KINEMATIC DIFFERENTIAL GEOMETRY AND SADDLE SYNTHESIS OF LINKAGES

DELUN WANG
WEI WANG

WILEY

KINEMATIC DIFFERENTIAL GEOMETRY AND SADDLE SYNTHESIS OF LINKAGES

Delun Wang and Wei Wang

Dalian University of Technology, China

WILEY

This edition first published 2015

Registered office
John Wiley & Sons Singapore Pte. Ltd., I Fusionopolis Walk, #07-01 Solaris South Tower, Singapore 138628.

For details of our global editorial offices, for customer services and for information about how to apply for permission to reuse the copyright material in this book please see our website at www.wiley.com.

Library of Congress Cataloging-in-Publication Data applied for.

ISBN: 9781118255049

Typeset in 10/12pt TimesLTStd by Laserwords Private Limited, Chennai, India

Printed and bound in Singapore by Markono Print Media Pte Ltd

1 2015

KINEMATIC DIFFERENTIAL GEOMETRY AND SADDLE SYNTHESIS OF LINKAGES

Preface

This book introduces the kinematic geometry of linkages in both analysis and synthesis, and builds up a theoretical system from planar, spherical, to spatial. The presentation differs from traditional ones in the approaches of both the differential geometry for kinematic geometry and the saddle point program for kinematic synthesis of linkages. Kinematic geometry provides the theoretical basis for the kinematic synthesis, both precise and approximated, of linkages by invariants.

The kinematic geometry of a rigid body, logically the combination of the kinematics of a rigid body and the geometry of graphs, tries to study the local geometrical properties of loci from the point of view of continuous motion along trajectories, while this continuous motion can certainly be visualized as the differential of the Frenet frame of the trajectories with respect to its arc length. Therefore, differential geometry, of course, may be the first choice in research on the kinematic geometry of a rigid body. However, the current research situation is unfortunately quite different, and this is one of the reasons for the authors writing this book.

There are currently many methods to study the kinematic geometry of a rigid body, such as geometry, algebra, screws, matrices, complex numbers, vectors, etc., each with their own merit in different application cases. In fact, this originates from the geometry by Burmester, which converts the displacement (or movement) of a lamina at several finite separated planar positions into a geometrical graph by means of corresponding poles of rotation. The algebraic equations are then built up to analyze the properties of geometrical graphs, which expand the object of research to all graphs of the lamina. For modern mathematics with expressions of vector algebra and invariants of geometrical graphs, it is difficult to identify them as belonging to the traditional geometry or algebra. For example, the differential geometry of curves and surfaces, both persists in geometrical significance and avoids the effects of external factors on geometrical graphs. In particular, a moving Frenet frame with three mutually orthogonal axes, or the natural trihedron of a curve or ruled surface, moving along the curve or surface is introduced to examine the intrinsic geometrical properties in differential geometry, whose derivatives can be viewed as the motion conversion for a rigid body at infinitesimally separated positions, just like the poles in finite separated positions, which is believed to be a powerful tool of kinematic geometry for a rigid body, in both planar and spatial motion. The kinematic geometry of a rigid body with multiple degrees of freedom is studied in multi-dimensional space. Of course, there is a natural extension from two or three dimensions to multiple dimensions while the classical differential geometry is developed into modern differential geometry, such as differential manifolds, Lie groups, and Lie algebras, although these are much more non-representational mathematical methods and the reader may have more difficulty understanding them.

The discrete kinematic geometry of a rigid body, naturally combining the discrete kinematics of a rigid body and the geometry of discrete graphs, studies the global geometrical properties of discrete trajectories, comprised of a series of discrete points or lines, which are globally compared with constraint curves or surfaces, while the differences between them or their errors have to be defined and estimated in terms of their invariants. Hence, the best uniform approximation in multi-dimensional space, or the saddle point programming approach, may be adopted first since it developed from one-dimensional space, or the interpolating approach of the Chebyshev polynomial originally, initially applied in the functional synthesis of linkages. The saddle point programming approach has been applied widely in geometrical error evaluations for manufacturing and measuring. However, the current objective function in the optimal synthesis of linkages for multiple positions, or the error evaluation method, is the least square structural error or the best square approximation, which intensively depends on the initial values and may be valid for the special cases but invalid for the general problems since the structural error is not uniformly defined and the design variables are redundant, other than the invariants in the approach of the saddle point program. This is another reasons for authors to write the book.

The book has seven chapters and two appendices in the order of planar, spherical, and spatial kinematic geometry of a rigid body and synthesis of linkages, so it is easy for readers to gain familiarity with the differential geometry and gradually build up the theoretical system. Also, for the reader's convenience, the required elemental knowledge of differential geometry is partly arranged in Chapter 1 for planar curves and Chapter 3 for space curves and surfaces. Chapters 1 and 2 describe the kinematic geometry and synthesis of planar linkages. Chapters 4 and 5 state the kinematic geometry and synthesis of spherical linkages, which is the bridge between the planar and spatial motion and a transition, even though it can be visualized as a special case of spatial motion. The kinematic geometry and synthesis of spatial linkages are respectively discussed in Chapters 6 and 7 in detail. In the appendices, the displacements of the spatial linkage RCCC are solved to provide the data for the numerical examples of kinematic geometry and synthesis of spatial linkages in the book.

Acknowledgments

Time goes so fast, and it has been over two decades since I first started research on the kinematic geometry of linkages as a PhD student under the guidance of Professor Dazhun Xiao and Professor Jian Liu (Dalian University of Technology). I can still remember the day when Professor Xiao told me honestly how arduous and challenging the study of the kinematic geometry of mechanisms was. I was deeply intrigued, and keen to discover the challenges and problems in the field of mechanisms. From that day on, I have been fully aware of the direction my research would take. Thanks to those unforgettable discussions with Professor Liu, I have gained many wonderful research ideas; he always supported and inspired me. I was also enlightened by many classical books, such as *Kinematic Geometry of Mechanisms* written by K.H. Hunt. This book would not have come into being if I had not been supported and encouraged by previous generations of scholars, both domestic and abroad, in the field of mechanisms, such as Professor Q.X. Zhang (Beihang University), Professor Y.L. Xiong (Huazhong University of Science and Technology), Professor H.M. Li (Harbin Institute of Technology), Professor S.X. Bai (Beijing University of Technology), Professor Z. Huang (Yanshan University), Professor T.L. Yang (Sinopec Jinling Petrochemical Co., Ltd), Professor H.J. Zou (Shanghai Jiao Tong University), Professor H.S. Yan (National Cheng Kung University), Professor C. Zhang (Tianjin University), Professor J.S. Dai (King's College London), Professor J.M. McCarthy (University of California, Irvine), Professor Kwun-Lon Ting (Tennessee Tech University), Professor J.Q. Ge (Stony Brook University), and so on. I should also mention the new generation of mechanism scholars in China, such as Professor T. Huang (Tianjin University), Professor F. Gao (Shanghai Jiao Tong University), Professor Z.Q. Deng (Harbin Institue of Technology), Professor Y.Q. Yu (Beijing University of Technology), Professor J. Xie (Southwest Jiaotong University), Professor X.L. Ding (Beihang University), Professor Y.H. Yang (Tianjin University), Professor S. Lin (Tongji University), Professor S.J. Li (Northeastern University), and Professor W.Z. Guo (Shanghai Jiao Tong University).

Many thanks go to my students and colleagues, because the book contains not just my own PhD dissertation but also those of three of my students – Mr. Wei Wang (co-author), Dr. Tao Li, and Dr. Shufen Wang – and eight Master's dissertations by Ms. Lihua Xiao, Mr. Jincang Zhou, Ms. Tianjian Li, Mr. Pengcheng Zheng, Mr. Baoying Zhang, Mr. Jianjun Zhang, Ms. Jie Chai, and Mr. Jinlei Li. Professor Huimin Dong, my classmate in graduate school, has been working with and helping me for 30 years. Professor Shudong Yu (Ryerson University), my classmate in college, Professor Yimin Tong, Professor Huili Wang, and Dr. Jin Qiu (Dalian University of Technology) have also given me enormous help in writing this book.

I want to express my gratitude to the National Natural Science Foundation of China (grants 59305033 and 59675003) for supporting my research into the kinematic geometry of mechanisms.

I would also like to thank the staff of John Wiley & Sons for their warm collaboration in presenting this book as accurately as possible in all its details.

Contents

1

Planar Kinematic Differential Geometry

Kinematics, a branch of dynamics, deals with displacements, velocities, accelerations, jerks, etc. of a system of bodies, without consideration of the forces that cause them, while kinematic geometry deals with displacements or changes in position of a particle, a lamina, or a rigid body without consideration of time and the way that the displacements are achieved. As a combination of kinematic geometry and differential geometry both in content and approach, kinematic differential geometry describes and studies the geometrical properties of displacements.

There are a number of articles and books on kinematic geometry. Pioneers such as Euler (1765), Savary (1830), Burmester (1876), Ball (1871), Bobillier (1880), and Müller (1892) established the theoretical foundation and developed the classical geometrical and algebraic approaches for studying kinematic geometry in two dimensions some hundred years ago. The classical geometric and algebraic approaches are still in use today. Differential geometry is favored by many researchers studying the geometrical properties of positions of a planar object, changes in its positions, and their relationships. Invariants, independent of coordinate systems, are introduced to describe the geometric properties concisely. Thanks to the moving Frenet frame for describing infinitesimally small variations of successive positions, the positional geometry can be naturally and conveniently connected to the time-independent differential movement of a planar object.

This chapter deals with the kinematic characteristics of a two-dimensional object (a point, a line) in a plane without consideration of time by means of differential geometry. Though abstract, the explanation is judiciously presented step by step for ease of understanding and will be a necessary foundation for studying the kinematic characteristics of a three-dimensional object by means of differential geometry in later chapters.

Kinematic Differential Geometry and Saddle Synthesis of Linkages, First Edition. Delun Wang and Wei Wang.

1.1 Plane Curves

1.1.1 Vector Curve

A plane curve Γ is represented in rectangular coordinates as

$$\begin{cases} x = x(t) \\ y = y(t) \end{cases} \tag{1.1}$$

where t is a parameter. The above equation can be rewritten in the following way by eliminating the parameter t:

$$y = F(x) \tag{1.2}$$

or in implicit form as

$$F(x, y) = 0 \tag{1.3}$$

In a fixed coordinate frame $\{\boldsymbol{O};\ \boldsymbol{i}, \boldsymbol{j}\}$, the vector equation of curve Γ can be written as

$$\Gamma : \boldsymbol{R} = x(t)\boldsymbol{i} + y(t)\boldsymbol{j} \tag{1.4}$$

or

$$\boldsymbol{R} = \boldsymbol{R}(t) \tag{1.5}$$

Obviously, both the magnitude and direction of $\boldsymbol{R}$ in equation (1.5) vary.

To describe a curve in the vector form, a real vector function, represented by a unit vector $\boldsymbol{e}_{\mathrm{I}(\varphi)}$ with an azimuthal angle φ with respect to axis $\boldsymbol{i}$, measured counterclockwise, is defined as a vector function of a unit circle (see Fig. 1.1). A plane curve Γ can be denoted by the following vector function:

$$\boldsymbol{R} = r(\varphi)\boldsymbol{e}_{\mathrm{I}(\varphi)} \tag{1.6}$$

In the above equation, the magnitude and direction of vector $\boldsymbol{R}$ depend on the scalar function $r(\varphi)$ and the vector function of a unit circle $\boldsymbol{e}_{\mathrm{I}(\varphi)}$.

Another vector function of a unit circle $\boldsymbol{e}_{\mathrm{II}(\varphi)} = \boldsymbol{e}_{\mathrm{I}(\varphi+\pi/2)}$ can be obtained by rotating $\boldsymbol{e}_{\mathrm{I}(\varphi)}$ counterclockwise about $\boldsymbol{k}$ by $\pi/2$ (in Chapters 1 and 2, $\boldsymbol{k}$ is the unit vector normal to the paper and directed toward the reader).

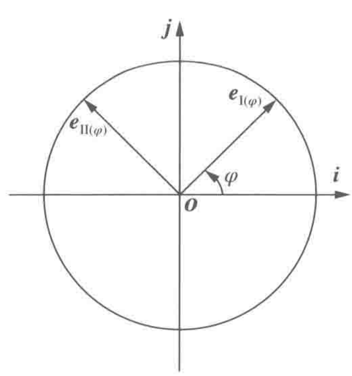

Figure 1.1 Vector function of a unit circle

The vector function of a unit circle has the following properties:

1. Expansion

$$\begin{cases} \boldsymbol{e}_{\mathrm{I}(\varphi)} = \cos\varphi \boldsymbol{i} + \sin\varphi \boldsymbol{j} \\ \boldsymbol{e}_{\mathrm{II}(\varphi)} = -\sin\varphi \boldsymbol{i} + \cos\varphi \boldsymbol{j} \end{cases} \tag{1.7}$$

2. Orthogonality

   For a unit orthogonal right-handed coordinate system $\{\boldsymbol{O}; \boldsymbol{e}_{\mathrm{I}(\varphi)}, \boldsymbol{e}_{\mathrm{II}(\varphi)}, \boldsymbol{k}\}$ consisting of $\boldsymbol{e}_{\mathrm{I}(\varphi)}$, $\boldsymbol{e}_{\mathrm{II}(\varphi)}$, and $\boldsymbol{k}$, we have the following identities:

$$\boldsymbol{e}_{\mathrm{I}(\varphi)} \cdot \boldsymbol{e}_{\mathrm{II}(\varphi)} = 0, \quad \boldsymbol{e}_{\mathrm{I}(\varphi)} \times \boldsymbol{e}_{\mathrm{II}(\varphi)} = \boldsymbol{k} \tag{1.8}$$

3. Transformation

$$\begin{cases} \boldsymbol{e}_{\mathrm{I}(\theta+\varphi)} = \cos(\theta+\varphi)\boldsymbol{i} + \sin(\theta+\varphi)\boldsymbol{j} = \cos\theta \boldsymbol{e}_{\mathrm{I}(\varphi)} + \sin\theta \boldsymbol{e}_{\mathrm{II}(\varphi)} \\ \boldsymbol{e}_{\mathrm{II}(\theta+\varphi)} = -\sin(\theta+\varphi)\boldsymbol{i} + \cos(\theta+\varphi)\boldsymbol{j} = -\sin\theta \boldsymbol{e}_{\mathrm{I}(\varphi)} + \cos\theta \boldsymbol{e}_{\mathrm{II}(\varphi)} \end{cases} \tag{1.9}$$

4. Differentiation

$$\frac{\mathrm{d}\boldsymbol{e}_{\mathrm{I}(\varphi)}}{\mathrm{d}\varphi} = \boldsymbol{e}_{\mathrm{II}(\varphi)}, \quad \frac{\mathrm{d}\boldsymbol{e}_{\mathrm{II}(\varphi)}}{\mathrm{d}\varphi} = -\boldsymbol{e}_{\mathrm{I}(\varphi)} \tag{1.10}$$

The descriptive form of a curve depends on the chosen parameters and coordinates. A curve may have many descriptive forms, which differ in complexity if the parameters and reference coordinates are chosen differently. Below are three examples.

Example 1.1 A circle with radius r and center point C is shown in Fig. 1.2. Write its equation in both vector and parameter forms.

Solution

The parameter equation of a circle in rectangular coordinates $\{\boldsymbol{O};\ \boldsymbol{i}, \boldsymbol{j}\}$ can be written as

$$\begin{cases} x = x_C + r\cos\varphi \\ y = y_C + r\sin\varphi \end{cases} \quad (0 \le \varphi < 2\pi) \tag{E1-1.1}$$

where (x_C, y_C) are the coordinates of the center of the circle in the reference frame $\{\boldsymbol{O};\ \boldsymbol{i}, \boldsymbol{j}\}$.

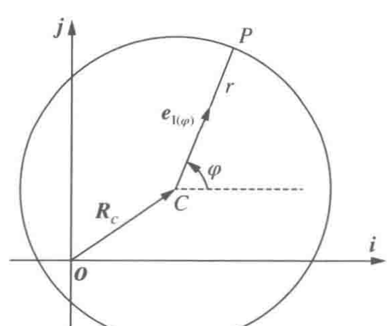

Figure 1.2 A circle

Alternatively, the same circle can be represented as a vector function of a unit circle:

$$\boldsymbol{R} = \boldsymbol{R}_C + r\boldsymbol{e}_{\mathrm{I}(\varphi)} \tag{E1-1.2}$$

Example 1.2 An involute is shown in Figs 1.3 and 1.4. Write its equation in both vector and parameter forms.

Solution

The equation of an involute can be written in three different forms using polar coordinates, rectangular coordinates, and a vector function of a unit circle, where r_b is the radius of the base circle.

1. Polar coordinates:

$$\begin{cases} r = \dfrac{r_b}{\cos\alpha} \\ \theta = \tan\alpha - \alpha \end{cases} \tag{E1-2.1}$$

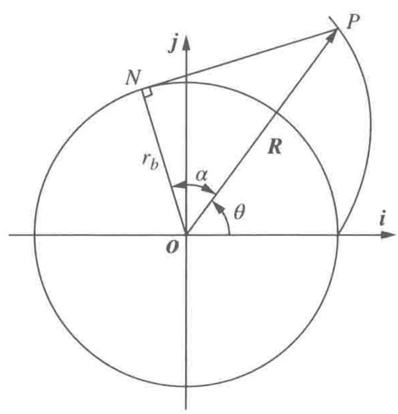

Figure 1.3 An involute

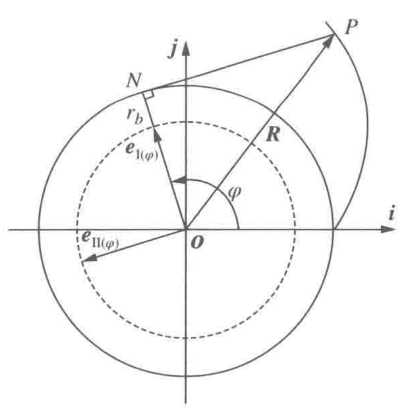

Figure 1.4 An involute with a unit circle vector function