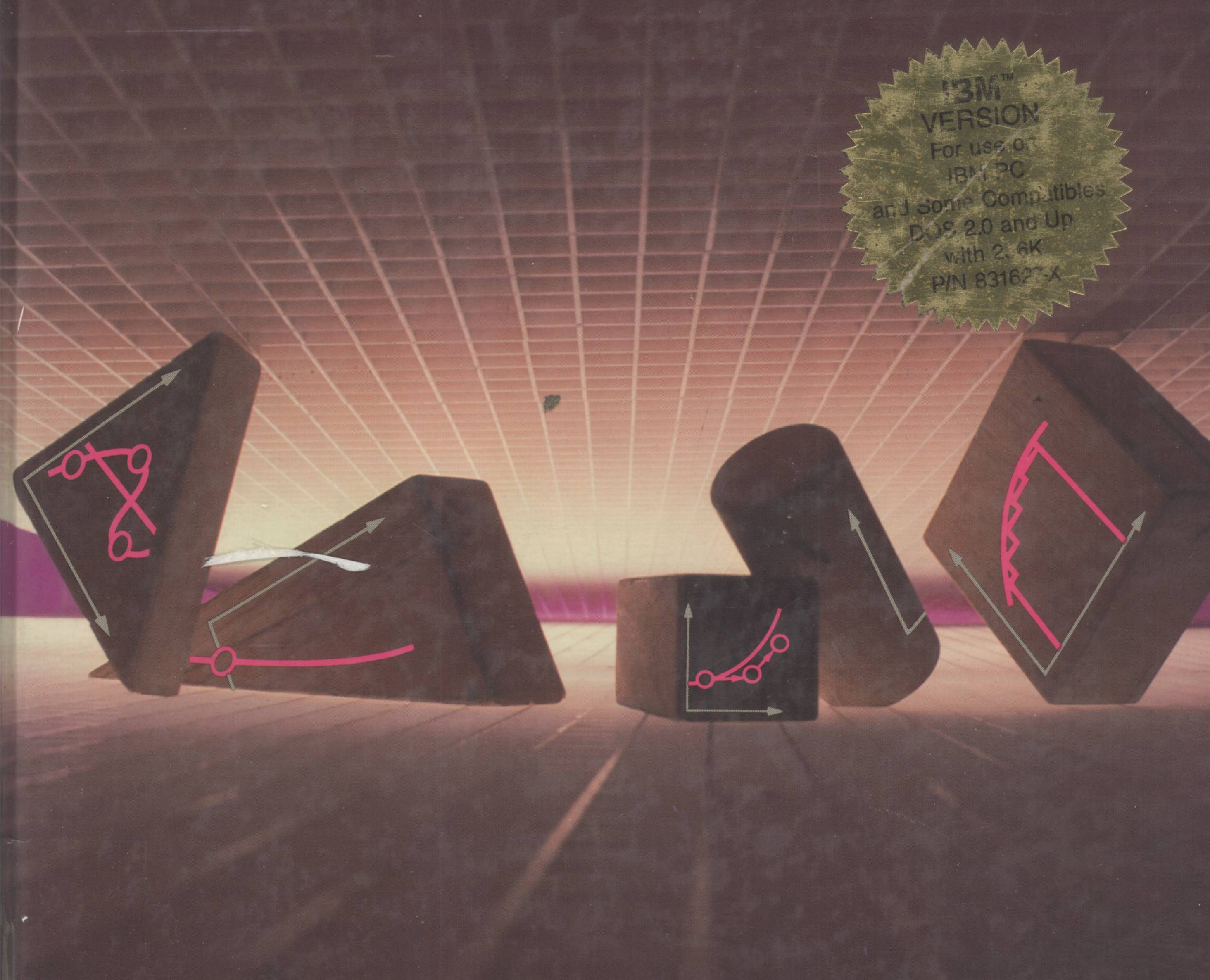
NUMERICAL METHODS FOR ENGINEERS

SECOND EDITION



STEVEN C. CHAPRA · RAYMOND P. CANALE

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To
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NUMERICAL METHODS FOR ENGINEERS

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Dr. Chapra received engineering degrees from Manhattan College and the University of Michigan. Before joining the faculty of the University of Colorado, he worked as an engineer and computer programmer for the Environmental Protection Agency and the National Oceanic and Atmospheric Administration and was an Associate Professor at Texas A&M University. Dr. Chapra has published, in addition to his books, several computer programs and numerous journal articles. All deal with mathematical modeling and the application of computers to engineering problem solving.

Dr. Chapra is active in a number of professional societies and serves on several governmental and professional committees related to applied modeling and computer applications in engineering education. He is the recipient of a number of awards including the Tenneco Award for Teaching Excellence and the Meriam/Wiley Distinguished Author Award.

Raymond P. Canale is a professor of civil and environmental engineering at the University of Michigan, where he teaches courses on computers and numerical methods. He also teaches and performs extensive research in the area of mathematical and computer modeling of environmental systems. He has authored books on mathematical modeling of aquatic ecosystems and is a coauthor of *Introduction to Computing for Engineers*. He has published over 100 scientific papers and reports, and designed and developed personal computer software for engineers. He has been given the Meriam/Wiley Distinguished Author Award by the American Society for Engineering Education for his books and software.

Professor Canale is also a practicing professional engineer. He is an active consultant for engineering firms as well as for industry and governmental agencies. He has served as an expert technical witness on numerous occasions.

Overall, Professor Canale has devoted over twenty years to his profession as a teacher, researcher, author, and practicing engineer.

PREFACE

A lot of water has passed under the bridge in the three years since the publication of the first edition of *Numerical Methods for Engineers*. Our contention that numerical methods and computers would figure more prominently in the engineering curriculum—particularly in the early parts—has been dramatically born out. Many universities now offer freshman, sophomore, and junior courses in both introductory computing and numerical methods. In addition, many of our colleagues are integrating computer-oriented problems into other courses at all levels of the curriculum. Thus, this new edition is still founded on the basic premise that student engineers should be provided with a strong and early introduction to numerical methods. Consequently, although we have expanded our coverage in the new edition, we have tried to maintain many of the features that made the first edition accessible to both lower- and upper-level undergraduates. These include:

1. *Boxed material.* We have endeavored to include important derivations and error analyses in order to enrich the presentation. However, such material sometimes represents a stumbling block for the beginning student. Consequently, we have sequestered the more complicated mathematical material in boxes. Many students will find that they can apply the numerical methods without completely mastering the boxed material.
2. *Introductory material and mathematical background.* Every part of the book includes an introductory section. After a brief statement of the general mathematical problem under study, motivation is provided by describing how the problem would be approached in the absence of computers and how the method would be used in engineering practice. This material is followed by a review of the mathematics required to successfully master the subject at hand. For example, matrix algebra is reviewed prior to introducing linear algebraic equations, and statistics is reviewed prior to regression. Finally, an outline and study objectives are listed to provide some orientation to subsequent materials.
3. *Epilogues.* Just as the introduction is designed to provide motivation and orientation, we include an epilogue at the end of each part of the book to consolidate the newly acquired concepts. An important feature of this epilogue is a section devoted to the

trade-offs involved in choosing the appropriate numerical methods for a particular problem. In addition, important formulas and references for advanced methods are summarized.

4. *Sequential presentations*. Each major part of the book consists of several chapters—the initial ones devoted to theory and the last to case studies. Wherever possible, the theory chapters are structured sequentially; that is, the more elementary and straightforward approaches are presented first. Because many of the more advanced methods build on the simpler ones, this development is intended to provide a sense of the evolution of the techniques. In addition, by first pushing the simpler approaches to their limits, incentive is provided to pursue the higher-order or more complicated methods.
5. *Case studies*. Case studies have been included in each part of the book to demonstrate the practical utility of the numerical methods. A significant effort was made to incorporate examples from the early courses in a typical engineering curriculum. When this was not possible, the theoretical basis and motivation for the problems have been provided.

Although the above features allow the book to be employed at the lower end of the undergraduate engineering curriculum, we have expanded our coverage by adding several advanced topics (Fig. P.1). There are three primary reasons for this expansion. First, although more universities are teaching the subject at lower levels, many institutions still offer numerical methods as an upper undergraduate or graduate course. We wanted to ensure that our book would be flexible enough to be useful at these levels. Second, we wanted to make the book a more comprehensive reference for both students and professionals. Finally, we have found that even when the book is used at the lower levels, introduction to more advanced topics such as eigenvalues and partial differential equations can be presented effectively. This has been facilitated by a number of trends including the growing computer sophistication of our students and the increased credit hours allocated to numerical methods courses at many institutions over the past few years.

The expanded coverage is depicted schematically in Fig. P.1. Notice that most of the new material is added at the end of each part prior to the case studies. In addition, a new part on partial differential equations has been included at the end of the book. The material was incorporated in this way to conform with our sequential approach of presenting numerical methods. Thus, because most of the new topics are of a more advanced nature, they could be very naturally appended to the already existing material. This configuration has the added benefit that those who do not want to explicitly include the new chapters in existing courses can more easily pass over them.

Along with the new material, other modifications have been added to enhance the new edition. Although *Part One* is still intended as an introductory section on modeling, computers, and error analysis, the content has been expanded. *Chapter 1* includes additional material on the major conservation laws employed in engineering. This material was included to broaden the student's conception of modeling and to reinforce the connection with the major case study chapters in the remainder of the book. *Chapter 2* has been completely rewritten to reflect changes in the state-of-computing with particular emphasis on structured programming techniques. *Chapter 3* on errors is essentially intact. However, we have moved some of the formulas on numerical

PART ONE Modeling, Computers and Error Analysis	PART TWO Roots of Equations	PART THREE Systems of Linear Algebraic Equations	PART FOUR Curve Fitting	PART FIVE Numerical Differentiation and Integration	PART SIX Ordinary Differential Equations	PART SEVEN Partial Differential Equations
INTRODUCTION	INTRODUCTION	INTRODUCTION	INTRODUCTION	INTRODUCTION	INTRODUCTION	INTRODUCTION
CHAPTER 1 Modeling and Problem Solving	CHAPTER 4 Bracketing Methods	CHAPTER 7 Gauss Elimination	CHAPTER 11 Regression	CHAPTER 15 Newton-Cotes Integration	CHAPTER 19 One-step Methods	CHAPTER 23 Finite Difference: Elliptic
CHAPTER 2 Computer Programming and Software	CHAPTER 5 Open Methods	CHAPTER 8 Matrix Inversion and Gauss-Seidel	CHAPTER 12 Interpolation	CHAPTER 16 Integration of Equations	CHAPTER 20 Adaptive Step Size Control	CHAPTER 24 Finite Difference: Parabolic
CHAPTER 3 Approximation and Errors	CHAPTER 6 Case Studies	CHAPTER 9 LU Decomposition	CHAPTER 13 Fourier Approximation	CHAPTER 17 Numerical Differentiation	CHAPTER 21 Boundary-value and Eigenvalue Problems	CHAPTER 25 Finite-Element Method
EPILOGUE	EPILOGUE	CHAPTER 10 Case Studies	CHAPTER 14 Case Studies	CHAPTER 18 Case Studies	CHAPTER 22 Case Studies	CHAPTER 26 Case Studies
		EPILOGUE	EPILOGUE	EPILOGUE	EPILOGUE	EPILOGUE

FIGURE P.1
An outline of the present edition. The shaded areas represent new material. In addition, many of the original chapters have been supplemented with new topics, and homework problems and case studies have been revised and new ones added.

differentiation to a new Chapter 17 and have included some new material on computer round-off errors and error propagation.

Part Two on roots of equations includes one major addition. A section on *systems of nonlinear equations* has been appended to *Chapter 5*.

The major addition to *Part Three* is *Chapter 9* on *LU decomposition methods* for solving systems of linear algebraic equations. This chapter begins by illustrating how Gauss elimination can be formulated as an LU decomposition solution. In addition, alternative approaches such as *Crout* and *Cholesky decomposition* are introduced. Aside from these significant additions, former material on banded systems has been expanded into a major section in *Chapter 9*.

Part Four has been augmented in two important ways. First, *Chapter 11* has been supplemented with a section on *nonlinear regression*. Second, material on *Fourier approximation* has been included in a new *Chapter 13*.

As mentioned previously, most of the material on numerical differentiation has been removed from Chapter 3 and expanded into *Chapter 17*. Thus, *Part Five* now treats this topic along with numerical integration. This allows a more satisfying and unified discussion of the effect of data error on these two fundamental mathematical operations. In addition, we have included a new section on improper integrals in *Chapter 16*.

The major addition to *Part Six* is that *boundary-value problems* are treated explicitly in *Chapter 21*. The material also includes an introduction to techniques for determining *eigenvalues*.

Finally, *Part Seven* consists of entirely new material on *partial differential equations*. *Chapter 23 and 24* deal with *finite-difference solutions* for elliptic and parabolic equations. *Chapter 25* provides an introduction to the *finite-element method*.

Aside from the expanded scope, the second edition has been upgraded in a number of ways. For example, many of the homework problems and case studies have been revised or replaced. In addition, our treatment of computer-related material has been significantly modified and improved. This has been necessitated in part by the pronounced changes that have occurred in the computing environments within which both students and faculty work.

Although microcomputers such as the Apple II were beginning to gain acceptance when the first edition was written five years ago, centralized mainframe computing installations represented the major computing environment. Today, although mainframes are still significant, a wide variety of more personalized alternatives are available. These range from local networks of minicomputers or workstations to microcomputer labs. In addition, more of our students are acquiring their own personal computers.

For this reason, and also because we anticipate that students will be using different types of machines to learn numerical methods, we have designed this book so that it can be used with any of these systems. Thus, most of the material can be implemented on machines ranging from personal computers to mainframe systems. At the same time, we believe that the "personal" aspect of the microcomputer revolution lies at the root of the present excitement over computing among students and professionals alike. Therefore, we have tried wherever possible to acknowledge and explore some of the exciting new developments that are directly associated with personal computers. For example, case studies on spreadsheet applications have been added to each part of the book.

Our treatment of computer algorithms and programs has also been upgraded to reflect new developments. Wherever possible, we have expressed algorithms in pseudocode. Additionally, all computer programs have been rewritten in structured format employing Microsoft BASIC, FORTRAN 77, and Turbo Pascal.

The first edition included computer examples and problems using our NUMERICOMP software package. These features have been retained in the second edition and can be implemented with either NUMERICOMP or an enhanced software package—the Electronic TOOLKIT.* In addition, we have developed a completely new computer software supplement for the second edition. The supplement, which is available at no cost to students, consists of several example spreadsheets. These are related to the case studies that we have added on numerical applications of spreadsheets. The supplement is designed to allow students to interactively explore the advantages and disadvantages of different numerical methods. Among other things, students can investigate the effects of changing step size, error criteria, and system parameters on numerical efficiency.

Finally, as with the previous edition, we have exerted a conscious effort to make this book as user-friendly as possible. Thus, we have endeavored to keep our explana-

*The Electronic TOOLKIT includes the numerical methods programs of NUMERICOMP along with a spreadsheet, statistics package, and graphics generator.

tions straightforward and oriented practically. Although our primary intent is to provide students with a sound introduction to numerical methods, we have the ancillary objective of making this introduction a pleasurable experience. We believe that students who enjoy numerical methods, computers, and mathematics will, in the end, make better engineers. If our book fosters an enthusiasm for these subjects, we will consider our efforts a success.

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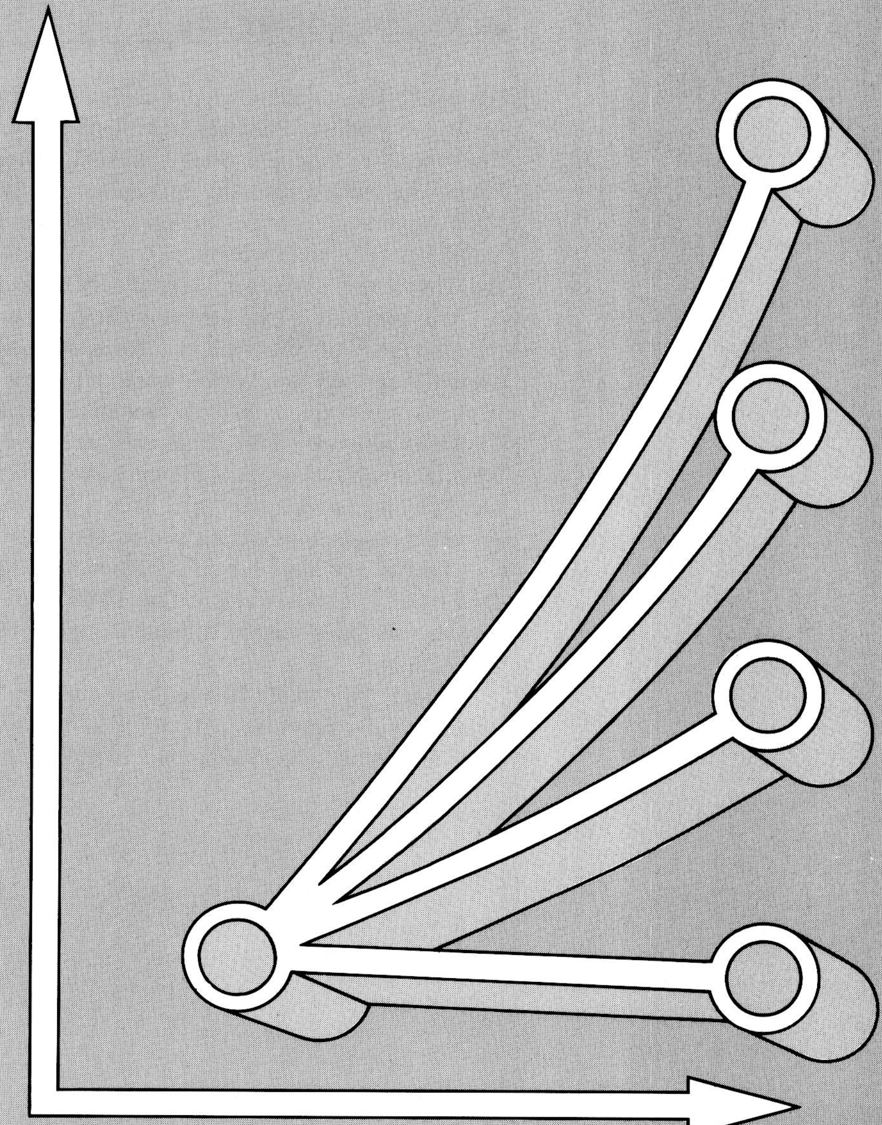
Steven C. Chapra
Raymond P. Canale

PART ONE



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