



LUIS CASTAÑER

UNDERSTANDING MEMS

PRINCIPLES AND APPLICATIONS



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WILEY

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To my wife Pamen, daughters Maya and Olga, grandsons Teo, Raul and
Marc-Eric and granddaughter Claudia

Preface

The field of microelectromechanical systems (MEMS) has greatly expanded in recent years since Richard Feynman's speech – 'there is plenty of room at the bottom' – opened the minds of researchers and companies to the possibility of exploring the potential of microstructures of minute dimensions.

The need for skilled professionals has steadily grown as businesses and research labs engage in challenging projects. Students are motivated because they are aware that they have, in their phones, watches or tablets, accelerometers, compasses and sensors and useful applications relying on them. Students see in the MEMS field, job opportunities and exciting career prospects.

Ultimately those devices have to work together with front-end electronics and digital processing to interface with the user. Students in electrical engineering and computer science departments in universities around the world are probably the most exposed to the field. Simultaneously, engineers and professionals working in the electronics and semiconductor industry face the challenge of integrating MEMS devices into chips, systems on chips or, broadly speaking, into electronic systems. The added value of those devices enables the expansion of high tech businesses.

In my experience of teaching MEMS in an electronic engineering department to engineering students from many countries I have faced the difficulty of selecting material for one-semester course and having to decide on the depth and breadth of the subjects covered.

The field is inherently multidisciplinary, and if the basics are not sufficiently covered, students will not achieve the intellectual satisfaction of a full understanding. However, if the coverage is too complex it cannot be extended to the various fundamental domains underlying the field. Solving problems is an important part of the learning process as it allows for concepts to be reviewed.

Those are the reasons why I have chosen to approach the subject using analytical solutions as far as possible, but with the help of two software tools: one very popular among science and engineering students, Matlab; and the other, very popular among electrical engineering students, PSpice. I have used Matlab to solve ordinary differential equations subject to boundary or initial conditions, applied to mechanical, electromechanical, electrokinetic and thermal problems. This allows numerical results to be found quickly which can then be discussed and put into context.

I have used PSpice to solve Laplace transforms of transfer functions and to solve electrical equivalent circuits of lumped thermal problems. Apart from the clarity of analytical solutions, this approach places the subject of MEMS in the same tool environment as other subjects the

student will already have taken. Commonality of tools is important at this level of the learning process, because it means that students do not have to spend a significant amount of time learning how to use new software.

The book includes 52 worked examples in the text and 100 solved problems in the appendices, organized by chapters. In my view this allows this textbook to be used not only as support material for a conventional course but also as a self-study resource for distance learning. I am very grateful to faculty colleagues, researchers and students with whom I have interacted all these years that have taken me to complete this book.

Luis Castañer
April 2015
Barcelona, Spain

Contents

Preface	xiii
About the Companion Website	xv
1 Scaling of Forces	1
1.1 Scaling of Forces Model	1
1.2 Weight	2
1.2.1 <i>Example: MEMS Accelerometer</i>	2
1.3 Elastic Force	3
1.3.1 <i>Example: AFM Cantilever</i>	4
1.4 Electrostatic Force	4
1.4.1 <i>Example: MEMS RF Switch</i>	6
1.5 Capillary Force	6
1.5.1 <i>Example: Wet Etching Force</i>	8
1.6 Piezoelectric Force	8
1.6.1 <i>Example: Force in Film Embossing</i>	9
1.7 Magnetic Force	10
1.7.1 <i>Example: Compass Magnetometer</i>	10
1.8 Dielectrophoretic Force	11
1.8.1 <i>Example: Nanoparticle in a Spherical Symmetry Electric Field</i>	12
1.9 Summary	13
Problems	13
2 Elasticity	15
2.1 Stress	15
2.2 Strain	18
2.3 Stress–strain Relationship	20
2.3.1 <i>Example: Plane Stress</i>	21
2.4 Strain–stress Relationship in Anisotropic Materials	22
2.5 Miller Indices	23
2.5.1 <i>Example: Miller Indices of Typical Planes</i>	24
2.6 Angles of Crystallographic Planes	25
2.6.1 <i>Example</i>	25
2.7 Compliance and Stiffness Matrices for Single-Crystal Silicon	26
2.7.1 <i>Example: Young’s Modulus and Poisson Ratio for (100) Silicon</i>	27

2.8	Orthogonal Transformation	29
2.9	Transformation of the Stress State	31
2.9.1	Example: Rotation of the Stress State	31
2.9.2	Example: Matrix Notation for the Rotation of the Stress State	32
2.10	Orthogonal Transformation of the Stiffness Matrix	32
2.10.1	Example: C_{11} Coefficient in Rotated Axes	33
2.10.2	Example: Young's Modulus and Poisson Ratio in the (111) Direction	34
2.11	Elastic Properties of Selected MEMS Materials	36
	Problems	36
3	Bending of Microstructures	37
3.1	Static Equilibrium	37
3.2	Free Body Diagram	38
3.3	Neutral Plane and Curvature	39
3.4	Pure Bending	40
3.4.1	Example: Neutral Plane for a Rectangular Cross-section	41
3.4.2	Example: Cantilever with Point Force at the Tip	42
3.5	Moment of Inertia and Bending Moment	43
3.5.1	Example: Moment of Inertia of a Rectangular Cross-section	43
3.6	Beam Equation	44
3.7	End-loaded Cantilever	45
3.8	Equivalent Stiffness	47
3.9	Beam Equation for Point Load and Distributed Load	48
3.10	Castigliano's Second Theorem	48
3.10.1	Strain Energy in an Elastic Body Subject to Pure Bending	50
3.11	Flexures	51
3.11.1	Fixed-fixed Flexure	51
3.11.2	Example: Comparison of Stiffness Constants	53
3.11.3	Example: Folded Flexure	53
3.12	Rectangular Membrane	54
3.13	Simplified Model for a Rectangular Membrane Under Pressure	55
3.13.1	Example: Thin Membrane Subject to Pressure	57
3.14	Edge-clamped Circular Membrane	58
	Problems	60
4	Piezoresistance and Piezoelectricity	65
4.1	Electrical Resistance	65
4.1.1	Example: Resistance Value	66
4.2	One-dimensional Piezoresistance Model	67
4.2.1	Example: Gauge Factors	68
4.3	Piezoresistance in Anisotropic Materials	69
4.4	Orthogonal Transformation of Ohm's Law	70
4.5	Piezoresistance Coefficients Transformation	71
4.5.1	Example: Calculation of Rotated Piezoresistive Components π'_{11} , π'_{12} and π'_{16} for unit axes X' [110], Y' [$\bar{1}$ 10] and Z' [001]	72
4.5.2	Analytical Expressions for Some Rotated Piezoresistive Components	74

4.6	Two-dimensional Piezoresistors	74
4.6.1	<i>Example: Accelerometer with Cantilever and Piezoresistive Sensing</i>	76
4.7	Pressure Sensing with Rectangular Membranes	79
4.7.1	<i>Example: Single-resistor Pressure Sensor</i>	82
4.7.2	<i>Example: Pressure Sensors Comparison</i>	85
4.8	Piezoelectricity	86
4.8.1	<i>Relevant Data for Some Piezoelectric Materials</i>	88
4.8.2	<i>Example: Piezoelectric Generator</i>	89
	Problems	91
5	Electrostatic Driving and Sensing	93
5.1	Energy and Co-energy	93
5.2	Voltage Drive	97
5.3	Pull-in Voltage	97
5.3.1	<i>Example: Forces in a Parallel-plate Actuator</i>	99
5.4	Electrostatic Pressure	101
5.5	Contact Resistance in Parallel-plate Switches	101
5.6	Hold-down Voltage	101
5.6.1	<i>Example: Calculation of Hold-down Voltage</i>	102
5.7	Dynamic Response of Pull-in-based Actuators	102
5.7.1	<i>Example: Switching Transient</i>	103
5.8	Charge Drive	105
5.9	Extending the Stable Range	105
5.10	Lateral Electrostatic Force	106
5.11	Comb Actuators	106
5.12	Capacitive Accelerometer	108
5.13	Differential Capacitive Sensing	108
5.14	Torsional Actuator	110
	Problems	111
6	Resonators	115
6.1	Free Vibration: Lumped-element Model	115
6.2	Damped Vibration	116
6.3	Forced Vibration	117
6.3.1	<i>Example: Vibration Amplitude as a Function of the Damping Factor</i>	120
6.4	Small Signal Equivalent Circuit of Resonators	121
6.4.1	<i>Example: Series and Parallel Resonances</i>	125
6.4.2	<i>Example: Spring Softening</i>	125
6.5	Rayleigh–Ritz Method	126
6.5.1	<i>Example: Vibration of a Cantilever</i>	128
6.5.2	<i>Example: Gravimetric Chemical Sensor</i>	129
6.6	Resonant Gyroscope	130
6.7	Tuning Fork Gyroscope	133
6.7.1	<i>Example: Calculation of Sensitivity in a Tuning Fork Gyroscope</i>	134
	Problems	135

7	Microfluidics and Electrokinetics	137
7.1	Viscous Flow	137
7.2	Flow in a Cylindrical Pipe	140
7.2.1	<i>Example: Pressure Gradient Required to Sustain a Flow</i>	141
7.3	Electrical Double Layer	142
7.3.1	<i>Example: Debye Length and Surface Charge</i>	144
7.4	Electro-osmotic Flow	144
7.5	Electrowetting	146
7.5.1	<i>Example: Droplet Change by Electrowetting</i>	148
7.5.2	<i>Example: Full Substrate Contacts</i>	149
7.6	Electrowetting Dynamics	151
7.6.1	<i>Example: Contact-angle Dynamics</i>	153
7.7	Dielectrophoresis	153
7.7.1	<i>Electric Potential Created by a Constant Electric Field</i>	154
7.7.2	<i>Potential Created by an Electrical Dipole</i>	155
7.7.3	<i>Superposition</i>	156
	Problems	157
8	Thermal Devices	159
8.1	Steady-state Heat Equation	159
8.2	Thermal Resistance	161
8.2.1	<i>Example: Temperature Profile in a Heated Wire</i>	162
8.2.2	<i>Example: Resistor Suspended in a Bridge</i>	165
8.3	Platinum Resistors	166
8.4	Flow Measurement Based on Thermal Sensors	166
8.4.1	<i>Example: Micromachined Flow Sensor</i>	169
8.5	Dynamic Thermal Equivalent Circuit	171
8.6	Thermally Actuated Bimorph	172
8.6.1	<i>Example: Bimorph Actuator</i>	174
8.7	Thermocouples and Thermopiles	175
8.7.1	<i>Example: IR Detector</i>	175
	Problems	176
9	Fabrication	181
9.1	Introduction	181
9.2	Photolithography	182
9.3	Patterning	183
9.4	Lift-off	184
9.5	Bulk Micromachining	184
9.5.1	<i>Example: Angle of Walls in Silicon (100) Etching</i>	185
9.6	Silicon Etch Stop When Using Alkaline Solutions	186
9.6.1	<i>Example: Boron drive-in at 1050°C</i>	186
9.7	Surface Micromachining	186
9.7.1	<i>Example: Cantilever Fabrication by Surface Micromachining</i>	187
9.8	Dry Etching	188

9.9	CMOS-compatible MEMS Processing	188
9.9.1	<i>Example: Bimorph Actuator Compatible with CMOS Process</i>	189
9.10	Wafer Bonding	190
9.11	PolyMUMPs Foundry Process	190
9.11.1	<i>Example: PolyMUMPs Cantilever for a Fabry–Perot Pressure Sensor</i>	191
	Problems	192
APPENDICES		195
A	Chapter 1 Solutions	197
B	Chapter 2 Solutions	207
C	Chapter 3 Solutions	221
D	Chapter 4 Solutions	239
E	Chapter 5 Solutions	249
F	Chapter 6 Solutions	267
G	Chapter 7 Solutions	277
H	Chapter 8 Solutions	285
I	Chapter 9 Solutions	299
References		307
Index		311

1

Scaling of Forces

There are a number of important forces in the field of microelectromechanical systems (MEMS). However, their relative importance does not necessarily match the importance they have in the macroworld. This chapter is concerned with the scaling of these forces to small dimensions. Weight, elastic, electrostatic, capillary, piezoelectric, magnetic and dielectrophoretic forces are examined and a scaling factor identified for all of them.

1.1 Scaling of Forces Model

The integration of complex and powerful systems in silicon for a large variety of applications stems from the miniaturization of electronic devices and components. Electromechanical components that were bulky, heavy and inefficient can now be miniaturized using MEMS technology. Here, mechanical moving parts are used both for sensing devices and actuators. The main forces present in the operation of these components depend on the geometrical dimensions, and thereby, when the dimensions are scaled down, the magnitudes of these forces change, creating a different scenario compared to the macroworld.

Given a force F that depends on a number of geometrical dimensions a_i and on a number of parameters γ_j , we have

$$F = F(a_i, \gamma_j). \quad (1.1)$$

When all dimensions are scaled by the same factor α , the force changes to

$$F_\alpha = F(\alpha a_i, \gamma_j), \quad (1.2)$$

provided all the parameters γ_j do not depend on the geometrical dimensions. The ratio of the forces before and after the dimension scaling is given by

$$\frac{F_\alpha}{F} = \frac{F(\alpha a_i, \gamma_j)}{F(a_i, \gamma_j)}. \quad (1.3)$$

Generally, when analytical models are used in simplified cases, the result of equation (1.3) provides a direct relation to a power n of the scaling factor α ,

$$\frac{F_\alpha}{F} = \alpha^n, \quad (1.4)$$

meaning that when the dimensions are scaled down by a factor α , the force scales down by a factor α^n .

1.2 Weight

As our first application of the rule provided by equation (1.3) we consider the scaling of weight. Imagine that we have a body of length L , width W and thickness t . The weight of this body is given by

$$F = \rho_m g L W t, \quad (1.5)$$

where ρ_m is the material density and g the acceleration due to gravity. If all dimensions are scaled by a factor α , the length becomes αL , the width becomes αW and the thickness becomes αt , and so the scaled weight is

$$F_\alpha = \rho_m g \alpha L \alpha W \alpha t = \alpha^3 \rho_m g L W t, \quad (1.6)$$

and the ratio of forces after and before scaling is given by

$$\frac{F_\alpha}{F} = \alpha^3. \quad (1.7)$$

Equation (1.7) tells us that the weight scales down as the third power of the scaling factor, so if we reduce all dimensions by a factor of 10 ($\alpha = 0.1$), the weight is multiplied by a factor of $\alpha^3 = 0.001$.

It will become clear in the next sections that when electromechanical structures are miniaturized, the weight loses the importance it has in the macroworld and other forces become the main players.

1.2.1 Example: MEMS Accelerometer

A MEMS accelerometer has an inertial mass made up of a plate of silicon bulk material of $500\text{ }\mu\text{m}$ side and $500\text{ }\mu\text{m}$ thickness. Calculate the force developed when subject to an acceleration ten times that due to gravity.

Taking into account that the density of silicon is 2329 kg/m^3 and that the volume is $500 \times 500 \times 500 \times 10^{-18} \text{ m}^3$, the force is given by

$$F = 2.85 \times 10^{-5} \text{ N}.$$

If all dimensions are reduced by a factor of 10 ($\alpha = 0.1$) the weight reduces to $F_\alpha = 2.85 \times 10^{-8} \text{ N}$.

1.3 Elastic Force

A body is deformed when it is subject to an external force. In equilibrium, the elastic force is the restoring force that compensates the external force. If the deformation is elastic, the initial dimensions of the body are recovered after the external force disappears. In a one-dimensional geometry and according to Hooke's law [1], the elastic force, F , is proportional to the deformation length δ , collinear with the force,

$$F = k\delta, \quad (1.8)$$

where k is the stiffness constant that is not independent of the geometry as will be shown in Chapter 3; for example, for a cantilever of rectangular cross-section with length L , width W and thickness t , subject to a force applied at the tip (see Figure 1.1), the stiffness is given by

$$k = \frac{EWt^3}{4L^3}, \quad (1.9)$$

where E is Young's elasticity modulus. We now proceed as in Section 1.1 and calculate the forces F and F_α before and after scaling:

$$F = \frac{EWt^3}{4L^3} \delta, \quad F_\alpha = \frac{E\alpha W\alpha^3 t^3}{4\alpha^3 L^3} \alpha \delta = \alpha^2 \frac{EWt^3}{4L^3} \delta. \quad (1.10)$$

The ratio between these two quantities is therefore

$$\frac{F_\alpha}{F} = \alpha^2. \quad (1.11)$$

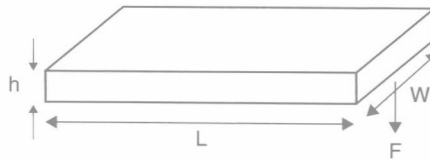


Figure 1.1 Geometry of a cantilever loaded at the tip

1.3.1 Example: AFM Cantilever

In atomic force microscopy tiny cantilevers with a very sharp tip are used to detect the force. The cantilever acts as a soft spring. Calculate the force that will deflect the cantilever by $1\text{ }\mu\text{m}$ for $L = 200\text{ }\mu\text{m}$, $W = 5\text{ }\mu\text{m}$ and $h = 2\text{ }\mu\text{m}$.

By equation (1.9), $k = 0.081\text{ N/m}$, and by equation (1.8),

$$F = 8.1 \times 10^{-8}\text{ N}.$$

Applying a dimension scaling with $\alpha = 0.1$, the force reduces to $F_\alpha = 8.1 \times 10^{-10}\text{ N}$.

1.4 Electrostatic Force

The electrostatic force between two plates is due to the electric field, \mathbf{E} , that builds up when an electric potential V is applied between them.¹ This is a very common way to make mechanical parts move in today's microelectromechanical devices.

If we consider one of the two plates charged with a charge density σ , as shown in Figure 1.2(a), Gauss's law [2] allows to calculate the electric field created by the charged sheet as

$$\oint \vec{D} d\vec{S} = \int \sigma dS = Q. \quad (1.12)$$

Signs in equation (1.12) are taken as positive for an electric field directed outward from the differential volume, and \vec{dS} is taken positive also directed outward from the face. As the electric field is normal to the charged surface, only integrals extending over the top and bottom surfaces of the volume are different from zero, so that

$$\int_{\text{top}} \epsilon \mathbf{E} dS + \int_{\text{bottom}} \epsilon \mathbf{E} dS = Q, \quad (1.13)$$

$$\epsilon \mathbf{E} A + \epsilon \mathbf{E} A = Q, \quad (1.14)$$

where A is the area of the surface. Then

$$\mathbf{E} = \frac{Q}{2\epsilon A}. \quad (1.15)$$

The Coulomb force that such a field exerts on the parallel plate with a charge of $-Q$ and at a distance g is

$$F = -QE = -\frac{Q^2}{2\epsilon A}. \quad (1.16)$$

¹ We denote the electric field by \mathbf{E} to distinguish it from the Young's modulus E .

1.2.1 Example: MEMS Accelerometer

A MEMS accelerometer has an inertial mass made up of a plate of silicon bulk material

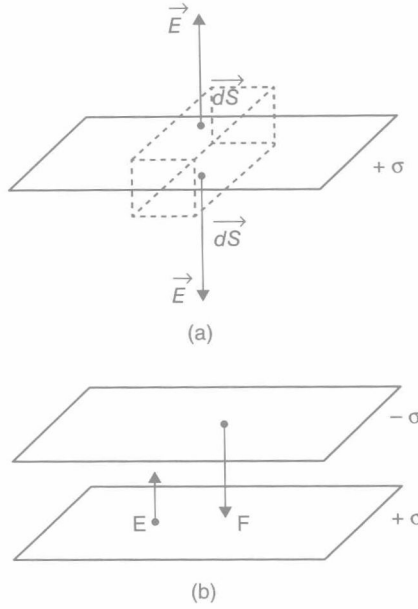


Figure 1.2 (a) Gauss's law for a sheet of charge σ , and (b) electric field and Coulomb force exerted on the upper plate

Since $Q = CV$ and $C = \epsilon A/g$,

$$F = \frac{\epsilon A V^2}{2g^2}. \quad (1.17)$$

As can be seen the force is downwards, that is to say, it is attractive between the plates and does not depend on the sign of the applied voltage as it is squared in the force equation (1.17). When we apply the scaling method we find that

$$F_\alpha = \frac{\epsilon A \alpha^2 V^2}{2g^2 \alpha^2}. \quad (1.18)$$

In equation (1.18), A is the area of the plates which scales as α^2 , and g is the value of the gap between plates. The scaling factor of the force is

$$\frac{F_\alpha}{F} = \alpha^0 = 1. \quad (1.19)$$

This is a very important result showing that the electrostatic force is independent of the scaling factor and can be very high compared to other forces in the microworld. However, it can be correctly argued that reducing the distance between plates increases the electric field and the devices may be damaged by breakdown. To prevent this situation, we can consider a different

scaling scenario in which the value of the electric field is kept constant. As the electric field is $E = V/g$, equation (1.18) can be written as

$$F = \frac{\epsilon AV^2}{2g^2} = \frac{\epsilon AE^2 g^2}{2g^2} = \frac{\epsilon AE^2}{2}, \quad F_\alpha = \frac{\epsilon A\alpha^2 E^2}{2} \quad (1.20)$$

and hence

$$\frac{F_\alpha}{F} = \alpha^2. \quad (1.21)$$

Here we see that in this scenario the scaling follows an α^2 rule instead.

1.4.1 Example: MEMS RF Switch

In a MEMS RF switch two metal plates $250 \times 250 \mu\text{m}^2$ are driven by a voltage of 9 V. Calculate the force required to close the $5 \mu\text{m}$ gap between them.

If we suppose that between the two plates there is air, the permittivity is $\epsilon = 8.85 \times 10^{-12}$ F/m, and the force can be calculated from equation (1.18):

$$F = \frac{\epsilon AV^2}{2g^2} = 8.85 \times 10^{-12} \frac{250 \times 10^{-6} \times 250 \times 10^{-6} \times 9^2}{2 \times 5^2 \times 10^{-12}} = 8.96 \times 10^{-7} \text{ N}.$$

If the dimensions are scaled by a factor of $\alpha = 0.1$ the force remains equal if equation (1.19) applies or 8.96×10^{-9} N if equation (1.21) applies.

1.5 Capillary Force

On the surface of a liquid the molecules are attracted by the other molecules inside the volume but do not have the attraction from the surroundings above the surface. This creates a situation where the molecules rearrange in order to expose the minimum surface. If an observer wants to increase the surface exposed to the ambient, he necessarily has to do some work. This work, dW , is proportional to the increase in area, dA [3]:

$$dW = \gamma dA. \quad (1.22)$$

The proportionality constant γ is the surface tension and has units of J/m^2 or, equivalently, N/m . Thus the surface tension is a measure of the surface energy per unit area.

When a liquid drop is in equilibrium, there is a pressure increase ΔP inside the drop, known as Laplace pressure, to prevent collapse. ΔP is related to the surface tension by

$$\Delta P = \gamma C, \quad (1.23)$$