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Preface

Aims of This Book

1. To present calculus and elementary differential equations with a minimum of fuss—through practice, not theory.
2. To stress techniques, applications, and problem solving, rather than definitions, theorems, and proofs.
3. To emphasize numerical aspects such as approximations, order of magnitude, and concrete answers to problems.
4. To organize the topics consistent with the needs of students in their concurrent science and engineering courses.
5. To illustrate the usefulness of computers in applications of calculus.
6. To introduce vector methods and their applications in physical problems.

Why This Approach?

Calculus can be an exciting subject; no other gives so much new scope and power. Yet painful experience has shown that theory and rigor tend to stifle the excitement. The teaching of real variables to freshmen and sophomores has generally been a failure, a great disservice to students, and a source of well-deserved criticism from science and engineering departments.

Our presentation is informal; we reject the practice of writing calculus texts with the style and precision of research papers. Instead of formal definitions, theorems, and proofs, we include intuitive discussions, rules of procedure, and realistic problems. Occasionally we allow ourselves the liberties of circular arguments or slight inconsistencies when expedient. We omit technicalities that almost never occur in practice, rather than clutter the exposition "for the sake of completeness."

The thoughtful student who wants to know more theory will find in Chapter 36 a sketch of some theoretical high points and references to more detailed discussions.

We stress explicit computation, and when appropriate, indicate the value of computers in numerical work. However our book is not a computerized calculus; it is a calculus text that recognizes the increasing importance of computers in all branches of science. (The material on computer applications can be omitted without loss of continuity.)

Organization

This book presupposes reasonable skill in algebraic manipulation, familiarity with the trigonometric functions, and a bit of analytic geometry —graphs of functions and basic facts about straight lines and conic sections. However, some of these topics are reviewed briefly as needed.

The text is divided into three parts, corresponding more or less to a three-semester course. This division and the order of topics are merely guidelines, and can be modified if desired.

Part I presents an elementary introduction to most of the basic material of calculus: derivatives, direction fields, antiderivatives, integrals, volumes by slicing, partial derivatives, low order Taylor polynomials, exponential and trigonometric functions.

Part I is specifically designed for the typical student who needs the basic topics—but not in great depth—in his physics, chemistry, and engineering courses soon after he begins calculus. In particular he needs differentiation and integration of a few standard functions, the most elementary aspects of differential equations, and the concept of partial derivatives.

Part II includes a deepening of the material of Part I and several new topics: inverse functions, interpolation, numerical integration, first and second order differential equations, vectors, double integrals over rectangular regions.

The student should acquire in Parts I and II a working knowledge of the functions he will need in real life—their graphs, rates of growth, orders of magnitude, and interrelations.

Part III completes calculus with harder topics on approximations and several variables: Taylor series, approximate solutions of differential equations, complex functions, double and triple integrals with applications.

The final chapter is a brief introduction to theory.

Order of Topics

Considerable flexibility is possible in the order of topics, particularly after Chapter 19.

The material in Part I will move along briskly; there may be time (for those on a semester system) to include a few chapters from Part II. These can be inserted anywhere after Chapter 7. In Part I three sections are marked [optional].

There is even more leeway for rearrangement of topics in Parts II and III. After 19, several chapters can be omitted, permuted, or postponed. For example, Chapters 21 and 22 on differential equations can very well be left until much later. Chapters 30 (Approximate solutions of differential equations), 31 (Complex numbers), and 35 (Applications of multiple integrals) can be omitted entirely. Furthermore, Parts II and III include many optional sections. Chapter 36 (Calculus theory) may be studied at any time.

A few optional sections and a few exercises involve the use of matrices. These will provide meaningful applications for a previous or concurrent course in linear algebra, but they are not an integral part of the text.

Examples and Exercises

The worked examples are the core of the text. Many times, methods are explained more through choice examples than elaborate discussions. We are always result-oriented and insist on explicit numerical answers.

A number of topics are included as much because they are a source of meaningful, non-contrived problems as they are of interest in themselves.

Physical examples occur in some sections and in many exercises. The science or engineering student is usually impressed by this material, and motivated to learn it.

Students have difficulty with calculus problems for several reasons: (a) inability to perform lengthy algebraic and numerical calculations, (b) too many steps involved (the beginning student often does not know where to start a problem involving several steps), and (c) lack of space perception.

The exercises in Part I are easy and involve few steps. Each exercise set in Parts II and III begins with easy exercises but continues to harder ones, the level of difficulty increasing as the book progresses, so that by Part III the student is solving substantial calculus problems. Altogether, about thirty-one hundred exercises are included.

Those exercise sets containing problems for the computer also contain parallel problems for hand computation.

We recommend use of a slide rule and a book of tables (such as the C. R. C. Standard Mathematical Tables). We include some tables on pages 913–926, and basic formulas in the inside covers.

Illustrations

To solve space problems, the student must be able to make clear drawings. We have purposely restricted the illustrations in this book to simple line drawings, the kind the student can make himself (in fact, we include an optional section on how to do so). We emphasize accuracy in our drawings. For example, a tangent to a sphere which is parallel to the y-axis must actually appear so in a plane projection.

Acknowledgments

It remains to thank our typists, Allene Fritsch, Helen Sutton, Kathy Smith, and Elizabeth Young. Our reviewers, Peter Balise, Ettore Infante, Paul Mielke, and three unknown reviewers, contributed numerous valuable suggestions for which we are most grateful.

Mistakes in the book are entirely our fault and we shall appreciate corrections.

Lafayette, Indiana

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1. INTRODUCTION

The processes of nature are dynamic. Living matter grows; a planet moves in its orbit; a chemical reaction occurs at a certain rate; a rocket accelerates; a heavy object falls at increasing speed; a quantity of radioactive matter decays; a particle of fluid in a stream flows along its path with varying speed. Differential Calculus is the precise scientific theory that unifies the study of most situations in which there is dynamic change. It is indispensable for your further study of mathematics, physics, chemistry, engineering, modern biology, economics, virtually every exact science, both the theory and the application.

The main objects of study in Differential Calculus are functions. A function is a law which tells how one variable quantity is related to another. Given a particular function, Differential Calculus shows us the precise *rate of change* of the dependent variable relative to change in the independent variable. For example, if the dependent variable x is the distance a particle moves in time t, where t is the independent variable, then $x = x(t)$ is the law of motion of the particle; Calculus tells us the rate of change of distance per unit time, or instantaneous speed. As another example, the pressure P (dependent variable) of a gas confined in a cylinder of variable volume V (independent variable) varies according to the gas law

$$P = \frac{c}{V} \qquad (c \text{ a constant}).$$

From Calculus we learn the rate at which the pressure P changes relative to change in the volume V.

2. SLOPE

Differential Calculus deals with functions whose graphs are smooth curves. The graph of such a function $y = f(x)$ is usually drawn on rectangular graph paper (Fig. 2.1).