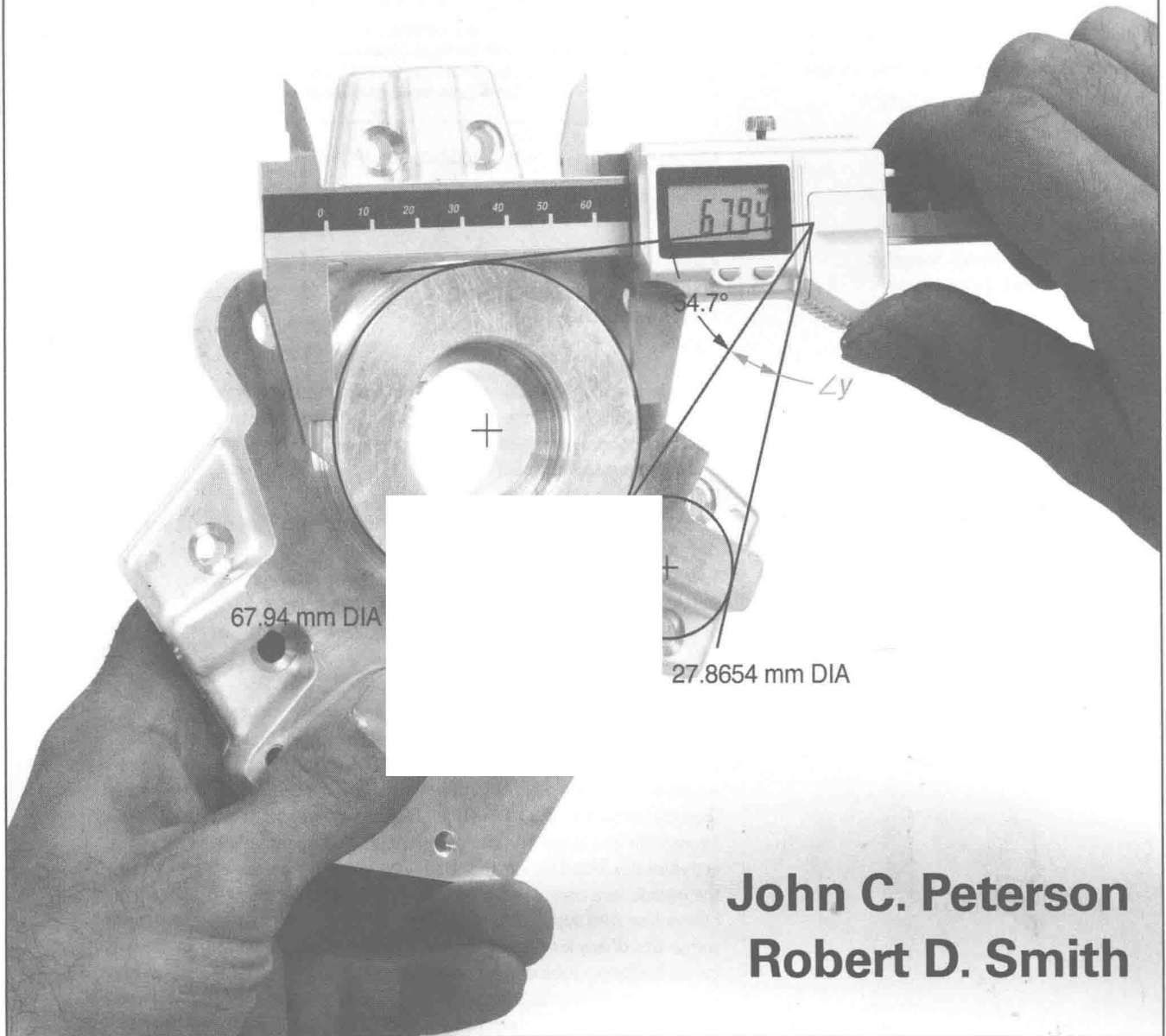
SEVENTH EDITION



**John C. Peterson**  
**Robert D. Smith**

# MATHEMATICS FOR Machine Technology

SEVENTH EDITION



**John C. Peterson**  
**Robert D. Smith**

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Library of Congress Control Number: 2014932903

ISBN: 978-1-133-28145-0

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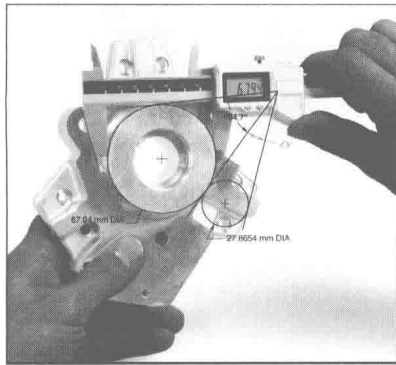
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# PREFACE

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“I can’t think of any job in my sheet metal fabrication company where math is not important. If you work in a manufacturing facility, you use math every day; you need to compute angles and understand what happens to a piece of metal when it’s bent to a certain angle.”<sup>1</sup>

Traci Tapani, Wyoming Machine

*Mathematics for Machine Technology* is written to overcome the often mechanical “plug in” approach found in many trade-related mathematics textbooks. An understanding of mathematical concepts is stressed in all topics ranging from general arithmetic processes to oblique trigonometry, compound angles, and numerical control.

Both content and method are those that have been used by the authors in teaching applied machine technology mathematics classes for apprentices in machine, tool-and-die, and tool design occupations. Each unit is developed as a learning experience based on preceding units—making prerequisites unnecessary.

Presentation of basic concepts is accompanied by realistic industry-related examples and actual industrial applications. The applications progress from the simple to those with solutions that are relatively complex. Many problems require the student to work with illustrations such as are found in machine technology handbooks and engineering drawings.

Great care has been taken in presenting explanations clearly and in providing easy-to-follow procedural steps in solving exercise and problem examples. The book contains a sufficient number of exercises and problems to permit the instructor to selectively plan assignments.

An analytical approach to problem solving is emphasized in the geometry, trigonometry, compound angle, and numerical control sections. This approach is necessary in actual practice in translating engineering drawing dimensions to machine working dimensions. Integration of algebraic and geometric principles with trigonometry by careful sequence and treatment of material also helps the student in solving industrial applications. The Instructor’s Guide provides answers and solutions for all problems.

A majority of instructors state that their students are required to perform basic arithmetic operations on fractions and decimals prior to calculator usage. Thereafter, the students use the calculator almost exclusively in problem-solving computations. The structuring of calculator instructions and examples in this text reflects the instructors’ preferences. Calculator instructions and examples have been updated and greatly expanded in this edition. The scientific calculator is introduced in the Preface. Extensive calculator instruction and examples are given directly following the units on fractions and mixed numbers and the units on decimals. Further calculator instruction and examples are given throughout the text wherever calculator applications are appropriate to the material presented. A Calculator Applications Index is provided at the end of the Preface. It provides a convenient reference for all the material in the text for which calculator usage is presented. Often there are differences in the

<sup>1</sup>Source: Thomas Friedman, “If You’ve Got the Skills, She’s Got the Job” *New York Times*, November 17, 2012, accessed November 18, 2012, <http://www.nytimes.com/2012/11/18/opinion/sunday/Friedman-You-Got-the-Skills.html>

methods of computation among various makes and models of calculators. Where there are two basic ways of performing calculations, both ways are shown.

Changes from the previous edition have been made to improve the presentation of topics and to update material.

A survey of instructors using the sixth edition was conducted. Based on their comments and suggestions, changes were made. The result is an updated and improved seventh edition that includes the following revisions:

Three major changes were made in this edition.

- The United States is the only major industrial country in the world that does not use the metric system. We need to acquaint our students with the prevailing system in the rest of the world. This is particularly true if we want to market our machined products outside the United States. To meet this challenge, a unit was added on metric calipers and height gages and a unit was added on metric micrometers.
- Electronic digital instruments are replacing hand-read/analog instruments. Two units were added to acquaint student with using electronic digital instruments. One unit was added on digital calipers and height gages and a unit was added on digital micrometers.
- Students have a tendency to forget a concept if it is not used frequently. To help students retain an idea most sets of applications begin with six “Tooling Up” exercises that use concepts from earlier units. These “Tooling Up” exercises are in each unit except for Unit 1 and the nine Achievement Review units at the end of each section.

## ABOUT THE AUTHORS

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**John C. Peterson** is a retired professor of mathematics at Chattanooga State Technical Community College, Chattanooga, Tennessee. Before he began teaching, he worked on several assembly lines in industry. He has taught at the middle school, high school, two-year college, and university levels. Dr. Peterson is the author or coauthor of four other Cengage Learning books: *Introductory Technical Mathematics* (with Robert D. Smith), *Technical Mathematics*, *Technical Mathematics with Calculus*, and *Math for the Automotive Trade* (with William J. deKryger). In addition, he has had over 80 papers published in various journals, has given over 200 presentations, and has served as a vice president of the American Mathematical Association of Two-Year Colleges.

**Robert D. Smith** was Associate Professor Emeritus of Industrial Technology at Central Connecticut State University, New Britain, Connecticut. Mr. Smith had experience in the manufacturing industry as tool designer, quality control engineer, and chief manufacturing engineer. He also taught applied mathematics, physics, and industrial materials and processes on the secondary technical school level and machine technology applied mathematics for apprentices in machine, tool-and-die, and tool design occupations. He was the author of *Technical Mathematics 4e*, also published by Cengage Learning.

## ACKNOWLEDGMENTS

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The publisher wishes to acknowledge the following instructors for their detailed reviews of this text:

Dan Taylor  
Orange Coast College  
Costa Mesa, California

Edwin Thomas  
Central Carolina Community College  
Sanford, North Carolina

In addition, the publisher and author acknowledge Linda Willey for her tireless commitment to the technical review of the text, examples, applications, answers, and solutions.



## INTRODUCTION TO THE SCIENTIFIC CALCULATOR

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A scientific calculator is to be used in conjunction with the material presented in this textbook. Complex mathematical calculations can be made quickly, accurately, and easily with a scientific calculator.

Although most functions are performed in the same way, there are some variations among different makes and models of scientific calculators. In this book, generally, where there are two basic ways of performing a function, or sequencing, both ways are shown. However, not all of the differences among the various makes and models of calculators can be shown. It is very important that you become familiar with the operation of your scientific calculator. An owner's manual or user's guide is included with the purchase of a scientific calculator; it explains the essential features and keys of the specific calculator, as well as providing information on the proper use. *It is important that the owner's manual or user's guide be studied and referred to whenever there is a question regarding calculator usage.* Also, information can be obtained from the manufacturer's Internet website, which is often listed in the user's guide.

For use in this textbook, examples are shown and problems are solved with calculators having EOS™ (Equation Operating System), V.P.A.M. (Visually Perfect Algebraic Method), or D.A.L. (Direct Algebraic Logic). Key operations are performed following the mathematical expressions exactly as they are written.

Most scientific calculator keys can perform more than one function. Depending on the calculator, generally the **(2nd)** key or **(SHIFT)** key enable you to use alternate functions. The alternate functions are marked above the key. Alternate functions are shown and explained in the book where their applications are appropriate to specific content.

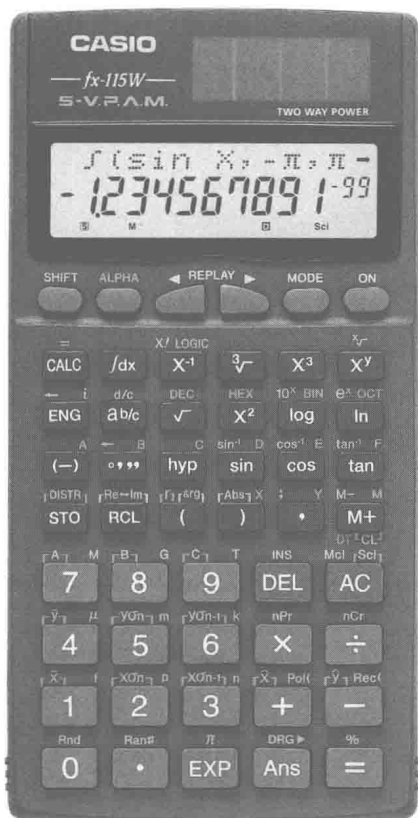
## DECISIONS REGARDING CALCULATOR USE

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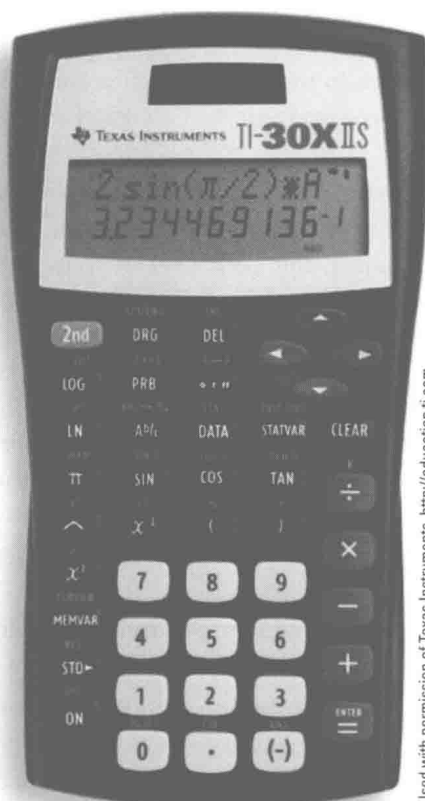
The exercises and problems presented throughout the text are well suited for solutions using a calculator. However, it is felt that decisions regarding calculator usage should be left to the discretion of the course classroom or shop instructor. The instructor best knows the unique learning environment and objectives to be achieved by the students in a course. Judgments should be made by the instructor as to the degree of emphasis to be placed on calculator applications, when and where a calculator is to be used, and the selection of specific problems for solution by calculator. Therefore, exercises and problems in this text are *not* specifically identified as calculator applications.

Calculator instruction and examples of the basic operations of addition, subtraction, multiplication, and division of fractions are presented in Unit 7. They are presented for decimals in Unit 16. Further calculator instruction and examples of mathematics operations and functions are given throughout the text wherever calculator applications are appropriate to the material presented.

The index that follows lists the mathematics operations or functions and the pages on which the calculator instruction is first given for the operations or functions. It provides a convenient reference for all material in the text for which calculator usage is presented. The operations and functions are listed in the order in which material is presented in the text.

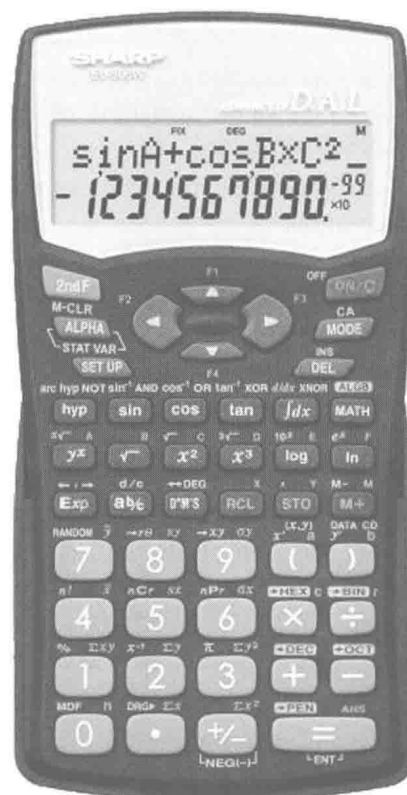


Casio FX-115W, S-V.P.A.M.



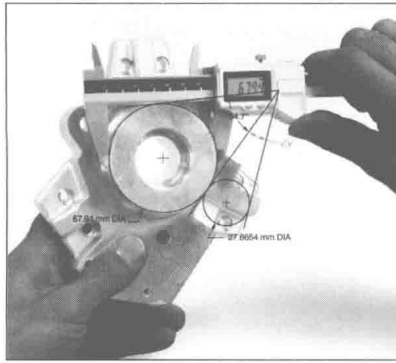
Texas Instruments TI-30XIIS

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Sharp EL-506W, advanced D.A.L.

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# Common Fractions and Decimal Fractions

SECTION

# 1

## UNIT 1

## Introduction to Common Fractions and Mixed Numbers

**OBJECTIVES** After studying this unit you should be able to

- Express fractions in lowest terms.
- Express fractions as equivalent fractions.
- Express mixed numbers as improper fractions.
- Express improper fractions as mixed numbers.

Most measurements and calculations made by a machinist are not limited to whole numbers. Dimensions are sometimes given as fractions and certain measuring tools are graduated in fractional units. The machinist must be able to make calculations using fractions and to measure fractional values.

### FRACTIONAL PARTS

A *fraction* is a value that shows the number of equal parts taken of a whole quantity or unit. The symbols used to indicate a fraction are the bar (—) and the slash (/).

Line segment AB as shown in Figure 1-1 is divided into 4 equal parts.

$$1 \text{ part} = \frac{1 \text{ part}}{\text{total parts}} = \frac{1 \text{ part}}{4 \text{ parts}} = \frac{1}{4} \text{ of the length of the line segment.}$$

$$2 \text{ parts} = \frac{2 \text{ parts}}{\text{total parts}} = \frac{2 \text{ parts}}{4 \text{ parts}} = \frac{2}{4} \text{ of the length of the line segment.}$$

$$3 \text{ parts} = \frac{3 \text{ parts}}{\text{total parts}} = \frac{3 \text{ parts}}{4 \text{ parts}} = \frac{3}{4} \text{ of the length of the line segment.}$$

$$4 \text{ parts} = \frac{4 \text{ parts}}{\text{total parts}} = \frac{4 \text{ parts}}{4 \text{ parts}} = \frac{4}{4} = 1, \text{ or unity (four parts make up the whole).}$$

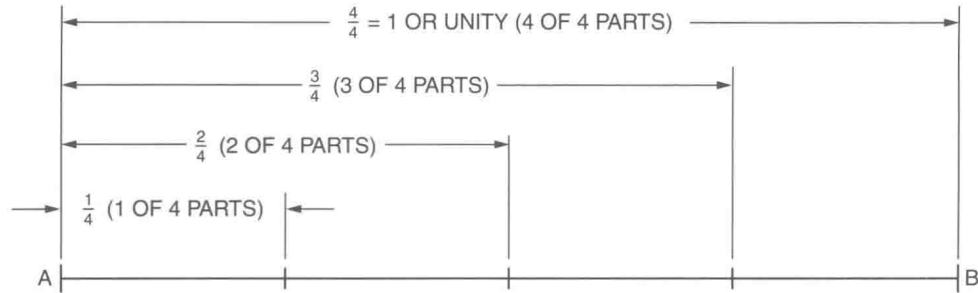


FIGURE 1-1

Each of the 4 equal parts of the line segment AB in Figure 1-2 is divided into eight equal parts. There is a total of  $4 \times 8$ , or 32, parts.

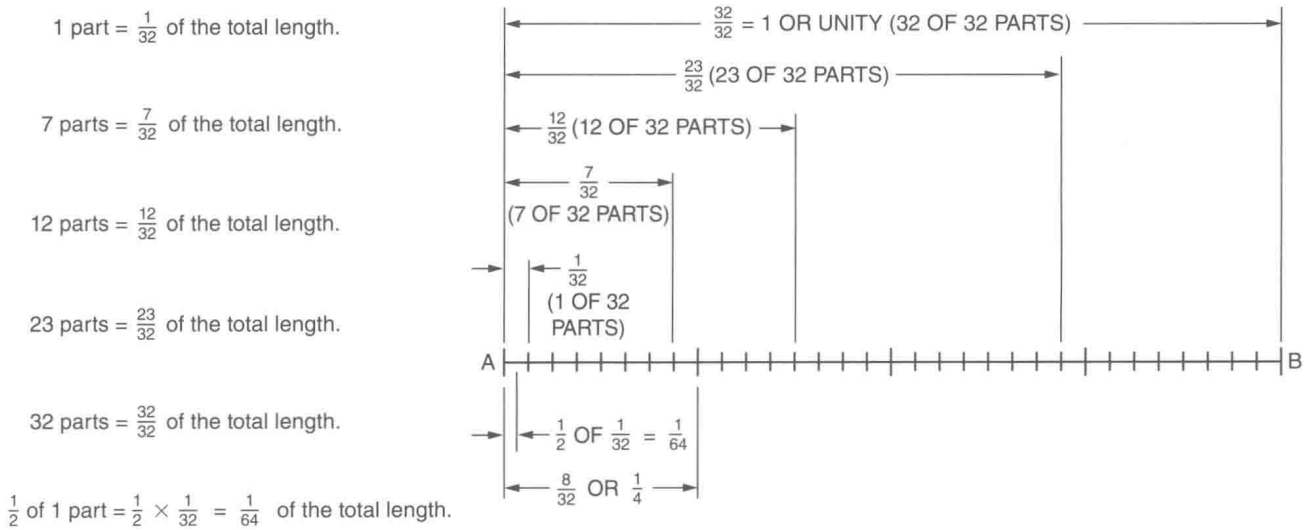


FIGURE 1-2

**Note:** 8 parts =  $\frac{8}{32}$  of the total length and also  $\frac{1}{4}$  of the total length.

Therefore,  $\frac{8}{32} = \frac{1}{4}$ .

## DEFINITIONS OF FRACTIONS

A *fraction* is a value that shows the number of equal parts taken of a whole quantity or unit.

Some examples of fractions are  $\frac{3}{4}$ ,  $\frac{5}{8}$ ,  $\frac{99}{100}$ , and  $\frac{17}{12}$ . These same fractions written with a slash are  $\frac{3}{4}$ ,  $\frac{5}{8}$ ,  $\frac{99}{100}$ , and  $\frac{17}{12}$ .

The *denominator* of a fraction is the number that shows how many equal parts are in the whole quantity. The denominator is written below the bar.

The *numerator* of a fraction is the number that shows how many equal parts of the whole are taken. The numerator is written above the bar.

The numerator and denominator are called the *terms* of the fraction.

$\frac{3}{4}$  ← numerator  
 $\frac{3}{4}$  ← denominator

A *common fraction* consists of two whole numbers.  $\frac{5}{5}$ ,  $\frac{3}{7}$ , and  $\frac{13}{4}$  are all examples of common fractions.

A *proper fraction* has a numerator that is smaller than its denominator. Examples of proper fractions are  $\frac{3}{4}$ ,  $\frac{5}{11}$ ,  $\frac{91}{92}$ , and  $\frac{247}{961}$ .

An *improper fraction* is a fraction in which the numerator is larger than or equal to the denominator, as in  $\frac{3}{2}$ ,  $\frac{5}{4}$ ,  $\frac{11}{8}$ ,  $\frac{6}{6}$ ,  $\frac{17}{17}$ .

A *mixed number* is a number composed of a whole number and a fraction, as in  $3\frac{7}{8}$ ,  $7\frac{1}{2}$ .

**Note:**  $3\frac{7}{8}$  means  $3 + \frac{7}{8}$ . It is read as three and seven-eighths.  $7\frac{1}{2}$  means  $7 + \frac{1}{2}$ . It is read as seven and one-half.

Writing fractions with a slash can cause people to misread a number. For example, some people might think that  $1\frac{1}{4}$  means  $\frac{11}{4} = \frac{11}{4}$  rather than  $1\frac{1}{4}$ . For this reason, the slash notation for fractions will not be used in this book.

A *complex fraction* is a fraction in which one or both of the terms are fractions or mixed numbers, as in  $\frac{\frac{3}{4}}{6}$ ,  $\frac{32}{\frac{15}{4}}$ ,  $\frac{8\frac{3}{4}}{3}$ ,  $\frac{\frac{7}{16}}{2\frac{2}{5}}$ ,  $\frac{4\frac{1}{4}}{7\frac{5}{8}}$ .

## EXPRESSING FRACTIONS AS EQUIVALENT FRACTIONS

The numerator and denominator of a fraction can be multiplied or divided by the same number without changing the value. For example,  $\frac{1}{2} = \frac{1 \times 4}{2 \times 4} = \frac{4}{8}$ . Both the numerator and denominator are multiplied by 4. Because  $\frac{1}{2}$  and  $\frac{4}{8}$  have the same value, they are *equivalent*. Also,  $\frac{8}{12} = \frac{8 \div 4}{12 \div 4} = \frac{2}{3}$ . Both numerator and denominator are divided by 4. Since  $\frac{8}{12}$  and  $\frac{2}{3}$  have the same value, they are equivalent. Equivalent fractions are necessary for comparing two fractions or for addition and subtraction of fractions.

A fraction is in its *lowest terms* when the numerator and denominator do not contain a common factor, as in  $\frac{5}{9}$ ,  $\frac{7}{8}$ ,  $\frac{3}{4}$ ,  $\frac{11}{12}$ ,  $\frac{15}{32}$ ,  $\frac{9}{11}$ . *Factors* are the numbers used in multiplying. For example, 2 and 5 are each factors of 10;  $2 \times 5 = 10$ . Expressing a fraction in lowest terms is often called *reducing* a fraction to lowest terms.

### ► PROCEDURE To reduce a fraction to lowest terms

- Divide both numerator and denominator by the greatest common factor (GCF).

**Example** Reduce  $\frac{12}{42}$  to lowest terms.

Both terms can be divided by 2.

$$\frac{12 \div 2}{42 \div 2} = \frac{6}{21}$$

**Note:** The fraction is reduced, but not to lowest terms.

Further reduce  $\frac{6}{21}$ .

Both terms can be divided by 3.

$$\frac{6 \div 3}{21 \div 3} = \frac{2}{7} \text{ Ans}$$

**Note:** The value  $\frac{2}{7}$  may be obtained in one step if each term of  $\frac{12}{42}$  is divided by  $2 \times 3$ , or 6. Six is the greatest common factor (GCF).

$$\frac{12 \div 6}{42 \div 6} = \frac{2}{7} \quad \text{Ans}$$

► **PROCEDURE** To express a fraction as an equivalent fraction with an indicated denominator that is larger than the denominator of the fraction

- Divide the indicated denominator by the denominator of the fraction.
- Multiply both the numerator and denominator of the fraction by the value obtained.

**Example** Express  $\frac{3}{4}$  as an equivalent fraction with 12 as the denominator.

Divide 12 by 4.

$$12 \div 4 = 3$$

Multiply both 3 and 4 by 3.

$$\frac{3 \times 3}{4 \times 3} = \frac{9}{12} \quad \text{Ans}$$

## EXPRESSING MIXED NUMBERS AS IMPROPER FRACTIONS

► **PROCEDURE** To express a mixed number as an improper fraction

- Multiply the whole number by the denominator.
- Add the numerator to obtain the numerator of the improper fraction.
- The denominator is the same as that of the original fraction.

**Example 1** Express  $4\frac{1}{2}$  as an improper fraction.

Multiply the whole number by the denominator.

Add the numerator to obtain the numerator for the improper fraction.

The denominator is the same as that of the original fraction.

$$\frac{4 \times 2 + 1}{2} = \frac{9}{2} \quad \text{Ans}$$

**Example 2** Express  $12\frac{3}{16}$  as an improper fraction.

$$\frac{12 \times 16 + 3}{16} = \frac{195}{16} \quad \text{Ans}$$

## EXPRESSING IMPROPER FRACTIONS AS MIXED NUMBERS

► **PROCEDURE** To express an improper fraction as a mixed number

- Divide the numerator by the denominator.
- Express the remainder as a fraction.

**Examples** Express the following improper fractions as mixed numbers.

$$\frac{11}{4} = 11 \div 4 = 2\frac{3}{4} \quad \text{Ans}$$

$$\frac{43}{3} = 43 \div 3 = 14\frac{1}{3} \quad \text{Ans}$$

$$\frac{931}{8} = 931 \div 8 = 116\frac{3}{8} \quad \text{Ans}$$

# APPLICATION

## Fractional Parts

1. Write the fractional part that each length, A through F, represents of the total shown on the scale in Figure 1-3.

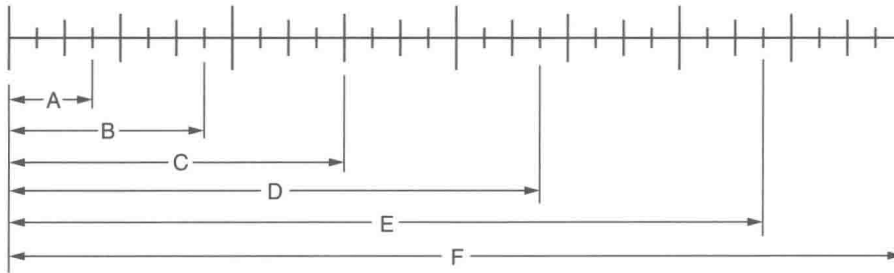


FIGURE 1-3

- A = \_\_\_\_\_  
 B = \_\_\_\_\_  
 C = \_\_\_\_\_  
 D = \_\_\_\_\_  
 E = \_\_\_\_\_  
 F = \_\_\_\_\_

2. A welded support base is cut into four pieces as shown in Figure 1-4. What fractional part of the total length does each of the four pieces represent? All dimensions are in inches.

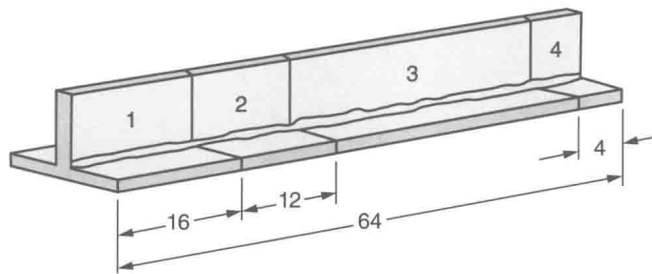


FIGURE 1-4

- Piece 1: \_\_\_\_\_  
 Piece 2: \_\_\_\_\_  
 Piece 3: \_\_\_\_\_  
 Piece 4: \_\_\_\_\_

3. The circle in Figure 1-5 is divided into equal parts. Write the fractional part represented by each of the following:

- |                            |       |                             |       |
|----------------------------|-------|-----------------------------|-------|
| a. 1 part                  | _____ | g. $\frac{1}{3}$ of 1 part  | _____ |
| b. 3 parts                 | _____ | h. $\frac{3}{4}$ of 1 part  | _____ |
| c. 7 parts                 | _____ | i. $\frac{1}{10}$ of 1 part | _____ |
| d. 5 parts                 | _____ | j. $\frac{1}{16}$ of 1 part | _____ |
| e. 16 parts                | _____ |                             |       |
| f. $\frac{1}{2}$ of 1 part | _____ |                             |       |

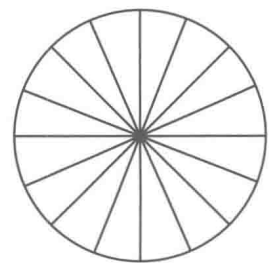


FIGURE 1-5

## Expressing Fractions as Equivalent Fractions

4. Reduce to halves.

- |                      |       |                     |       |
|----------------------|-------|---------------------|-------|
| a. $\frac{4}{8}$     | _____ | e. $\frac{25}{10}$  | _____ |
| b. $\frac{9}{18}$    | _____ | f. $\frac{18}{12}$  | _____ |
| c. $\frac{100}{200}$ | _____ | g. $\frac{126}{36}$ | _____ |
| d. $\frac{121}{242}$ | _____ | h. $\frac{225}{50}$ | _____ |



5. Reduce to lowest terms.

a.  $\frac{6}{8}$  \_\_\_\_\_

b.  $\frac{12}{4}$  \_\_\_\_\_

c.  $\frac{6}{10}$  \_\_\_\_\_

d.  $\frac{30}{5}$  \_\_\_\_\_

e.  $\frac{11}{44}$  \_\_\_\_\_

f.  $\frac{14}{6}$  \_\_\_\_\_

g.  $\frac{24}{8}$  \_\_\_\_\_

h.  $\frac{65}{15}$  \_\_\_\_\_

i.  $\frac{25}{150}$  \_\_\_\_\_

j.  $\frac{14}{105}$  \_\_\_\_\_

6. Express as thirty-seconds.

a.  $\frac{1}{4}$  \_\_\_\_\_

b.  $\frac{3}{4}$  \_\_\_\_\_

c.  $\frac{11}{8}$  \_\_\_\_\_

d.  $\frac{7}{16}$  \_\_\_\_\_

e.  $\frac{21}{16}$  \_\_\_\_\_

f.  $\frac{19}{2}$  \_\_\_\_\_

g.  $\frac{197}{16}$  \_\_\_\_\_

h.  $\frac{21}{8}$  \_\_\_\_\_

7. Express as equivalent fractions as indicated.

a.  $\frac{3}{4} = \frac{?}{8}$  \_\_\_\_\_

b.  $\frac{7}{12} = \frac{?}{36}$  \_\_\_\_\_

c.  $\frac{6}{15} = \frac{?}{60}$  \_\_\_\_\_

d.  $\frac{17}{14} = \frac{?}{42}$  \_\_\_\_\_

e.  $\frac{20}{9} = \frac{?}{45}$  \_\_\_\_\_

f.  $\frac{14}{3} = \frac{?}{18}$  \_\_\_\_\_

g.  $\frac{7}{16} = \frac{?}{128}$  \_\_\_\_\_

h.  $\frac{13}{8} = \frac{?}{48}$  \_\_\_\_\_

i.  $\frac{21}{16} = \frac{?}{160}$  \_\_\_\_\_

## Mixed Numbers and Improper Fractions

8. Express the following mixed numbers as improper fractions.

a.  $2\frac{2}{3}$  \_\_\_\_\_

b.  $1\frac{7}{8}$  \_\_\_\_\_

c.  $5\frac{2}{5}$  \_\_\_\_\_

d.  $3\frac{3}{8}$  \_\_\_\_\_

e.  $5\frac{9}{32}$  \_\_\_\_\_

f.  $8\frac{3}{7}$  \_\_\_\_\_

g.  $10\frac{1}{3}$  \_\_\_\_\_

h.  $9\frac{4}{5}$  \_\_\_\_\_

i.  $100\frac{1}{2}$  \_\_\_\_\_

j.  $4\frac{63}{64}$  \_\_\_\_\_

k.  $49\frac{3}{8}$  \_\_\_\_\_

l.  $408\frac{13}{16}$  \_\_\_\_\_