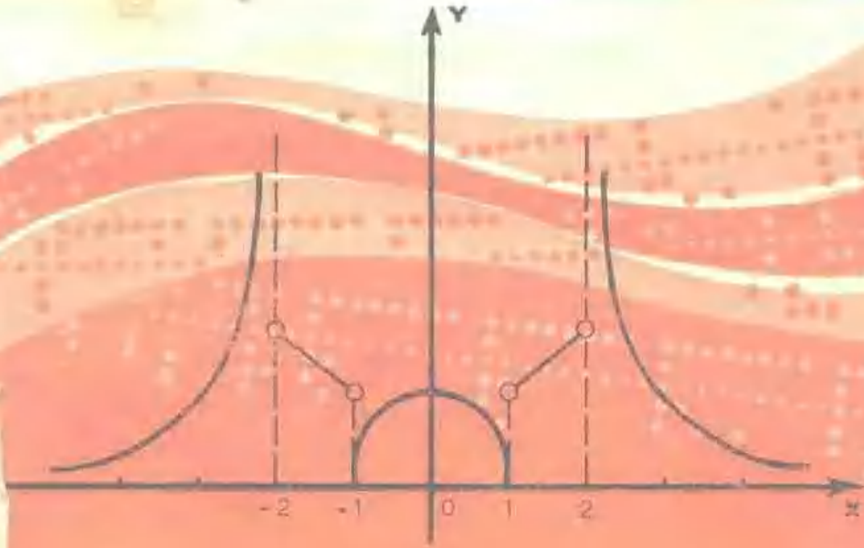
专业英语文选

数学专业英语文选

下册



ROFESSIONAL ENGLISH
ELECTED WORKS

数学专业英语文选

下册

南京大学外文系公共英语教研室编

商务印书馆
1979年·北京

数学专业英语文选

下　册

南京大学外文系
公共英语教研室编

商　务　印　书　馆　出　版
（北京王府井大街36号）
新华书店北京发行所发行
沔阳县印刷厂印刷

787×1092 毫米 1/32　$6\frac{5}{8}$ 印张　136 千字
1979 年 8 月第 1 版　　1979 年 8 月湖北第 1 次印刷
印数：1—61,000 册
统一书号：9017·859　　定价：0.46 元

CONTENTS

31. COMPARATIVE RATE OF FUNCTIONS AND INDEPENDENT VARIABLES

It is the primary object of the Differential Calculus to obtain a measure of the rate of increase of the function as compared with that of the independent variable.① For this purpose, we let Δx denote an increment in the value of x, so that x and $x+\Delta x$ are two values of the independent variable. Let Δy denote the movement in y consequent upon the increase of x to $x+\Delta x$. Then $y+\Delta y$ is the new value of the function; that is to say, it is the same function for $x+\Delta x$ that y is of x.② We shall first consider the ratio of the two increments Δy and Δx.

To begin with the simplest function, let

$$y=mx+b, \tag{1}$$

where m and b are constants. The graph of this function is a straight line, and the function is hence called the linear function. If x be increased by Δx, the new value of y will be

$$y+\Delta y=m(x+\Delta x)+b \tag{2}$$

Subtracting equation (1), we find

$$\Delta y=m\Delta x;$$

whence

$$\frac{\Delta y}{\Delta x}=m.$$

Thus the ratio between the corresponding increments of y

and x is, in this case, constant.

There are two things implied in this statement: first, that no matter how large x is taken, the ratio is unchanged, secondly, no matter whereon the line we take the point P, the ratio remains the same.

In this case, the ratio Δy: Δx ,is the measure of relative rates of increase of y and x. Thus, if $m=\frac{1}{2}$, y increases half as fast as x; if $m=2$, it increases twice as fast as x; if $m=-1$, it decreases with the rate with which x increases. If Φ denotes the angle the line makes with the axis of $x, m=\tan\Phi$. In the graph, $\tan\Phi$ is taken as the gradient or measure of the slope of the line, this slope being constant in the case of the straight line.

Let us next apply a similar process to the function $y=x^2$. When x is increased to $x+\Delta x$, the function becomes

$$y+\Delta y=(x+\Delta x)^2=x^2+2x\Delta x+(\Delta x)^2; \tag{1}$$

whence

$$\Delta y=2x\Delta x+(\Delta x)^2, \tag{2}$$

and

$$\frac{\Delta y}{\Delta x}=2x+\Delta x. \tag{3}$$

The ratio of the two increments is no longer constant. This is obviously due to the fact that③ the graph is not a straight line, and that in consequence the relation rate of increase of y is not constant; in other words, if x increases uniformly, y will not increase uniformly. We should therefore expect the measure of y's rate to contain x. Now the ratio equation in (3) is the slope of the straight line passing through P and P', which, with reference to the curve, we call a secant line. The slope of this secant is not an exact measure of the relative rate of increase in y at the point P, because it depends also upon the point P'. The ratio of increments Δy: Δx in fact depends not only upon the rate of y at P, but upon all the various values of the rate while the moving point goes from P to P'. It may be taken as the measure of the average rate for the whole interval of this motion, but it is not the measure of the rate (at the instant) when the point is at P.

This obviously applies whenever the graph is a curve.

词 汇

comparative [kəm'pærətiv] *a.* 比较的,相对的
primary ['praiməri] *a.* 主要的;初步的
consequent ['kɔnsikwənt] *a.* 因……而起的;*n.* 后项
imply [im'plai] *v.t.* 包含
secondly ['sekəndli] *ad.* 第二;其次
whereon [hwɛə'rɔn] *ad.* 在什么上面;在那上面
expect [iks'pekt] *v.t.* 期待,指望
contain [kən'tein] *v.t.* 包含
reference ['refrəns] *n.* 参看;提到
secant ['si:kənt] *a.* 割的;*n.* 正割;割线
average ['ævəridʒ] *n.* 平均率(数);平均;*a.* 平均的
instant ['instənt] *n.* 瞬间;时刻

词 组

in consequence 因此,结果
with reference to 关于
at the instant 在那一瞬间

注 释

① 此句中 it 为先行代词,代主语 to obtain ... 到句末。
that 为指示代词,代 the rate of increase。
② 在 that y is of x 句中,that 为关系代词,引导一定语从句,说明 function,它在从句里代 function 作表语。
③ 此句中两个 that 连接的为两并列的同位语从句,说明名词 fact。

32. DERIVATIVES

The Measure of the Relative Rate

To find the proper measure of the *relative rate* of y at the point P, we observe that, if P' were taken nearer to P, the slope of PP' would measure the average rate of y for a smaller interval, and thus come nearer to being① the measure of the rate at P. Moreover, if P' approaches P indefinitely and finally coincides with it, the secant line becomes a tangent line, and its slope then depends upon no value of the rate except that at P. The slope of the tangent line is therefore the proper measure of the rate at P, defining the expression slope of the curve at a point to mean slope of the tangent line at the point, this is expressed as follows: the measure of the relative rate of y compared with x is the slope of the graph of the function at the point representing the values of y and x in question.

The tangent is often called the limiting position of the secant line, but it is an actual position of the line; it is only limiting because the line ceases for a moment to be properly called a secant (since a secant is defined as line passing through two points of the curve). It is sometimes called a secant passing through two consecutive points of the curve, or through two coincident points of the curve, the latter phrase implying, of course, that the two points have come into coincidence by motion along the curve.

The Derivative

The analytical meaning of the statement above is that, when y and x diminish together, their ratio tends to a limiting value which is perfectly definite quantity; this value is reached just as the terms of the ratio vanish, and it is the measure of the relative rate of y and x. It is called limiting ratio because the ratio then ceases to be a fraction whose value could be obtained by finding how many times the numerator contains the denominator. For reasons which will be explained further on, the value of this limit is denoted by $\frac{dy}{dx}$. Thus, defining $\frac{dy}{dx}$ as the measure of the relative rate of y, we may write:

$$\text{Limit, when } \Delta x \to 0, \text{ of } \frac{\Delta y}{\Delta x} = \frac{dy}{dx}.$$

This is also frequently expressed by the equation

$$\frac{\Delta y}{\Delta x} = \frac{dy}{dx} + \varepsilon,$$

where it is understood that ε is a quantity which vanished with Δx.

The value of $\frac{dy}{dx}$ depending, as it does,② upon x (when y is any function of x except the linear), is a new function of x, which is called the derivative of the given function. Thus, from equation (3) we derive, by making $\Delta x \to 0$,

$$\frac{dy}{dx} = 2x;$$

hence $2x$ is the derivative of the function x^2.

It follows that a positive value of the derivative indicates an increasing function, and a negative value, a decreasing function.

词 汇

derivative [di'rivətiv] *n.* 导数; *a.* 导出的

coincide [kəuin'said] *v.i.* 与……一致,符合

tangent ['tændʒənt] *a.* 切线的; *n.* 切线

consecutive [kən'sekjutiv] *a.* 依次的;继续的

coincident [kəu'insidənt] *a.* 一致的,符合的

phrase [freiz] *n.* 短语;片语

tend [tend] *v.i.* 倾向

perfectly ['pə:fiktli] *ad.* 完全地;正确地

vanish ['væniʃ] *v.i.* 消失;化为零

further ['fə:ðə] *ad.* 更进一步

increasing function (递)增函数

decreasing function (递)减函数

词 组

for a moment 暂时

(to) tend to 趋向于

further on 更向前(进)

注 释

① 本句中 being 为动词 be 的动名词,作其前介词 to 的宾语。

② as it does 是插入语,说明主语: The value of $\frac{dy}{dx}$ depending upon x.

33. NEWTON'S INTEGRALS AND RIEMANN'S INTEGRALS

Areas, and the Differential and Integral Calculus

In the simplest case the process of integration is the adding together of areas of non-overlapping elementary figures, and then the taking① of some kind of a limit. The Greeks computed many simple areas, the methods being systematized through the years, and culminating in the method of exhaustions of Eudoxus (c. 408–355 B.C.) and Archimedes (c. 287–212 B.C.). This method was the first crude limit process, and they used the geometry of the figures to fit a sequence of non-overlapping triangles inside each main figure that finally exhausts the area. By this means they found the areas of the circle and sections of parabolas, for example, but could not define a general non-negative polynomial, and so could not compute the area under its curve.

The second approach to integration lies in inverting the result of differentiating a known function. The operation of differentiation was first systematized by I. Newton (1642—1727) and G.W. Leibnitz (1646—1716). To each of a certain class of functions f for which the derivative $Df=df/dx$ exists, say, for x in $a\leqslant x\leqslant b$, we make correspond that derivative,② so that we can regard D as an operator. It obeys the following rules. If f,g are differentiable functions of x in $a \leqslant x \leqslant b$, and if α, β are constants, then in $a \leqslant x \leqslant b$ we have:

$$D(\alpha f+\beta g)=\alpha Df+\beta Dg \tag{1.1}$$

$$D(fg)=(Df)g+f(Dg) \tag{1.2}$$

$$D\{f(g(x))\}=(df/dg)Dg \tag{1.3}$$

$$D\alpha=0 \tag{1.4}$$

The rule for division is obtained from (1.2); if $f=h/g$ then,

$$Dh=(Df)g+f(Dg)$$

$$D(h/g)=Df=\{Dh-(h/g)Dg\}/g$$

A function H of points x is an indefinite Newton integral of a known finite function f in $a \leqslant x \leqslant b$, if $DH=f$ in that interval. The functions that Newton integrated are all continuous, but we can ignore that limitation. Then the definite Newton integral in $a \leqslant x \leqslant b$ is $H(b)-H(a)$. We can write H as:

$$H=D^{-1}f=(NL)\int f\,dx,\ H(b)-H(a)=(NL)\int_a^b f\,dx$$

where NL stands for Newton-Leibnitz. This definition of the integral is descriptive. No method of construction is offered, but we are given its properties so that we can recognize it if it is produced in another way. Because of this we have to prove that if H and H_1 are both indefinite Newton integrals of the same function f in $a \leqslant x \leqslant b$, then:

$$H(b)-H(a)=H_1(b)-H_1(a) \tag{1.5}$$

To prove (1.5) we note that by (1.1)

$$D(H-H_1)=f-f=0$$

so that in particular $H-H_1$ is continuous, and then the mean value theorem gives (1.5).

From (1.1) we obtain the distributivity of the Newton integral, namely,

$$D^{-1}(\alpha f+\beta g)=\alpha D^{-1}f+\beta D^{-1}g \tag{1.6}$$

From (1.2; 1.6) we have the formula for integration by parts,

$$D^{-1}(gDf)+D^{-1}(fDg)=fg$$

$$(NL)\int\left(\frac{fdg}{dx}\right)dx=fg-(NL)\int\left(\frac{gdf}{dx}\right)dx \tag{1.7}$$

From (1.3) we have

$$f(g(x))=(NL)\int\frac{df}{dg}\cdot\frac{dg}{dx}dx$$

and replacing df/dg by f_1,

$$(NL)\int f_1(g)dg=(NL)\int f_1\cdot\frac{dg}{dx}dx \tag{1.8}$$

the formula for integration by substitution.

When we have defined more general integrals we will see that the formulae (1.5; 1.6; 1.7; 1.8) are in some sense still true for them.

The integration of a polynomial in x is now easy, but some simple functions cannot be integrated. It can be proved that if DH exists in $a\leqslant x\leqslant b$, and if γ is a number between $H'(a)$ and $H'(b)$, then there is a ξ in $a\leqslant\xi\leqslant b$ such that $H'(\xi)=\gamma$. It follows that if f is zero for x less than $\frac{1}{2}(a+b)$, and is 1 otherwise, then f does not have a Newton integral in $a\leqslant x\leqslant b$.

Riemann, Riemann-Stieltjes and Burkill Integration

G.F.B.Riemann (1826—66) gave the following definition of the definite integral of a function f in $a\leqslant x\leqslant b$:

Let

$$a = x_0 < x_1 < \cdots < x_n = b \tag{2.1}$$

be a division of $a \leqslant x \leqslant b$ into smaller intervals, let ξ_j be a point of the interval $x_{j-1} \leqslant x \leqslant x_j$, and consider the sum

$$S = \sum_{j=1}^{n} f(\xi_j)\ (x_j - x_{j-1}) \tag{2.2}$$

The number I is the definite Riemann integral of f in $a \leqslant x \leqslant b$, if to each $\epsilon > 0$ there is a $\delta > 0$ such that

$$|S - I| < \epsilon \tag{2.3}$$

whenever

$$x_{j-1} \leqslant \xi_j \leqslant x_j < x_{j-1} + \delta \quad (j = 1, 2, \cdots, n) \tag{2.4}$$

J.G. Darboux (1842—1917) made the following modification when f is real. He replaced $f(\xi_j)$ by the supremum (least upper bound) of f in $x_{j-1} \leqslant x \leqslant x_j$, and obtained an upper sum. For a lower sum he replaced $f(\xi_j)$ by the infimum (greatest lower bound) of f in $x_{j-1} \leqslant x \leqslant x_j$. If f is non-negative, with a given graph, and if we take a division (2.1) of $a \leqslant x \leqslant b$, then the upper Darboux sum is the sum of the areas of rectangles with bases the intervals $x_{j-1} \leqslant x \leqslant x_j$, and with just sufficient height to include the graph. The lower Darboux sum is the sum of the areas of rectangles with the same bases, but lying just below the graph. When f is real it is clear that for suitable choice of the ξ_j, the S of (2.2) can be taken arbitrarily near to the upper sum, and for another choice, arbitrarily near to the lower sum, so that the Darboux modification does not alter the Riemann integral of a real function. Thus if a real function has a Riemann integral in $a \leqslant x \leqslant b$ it must be bounded there. From this we can show that not every Newton integral is a Riemann integral. For

$$H(x)=x^2\cdot\sin(1/x^2)(x\neq0),\ H(0)=0 \qquad (2.5)$$

is differentiable everywhere, the derivative being unbounded in the neighbourhood of $x=0$. However, not every Riemann integral is a Newton integral, for the Riemann integral of the last function of section 2 exists in $a\leqslant x\leqslant b$, and is equal to $\frac{1}{2}(b-a)$. There is a common region, for the Riemann and Newton integrals of a continuous function exist and are equal. The Riemann integral cannot integrate every bounded function, for if

$$f(x)=\begin{cases}1(x\ \text{rational})\\ 0(x\ \text{irrational})\end{cases} \qquad (2.6)$$

then any upper Darboux sum is $b-a$, while any lower Darboux sum is 0. Thus f does not have a Riemann integral (nor a Newton integral).③

词 汇

Riemann ['ri:mən] 黎曼(人名)
integral calculus 积分学
integration [ˌinti'greiʃən] *n.* 积分,集成
non-overlap [nɔnˌəuvə'læp] *v.t.* & *v.i.* 不重叠
systematize ['sistimətaiz] *v.t.* 使有系统;分类
culminate ['kʌlmineit] *v.i.* & *v.t.* 到顶点;结束
exhaustion [ig'zɔ:stʃən] *n.* 用尽;穷举
Eudoxus [ju:'dɔksəs] 尤多克色斯(人名)
Archimedes [ˌɑ:ki'mi:di:z] 阿基米德(人名)
crude [kru:d] *a.* 天然的;浅薄的
fit [fit] *v.t.* 使适合
inside ['in'said] *prep.* 在……内部
non-negative ['nɔn'negətiv] *a.* 非负的
invert [in'və:t] *v.t.* 翻过来
differentiation [ˌdifərenʃi'eiʃən] *n.* 区别,微分法
Leibnitz ['laibnits] 莱布尼兹(人名)
operator ['ɔpəreitə] *n.* 算子;运算数
obey [ə'bei] *v.t.* & *v.i.* 遵守

differentiable function [ˌdifəˈrenʃiəbl ˈfʌŋkʃən] 可微函数

ignore [igˈnɔː] *v.t.* 不顾

limitation [ˌlimiˈteiʃən] *n.* 限制，限定

offer [ˈɔfə] *v.t.* 提供

mean value 平均数，平均值

distributivity [disˌtribjuˈtiviti] *n.* 分配法

namely [ˈneimli] *ad.* 换句话说

otherwise [ˈʌðəwaiz] *ad.* 否则；在其它方面

Stieltjes （人名）

Burkill （人名）

Darboux （人名）

modification [ˌmɔdifiˈkeiʃən] *n.* 变更；限制

supremum [ˈsjuːpriməm] *n.* 最小上限，上确界

upper bound [ˈʌpə baund] 上限

infimum [ˈinfiːməm] *n.* 最大下限，下确界

lower bound [ˈləuə baund] 下限

rectangle [ˈrekˌtæŋgl] *n.* 矩形、长方形

height [hait] *n.* 高度，高

arbitrarily [ˈɑːbitrərili] *ad.* 任意

alter [ˈɔːltə] *v.t.* 改变

unbounded [ˈʌnˈbaundid] *a.* 无限制的，无边的

neighbourhood [ˈneibəhud] *n.* 邻近

词　组

(to) culminate in 结果竟成

in some sense 在某种意义上

near to 靠近

注　释

① 本句中 the adding ... elementary figures 以及 the taking ... a limit 为两个动名词短语，在句中作两个并列表语。

② 本句中 that derivative 为 make 的宾语。 correspond 为宾语补足语，因在 make 的后面，所以不定式符号“to”省略。to each of a certain class of functions *f* 为状语，to 是 correspond 所要求的介词，其余为介词 to 的宾语。

③ nor 为连接词，意为“……也不(没有)……”，常和 not 或 neither 连用。本句中与 not 连用。

　　与 neither 连用的例子如：He can neither read nor write. 他既不会读，也不会写。