

Third Edition



Structural and Stress Analysis

T.H.G. Megson



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To the memory of my darling wife, Margaret

Preface to the First Edition

The purpose of this book is to provide, in a unified form, a text covering the associated topics of structural and stress analysis for students of civil engineering during the first two years of their degree course. The book is also intended for students studying for Higher National Diplomas, Higher National Certificates and related courses in civil engineering.

Frequently, textbooks on these topics concentrate on structural analysis or stress analysis and often they are lectured as two separate courses. There is, however, a degree of overlap between the two subjects and, moreover, they are closely related. In this book, therefore, they are presented in a unified form which illustrates their interdependence. This is particularly important at the first-year level where there is a tendency for students to ‘compartmentalize’ subjects so that an overall appreciation of the subject is lost.

The subject matter presented here is confined to the topics students would be expected to study in their first two years since third- and fourth-year courses in structural and/or stress analysis can be relatively highly specialized and are therefore best served by specialist texts. Furthermore, the topics are arranged in a logical manner so that one follows naturally on from another. Thus, for example, internal force systems in statically determinate structures are determined before their associated stresses and strains are considered, while complex stress and strain systems produced by the simultaneous application of different types of load follow the determination of stresses and strains due to the loads acting separately.

Although in practice modern methods of analysis are largely computer based, the methods presented in this book form, in many cases, the basis for the establishment of the flexibility and stiffness matrices that are used in computer-based analysis. It is therefore advantageous for these methods to be studied since, otherwise, the student would not obtain an appreciation of structural behaviour, an essential part of the structural designer’s background.

In recent years some students enrolling for degree courses in civil engineering, while being perfectly qualified from the point of view of pure mathematics, lack a knowledge of structural mechanics, an essential basis for the study of structural and stress analysis. Therefore a chapter devoted to those principles of statics that are a necessary preliminary has been included.

As stated above, the topics have been arranged in a logical sequence so that they form a coherent and progressive ‘story’. Hence, in Chapter 1, structures are considered in terms of their function, their geometries in different roles, their methods of support and the differences between their statically determinate and indeterminate forms. Also considered is the role of analysis in the design process and methods of idealizing structures so that they become amenable to analysis. In Chapter 2 the necessary principles of statics are discussed and applied directly to the calculation of support reactions. Chapters 3–6 are concerned with the determination of internal force distributions in statically determinate beams, trusses, cables and arches, while in Chapter 7 stress and strain are discussed and stress–strain relationships established. The relationships between the elastic constants are then derived and the concept of strain energy in axial tension and compression introduced. This is then applied to the determination of the effects of impact loads, the calculation of displacements in axially loaded members and the deflection of a simple truss. Subsequently, some simple statically indeterminate systems are analysed and the compatibility of displacement condition introduced. Finally, expressions for the stresses in thin-walled pressure vessels are derived. The properties of the different materials used in civil engineering are investigated in Chapter 8 together with an introduction to the phenomena of strain-hardening, creep and relaxation

and fatigue; a table of the properties of the more common civil engineering materials is given at the end of the chapter. Chapters 9, 10 and 11 are respectively concerned with the stresses produced by the bending, shear and torsion of beams while Chapter 12 investigates composite beams. Deflections due to bending and shear are determined in Chapter 13, which also includes the application of the theory to the analysis of some statically indeterminate beams. Having determined stress distributions produced by the separate actions of different types of load, we consider, in Chapter 14, the state of stress and strain at a point in a structural member when the loads act simultaneously. This leads directly to the experimental determination of surface strains and stresses and the theories of elastic failure for both ductile and brittle materials. Chapter 15 contains a detailed discussion of the principle of virtual work and the various energy methods. These are applied to the determination of the displacements of beams and trusses and to the determination of the effects of temperature gradients in beams. Finally, the reciprocal theorems are derived and their use illustrated. Chapter 16 is concerned solely with the analysis of statically indeterminate structures. Initially methods for determining the degree of statical and kinematic indeterminacy of a structure are described and then the methods presented in Chapter 15 are used to analyse statically indeterminate beams, trusses, braced beams, portal frames and two-pinned arches. Special methods of analysis, i.e. slope–deflection and moment distribution, are then applied to continuous beams and frames. The chapter is concluded by an introduction to matrix methods. Chapter 17 covers influence lines for beams, trusses and continuous beams while Chapter 18 investigates the stability of columns.

Numerous worked examples are presented in the text to illustrate the theory, while a selection of unworked problems with answers is given at the end of each chapter.

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Preface to the Second Edition

Since 'Structural and Stress Analysis' was first published changes have taken place in courses leading to degrees and other qualifications in civil and structural engineering. Universities and other institutions of higher education have had to adapt to the different academic backgrounds of their students so that they can no longer assume a basic knowledge of, say, mechanics with the result that courses in structural and stress analysis must begin at a more elementary stage. The second edition of 'Structural and Stress Analysis' is intended to address this issue.

Although the feedback from reviewers of the first edition was generally encouraging there were suggestions for changes in presentation and for the inclusion of topics that had been omitted. This now means, in fact, that while the first edition was originally intended to cover the first two years of a degree scheme, the second edition has been expanded so that it includes third- and fourth-year topics such as the plastic analysis of frames, the finite element method and yield line analysis of slabs. Furthermore, the introductions to the earlier chapters have been extended and in Chapter 1, for example, the discussions of structural loadings, structural forms, structural elements and materials are now more detailed. Chapter 2, which presents the principles of statics, now begins with definitions of force and mass while in Chapter 3 a change in axis system is introduced and the sign convention for shear force reversed.

Chapters 4, 5 and 6, in which the analysis of trusses, cables and arches is presented, remain essentially the same although Chapter 4 has been extended to include an illustration of a computer-based approach.

In Chapter 7, stress and strain, some of the original topics have been omitted; these are some examples on the use of strain energy such as impact loading, suddenly applied loads and the solutions for the deflections of simple structures and the analysis of a statically indeterminate truss which is covered later.

The discussion of the properties of engineering materials in Chapter 8 has been expanded as has the table of material properties given at the end of the chapter.

Chapter 9 on the bending of beams has been modified considerably. The change in axis system and the sign convention for shear force is now included and the discussion of the mechanics of bending more descriptive than previously. The work on the plastic bending of beams has been removed and is now contained in a completely new chapter (18) on plastic analysis. The introduction to Chapter 10 on the shear of beams now contains an illustration of how complementary shear stresses in beams are produced and is also, of course, modified to allow for the change in axis system and sign convention. Chapter 11 on the torsion of beams remains virtually unchanged as does Chapter 12 on composite beams apart from the change in axis system and sign convention. Beam deflections are considered in Chapter 13 which is also modified to accommodate the change in axis system and sign convention.

The analysis of complex stress and strain in Chapter 14 is affected by the change in axis system and also by the change in sign convention for shear force. Mohr's circle for stress and for strain are, for example, completely redrawn.

Chapters 15 and 16, energy methods and the analysis of statically indeterminate structures, are unchanged except that the introduction to matrix methods in Chapter 16 has been expanded and is now part of Chapter 17 which is new and includes the finite element method of analysis.

Chapter 18, as mentioned previously, is devoted to the plastic analysis of beams and frames while Chapter 19 contains yield line theory for the ultimate load analysis of slabs.

Chapters 20 and 21, which were Chapters 17 and 18 in the first edition, on influence lines and structural instability respectively, are modified to allow for the change in axis system and, where appropriate, for the change in sign convention for shear force.

Two appendices have been added. Appendix A gives a list of the properties of a range of standard sections while Appendix B gives shear force and bending moment distributions and deflections for standard cases of beams.

Finally, an accompanying Solutions Manual has been produced which gives detailed solutions for all the problems set at the end of each chapter.

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Preface to the Third Edition

After the encouraging response to the second edition, the main features of the third edition are an increase in the number of worked examples and end of chapter problems, together with an extension of the work on fatigue, to the prediction of the fatigue life of a structure in terms of the number of cycles to failure, and to the study of crack propagation and crack propagation rates.

The accompanying Solutions Manual gives detailed solutions for all the problems set at the end of each chapter. It can be found at: <http://booksite.elsevier.com/9780080999364>

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