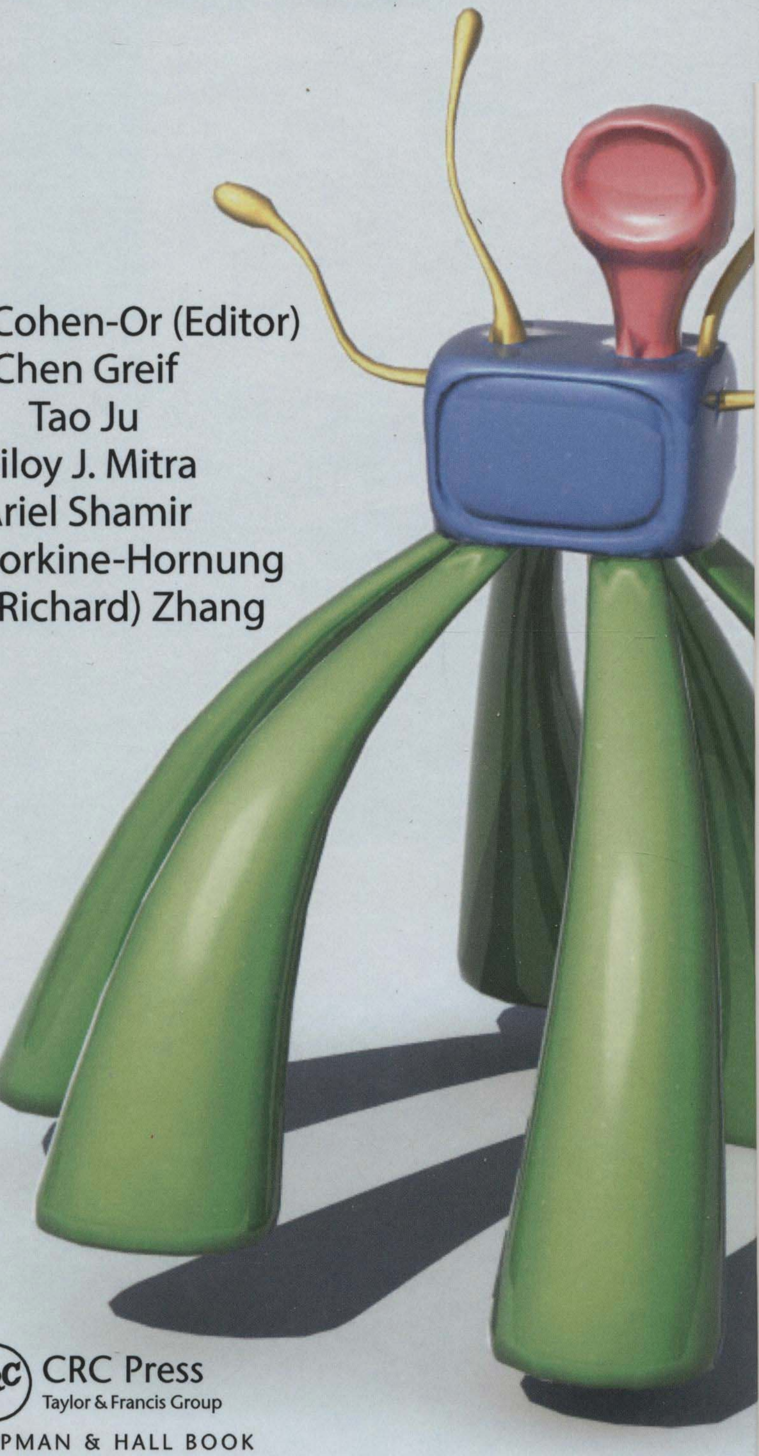
A Sampler of Useful …utational Tools for Applied Geometry, Computer Graphics, and Image Processing



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A Sampler of Useful Computational Tools for Applied Geometry, Computer Graphics, and Image Processing

About the Book

Many important topics in applied geometry cannot be solved efficiently without using mathematical tools. This book presents, in a rather light way, some mathematical tools that are useful in many domains, such as computer graphics, image processing, computer vision, digital geometry processing and geometry in general.

What the book includes

This book presents a collection of mathematical tools that correspond to a broad spectrum of applied mathematics and computer science. The book consists of thirteen chapters, each of which covers one topic. The topics are deliberately meant to be uncoupled and can be read or taught in any order. However, the first two chapters discuss more fundamental tools, and serve either to recall or to introduce essential elementary notions from analytical geometry and linear algebra. The remaining chapters cover a wide range of topics, from matrix decomposition to curvature analysis, principle component analysis to dimensionality reduction and more. The full list is presented below.

The presentation style

This book has a special presentation style that gives up excessive rigor and completeness, but strives to be useable, effective and well motivated. The mathematical tools are described and presented as solutions to specific applied problems such as image alignment, surface approximation, compression or image manipulation. The book is meant to be practical and handy, and we hope that reading it will be an enjoyable experience.

To whom the book will appeal

By avoiding detailed proofs and analysis, the book will appeal to those who wish to enrich their problem-solving arsenal. The book is ideal for people who do not have a very deep academic background in mathematics, and yet wish to use mathematics for work or research.

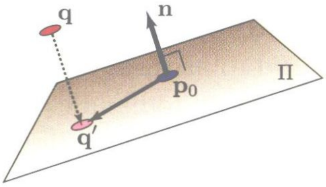
The book is structured for use as a one-semester, intermediate-level course in computer science, both in terms of its length and the required level of knowledge. In fact, the idea for this book was born out of a course that we have been teaching and improving over several years. Each chapter represents the material taught in a weekly class. The course's main objective was to quickly introduce the students to knowledge and tools useful in their advanced studies in a visual computing or applied geometry field. As such, the course primarily served faculty and senior researchers who wanted their students to acquire this arsenal. However, the book is also perfectly suited for individuals who wish to learn how to solve non-trivial geometric problems.

Is it a text book?

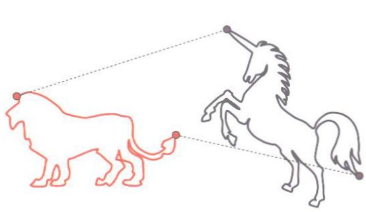
Not by standard definitions, but it can certainly accompany a course, as explained above. Each chapter can be taught within one session, and further reading material is indicated within the chapters, when appropriate.

Book Contents

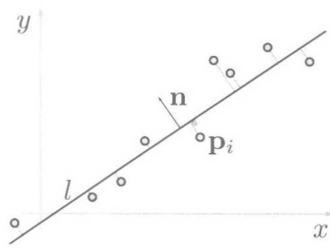
In the first chapter, we will familiarize ourselves with some basic geometric tools and see how we can put them to practical use to solve several geometric problems. Instead of describing the tools directly, we do it through an interesting discussion of two possible ways to approach the geometric problem at hand: we can employ our geometric intuition and use geometric reasoning, or we can directly formalize everything and employ our algebraic skills to write down and solve some equations. The discussion leads to a presentation of linear geometric elements (points, lines, planes), and the means to manipulate them in common geometric applications that we encounter, such as distances, transformations, projections and more.



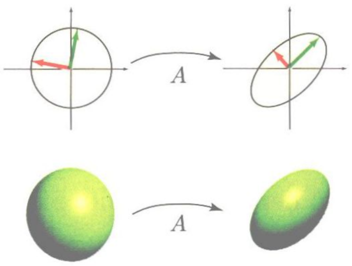
In this chapter, we will review basic linear algebra notions that we learned in a basic linear algebra course, including vector spaces, orthogonal bases, subspaces, eigenvalues and eigenvectors. However, our main goal here is to convince the readers that these notions are really useful. Furthermore, we will see the close relation between linear algebra and geometry. The chapter will be driven by an important tool called singular value decomposition (SVD), to which we will devote a separate full chapter. To understand what an SVD is, we first need to understand the notions of bases, eigenvectors, and eigenvalues and to refresh some fundamentals of linear algebra with examples in geometric context.



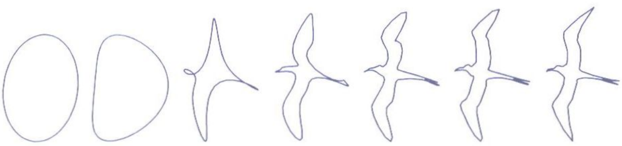
When dealing with real-world data, simple patterns can often be submerged in noise and outliers. In this chapter, we will learn about basic data fitting using the least-squares method, first starting with simple line fitting before moving on to fitting low-order polynomials. Beyond robustness to noise, we will also learn how to handle outliers and look at basic robust statics.



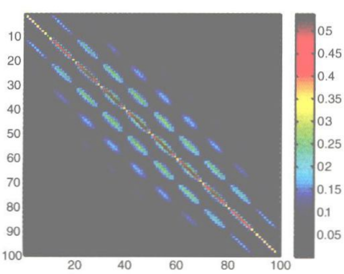
In this chapter, we introduce two related tools from linear algebra that have become true workhorses in countless areas of science: principal component analysis (PCA) and singular value decomposition (SVD). These tools are extremely useful in geometric modeling, computer vision, image processing, computer graphics, machine learning and many other applications. We will see how to decompose a matrix into several factors that are easy to analyze and reveal important properties of the matrix and hence the data, or the problem in which the matrix arises. As in the whole book, the presentation is rather light, emphasizing the main principles without excessive rigor.



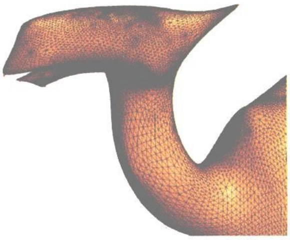
The use of signal transforms, such as the discrete Fourier or cosine transforms, is a classic topic in image and signal processing. In this chapter, we will learn how such transforms can be formulated and applied to the processing of 2D and 3D geometric shapes. The key concept to take away is the use of eigenvectors of discrete Laplacian operators as basis vectors to define spectral transforms for geometry. We will show how the Laplacian operators can be defined for 2D and 3D shapes, as well as a few applications of spectral transforms including geometry smoothing, enhancement and compression.



In the solution of problems discussed in this book, a frequent task that arises is the need to solve a linear system. Understanding the properties of the matrix associated with the linear system is critical for guaranteeing speed and accuracy of the solution procedure. In this chapter, we provide an overview of linear system solvers. We describe direct methods and iterative methods, and discuss important criteria for the selection of a solution method, such as sparsity and positive definiteness. Important notions such as pivoting and preconditioning are explained, and a recipe is provided that helps in determining which solver should be used.

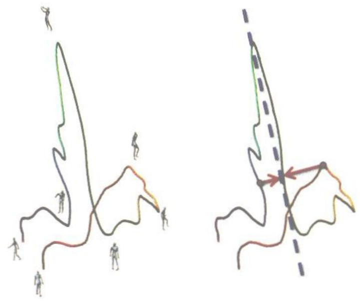


In this chapter, we make use of the well-known equations of Laplace and Poisson. The two equations have an extremely simple form, and they are very useful in many diverse branches of mathematical physics. However, in this chapter, we will interpret them in the context of image processing. We will show some interesting image editing and geometric problems and how they can be solved by simple means using these equations.

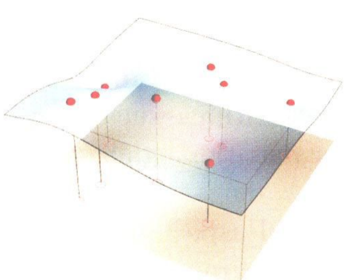


Local surface details, e.g., how "flat" a surface is locally, carry important information about the underlying object. Such information is critical for many applications in geometry processing, ranging from surface meshing, shape matching, surface reconstruction, scan alignment and detail-preserving deformation, to name only a few. In this chapter, we will cover the basics of differential geometry, particularly focusing on curvature estimates with some illustrative examples as an aid to geometry processing tasks.

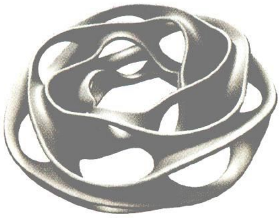
In this chapter, we will learn the concept, usefulness, and execution of dimensionality reduction. Generally speaking, we will seek to reduce the dimensionality of a given data set, mapping high-dimensional data into a lower-dimensional space to facilitate visualization, processing, or inference. We will present and discuss only a sample of dimensionality reduction techniques and illustrate them using visually intuitive examples, including face recognition, surface flattening and pose normalization of 3D shapes.



In this chapter, we visit the classical mathematical problem of obtaining a continuous function over a spatial domain from data at a few sample locations. The problem comes up in various geometric modeling scenarios, a good example of which is surface reconstruction. The chapter will eventually introduce the very useful radial basis functions (RBFs) as a smooth and efficient solution to the interpolation problem. However, to understand their usefulness, the chapter will go through a succession of methods with increasing sophistication, including piecewise linear interpolation and Shepherd's method.

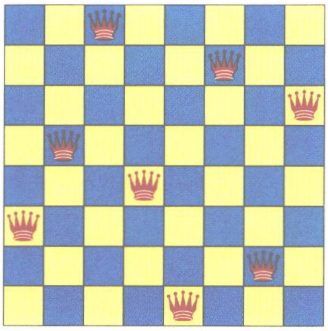


In Chapter 8, we learned about local differential analysis of surfaces. In this chapter, we focus on global aspects. We will learn about what is meant by orientable surfaces or manifold surfaces. Most importantly, we will learn about the Euler characteristic, which links local curvature properties to global connectivity constraints, and comes up in a surprising range of applications.



Graphs play an important role in many computer science fields and are also extensively used in imaging and graphics. This chapter concentrates on image processing and demonstrates how images can be represented by a graph. This allows translating problems of analysis and manipulation of images to well-known graph algorithms. Specifically, we will show how segmentation of images can be solved using region-growing algorithms such as watershed or partitioning algorithms using graph cuts. We will also show how intelligently changing the size and aspect ratio of images and video can be solved using dynamic programming or graph cuts.

In this chapter, we will show an example of the usefulness of number theory, or at least one of its known theorems. We will discuss mappings of numbers to a lattice, a problem that has practical applications in systems that require simultaneous, conflict-free access to elements distributed in different memory modules. Such mappings are also called *skewing schemes* since they skew the trivial mapping from element to memory. To understand these mappings, we will visit the notions of relatively prime numbers, and the greatest common divisor (gcd).



Chapter 1

Analytical Geometry

Olga Sorkine-Hornung and Daniel Cohen-Or

When dealing with geometric problems, we typically deal with 2D and 3D objects. These objects are represented mathematically in the computer. Therefore, the basic requirement is to have a good grasp of the mathematical entities that represent the objects, and be familiar with the tools that enable their manipulation. The basic geometric elements are linear (points, lines, planes), and the typical queries and manipulations we encounter in common geometric applications include distances, transformations, projections and so on.

All these manipulations require performing computations based on the mathematical representation of digital objects, by using recipes from analytical geometry and linear algebra. In this chapter, we will familiarize ourselves with some basic geometric tools and see how we can put them to practical use to solve several geometric problems. However, before we dive into the details, let us discuss how can we approach geometric problems in general.

Solving geometric problems

When we have a geometric problem at hand, there are two possible ways to look at it: we can employ our geometric intuition and use geometric reasoning first, or we can directly formalize everything and employ our algebraic skills to write down and solve some equations. We will often try to use geometric understanding first, which will possibly simplify the problem, and then use the algebra. Sometimes, when the problem is complex, it is hard to find intuitive geometric insights, and then we have to rely on algebraic tools to help us.

Let us look at our first example of a geometric problem and discuss the two possible ways to attack it.

Problem: *Given two straight lines in the 3D space, compute the distance between the two lines.*

Let us first try and think of the problem in pure geometric terms, without equations. The first observation is that the distance between two lines in 3D is the length of the shortest segment connecting them. This segment is perpendicular to both lines. We can look at the problem of finding that segment to try and simplify the distance problem. Indeed, imagine that we are looking at the installation of our two lines, so that the direction of our sight coincides with one of the lines, say, the first line. Then we will not see the first line at all, instead we will see just a single point! So the (hypothetical) image of the scene in our eye will be a line and a point, where the line is actually the projection image of the second line onto the plane that is perpendicular to the first line (see Figure 1.2). Thus, our problem becomes simpler: now we need to compute the distance between a point and a line.

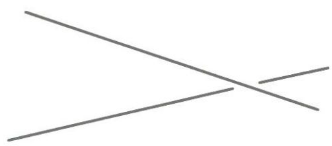


Figure 1.1: Two lines in 3D space, in general configuration.

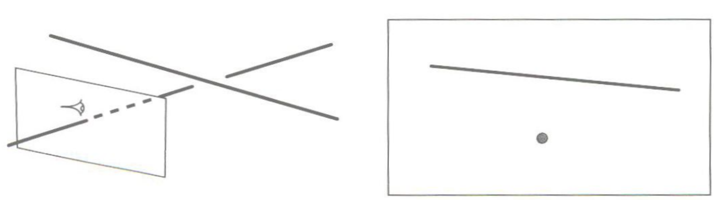


Figure 1.2: If we look at the scene in the direction of one line, that line becomes a single point.

We can simplify even further by using the same geometric argument: let us now imagine that we are looking at the result of the previous mental exercise (a point and a projected line), and our line of sight coincides with the line again (see Figure 1.3). As