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Quantum Chromodynamics on the Lattice: An Introductory Presentation

格点量子色动力学导论

(影印版)

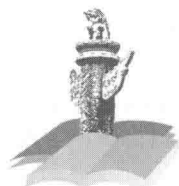
〔奥地利〕加特林格 (C. Gatttringer)

著

〔奥地利〕朗 (C. B. Lang)



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By Christof Gattringer and Christian B. Lang

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序 言

物理学是研究物质、能量以及它们之间相互作用的科学。她不仅是化学、生命、材料、信息、能源和环境等相关学科的基础,同时还是许多新兴学科和交叉学科的前沿。在科技发展日新月异和国际竞争日趋激烈的今天,物理学不仅囿于基础科学和技术应用研究的范畴,而且在社会发展与人类进步的历史进程中发挥着越来越关键的作用。

我们欣喜地看到,改革开放三十多年来,随着中国政治、经济、教育、文化等领域各项事业的持续稳定发展,我国物理学取得了跨越式的进步,做出了很多为世界瞩目的研究成果。今日的中国物理正在经历一个历史上少有的黄金时代。

在我国物理学科快速发展的背景下,近年来物理学相关书籍也呈现百花齐放的良好态势,在知识传承、学术交流、人才培养等方面发挥着无可替代的作用。从另一方面看,尽管国内各出版社相继推出了一些质量很高的物理教材和图书,但系统总结物理学各门类知识和发展,深入浅出地介绍其与现代科学技术之间的渊源,并针对不同层次的读者提供有价值的教材和研究参考,仍是我国科学传播与出版界面临的一个极富挑战性的课题。

为有力推动我国物理学研究、加快相关学科的建设与发展,特别是展现近年来中国物理学家的研究水平和成果,北京大学出版社在国家出版基金的支持下推出了“中外物理学精品书系”,试图对以上难题进行大胆的尝试和探索。该书系编委会集结了数十位来自内地和香港顶尖高校及科研院所的知名专家学者。他们都是目前该领域十分活跃的专家,确保了整套丛书的权威性和前瞻性。

这套书系内容丰富,涵盖面广,可读性强,其中既有对我国传统物理学发展的梳理和总结,也有对正在蓬勃发展的物理学前沿的全面展示;既引进和介绍了世界物理学研究的发展动态,也面向国际主流领域传播中国物理的优秀专著。可以说,“中外物理学精品书系”力图完整呈现近现代世界和中国物理

科学发展的全貌,是一部目前国内为数不多的兼具学术价值和阅读乐趣的经典物理丛书。

“中外物理学精品书系”另一个突出特点是,在把西方物理的精华要义“请进来”的同时,也将我国近现代物理的优秀成果“送出去”。物理学科在世界范围内的重要性不言而喻,引进和翻译世界物理的经典著作和前沿动态,可以满足当前国内物理教学和科研工作的迫切需求。另一方面,改革开放几十年来,我国的物理学研究取得了长足发展,一大批具有较高学术价值的著作相继问世。这套丛书首次将一些中国物理学者的优秀论著以英文版的形式直接推向国际相关研究的主流领域,使世界对中国物理学的过去和现状有更多的深入了解,不仅充分展示出中国物理学研究和积累的“硬实力”,也向世界主动传播我国科技文化领域不断创新的“软实力”,对全面提升中国科学、教育和文化领域的国际形象起到重要的促进作用。

值得一提的是,“中外物理学精品书系”还对中国近现代物理学科的经典著作进行了全面收录。20世纪以来,中国物理界诞生了很多经典作品,但当时大都分散出版,如今很多代表性的作品已经淹没在浩瀚的图书海洋中,读者们对这些论著也都是“只闻其声,未见其真”。该书系的编者们在这方面下了很大工夫,对中国物理学科不同时期、不同分支的经典著作进行了系统的整理和收录。这项工作具有非常重要的学术意义和社会价值,不仅可以很好地保护和传承我国物理学的经典文献,充分发挥其应有的传世育人的作用,更能使广大物理学人和青年学子亲身体会我国物理学研究的发展脉络和优良传统,真正领悟到老一辈科学家严谨求实、追求卓越、博大精深的治学之美。

温家宝总理在2006年中国科学技术大会上指出,“加强基础研究是提升国家创新能力、积累智力资本的重要途径,是我国跻身世界科技强国的必要条件”。中国的发展在于创新,而基础研究正是一切创新的根本和源泉。我相信,这套“中外物理学精品书系”的出版,不仅可以使所有热爱和研究物理学的人们从中获取思维的启迪、智力的挑战和阅读的乐趣,也将进一步推动其他相关基础科学更好更快地发展,为我国今后的科技创新和社会进步做出应有的贡献。

“中外物理学精品书系”编委会 主任
中国科学院院士,北京大学教授

王恩哥

2010年5月于燕园

C. Gattringer
C.B. Lang

Quantum Chromodynamics on the Lattice

An Introductory Presentation

To our wives,
who endured
absent-minded husbands
working on weird texts.

Preface

Quantum chromodynamics (QCD) is the fundamental quantum field theory of quarks and gluons. In order to discuss it in a mathematically well-defined way, the theory has to be regularized. Replacing space-time by a Euclidean lattice has proven to be an efficient approach which allows for both theoretical understanding and computational analysis. Lattice QCD has become a standard tool in elementary particle physics.

As the title already says: this book is introductory! The text is intended for newcomers to the field, serving as a starting point. We simply wanted to have a book which we can put into the hands of an advanced student for a first reading on lattice QCD. This imaginary student brings as a prerequisite knowledge of higher quantum mechanics, some continuum quantum field theory, and basic facts of elementary particle physics phenomenology.

In view of the wealth of applications in current research the topics presented here are limited and we had to make some painful choices. We discuss QCD but omit most other lattice field theory applications like scalar theories, gauge-Higgs models, or electroweak theory. Although we try to lead the reader up to present day understanding, we cannot possibly address all ongoing activities, in particular concerning the role of QCD in electroweak theory. Subjects like glueballs, topological excitations, and approaches like chiral perturbation theory are mentioned only briefly. This allows us to cover the other topics quite explicitly, including detailed derivations of key equations. The field is rapidly developing. The proceedings of the annual lattice conferences provide information on newer directions and up-to-date results.

As usual, completing the book took longer than originally planned and we thank our editor Claus Ascheron for his patience. We are very grateful to many of our colleagues, who offered to read one or the other piece. In particular we want to thank Vladimir Braun, Dirk Brömmel, Tommy Burch, Stefano Capitani, Tom DeGrand, Stephan Dürr, Georg Engel, Christian Hagen, Leonid Glozman, Meinulf Göckeler, Peter Hasenfratz, Jochen Heitger, Verena Hermann, Edwin Laermann, Markus Limmer, Pushan Majumdar, Daniel Mohler, Wolfgang Ortner, Bernd-Jochen Schaefer, Stefan Schaefer,

VIII Preface

Andreas Schäfer, Erhard Seiler, Stefan Sint, Stefan Solbrig, and Pierre van Baal.

It would be surprising if there were not mistakes in this text. We therefore set up a web companion to this book: <http://physik.uni-graz.at/qcdlatt/> On that page we document errata and provide further links and information.

Graz,
March 2009

Christof Gattringer
Christian B. Lang

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