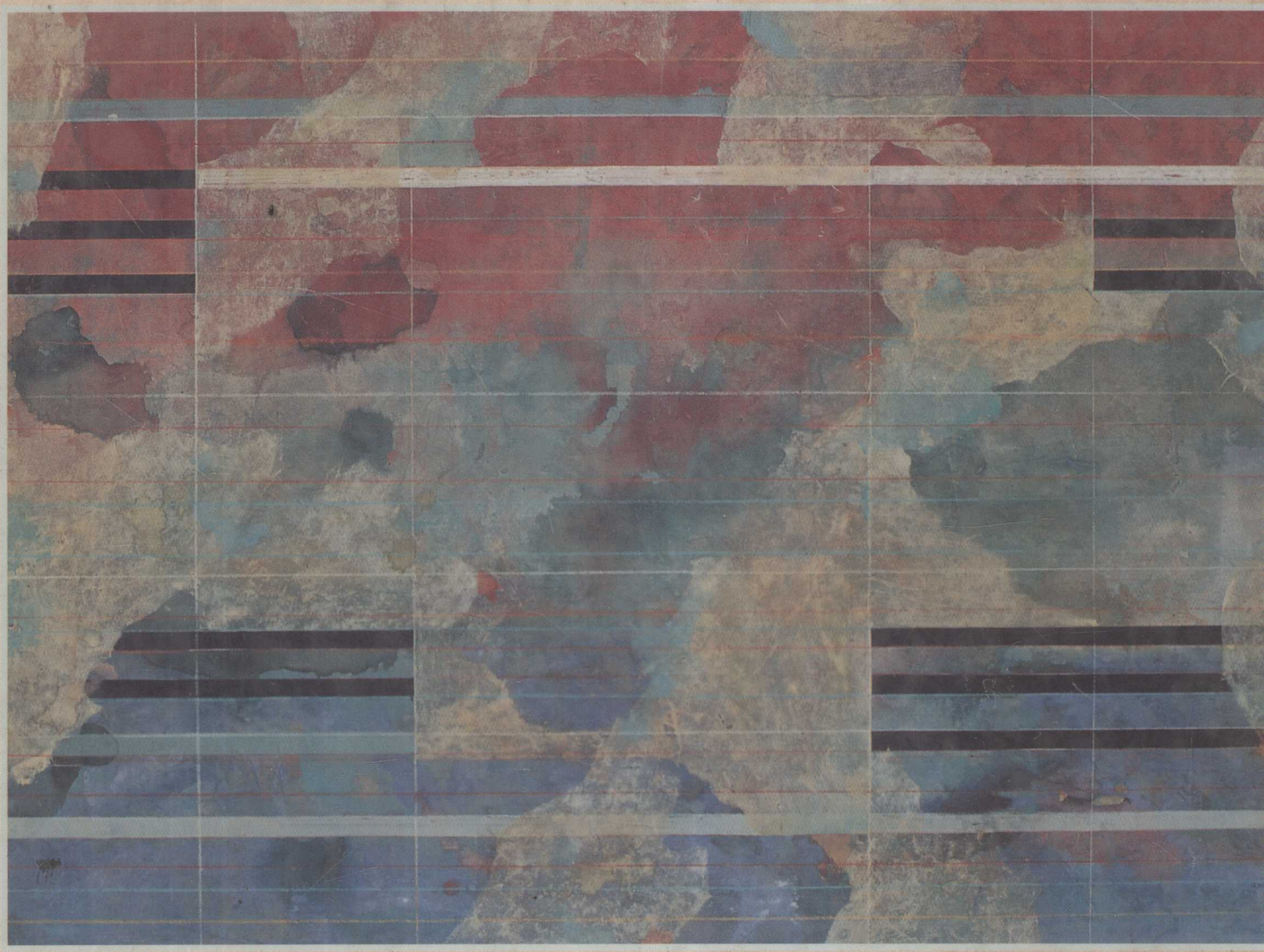
Understanding Elementary Algebra
A COURSE FOR COLLEGE STUDENTS

THIRD EDITION

GOODMAN HIRSCH



Understanding Elementary Algebra

A Course for College Students

Third Edition

Arthur Goodman
Queens College of the City University of New York

Lewis Hirsch
Rutgers University

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Preface to the Instructor

This third edition of *Understanding Elementary Algebra* retains the same basic structure and philosophy as the previous editions.

PURPOSE *Understanding Elementary Algebra*, 3rd edition, is an attempt on our part to offer a textbook that reflects our philosophy—that students can *understand* what they are doing in algebra and why.

We offer a view of algebra that takes every opportunity to explain why things are done in a certain way, and to show how supposedly "new" topics are actually just new applications of concepts already learned.

This book assumes only a basic knowledge of arithmetic. Appendix A includes a brief review of the arithmetic of decimals and percents.

PEDAGOGY We believe that a student can successfully learn elementary algebra by mastering a few basic concepts and being able to apply them to a wide variety of situations. Thus, each section begins by relating the topic to be discussed to material previously learned. In this way the students can see algebra as a whole rather than as a series of isolated ideas.

Basic concepts, rules, and definitions are motivated and explained via numerical and algebraic examples. Formal proofs have been avoided except for those occasions when they illuminate the discussion.

Concepts are developed in a series of carefully constructed illustrative examples. Through the course of these examples we compare and contrast related ideas, helping the student to understand the sometimes subtle distinctions among various situations. In addition, these examples strengthen a student's understanding of how this "new" idea fits into the overall picture.

Every opportunity has been taken to point out common errors often made by students and to explain the misconception that often leads to a particular error.

Basic rules and/or procedures are highlighted so that students can find important ideas quickly and easily.

A spiral approach has been used for the presentation of some more difficult topics. That is, a topic is first presented at an elementary level and then returned to at increasing levels of complexity.

For example,

Simple rational expressions are covered in Chapter 4, whereas more complex rational expressions are dealt with in Chapter 9.

Verbal problems are covered in Sections 2.5, 3.3, 4.6, 9.6, and 11.6.

Factoring is covered in Section 2.3, and in Chapters 8 and 9.

FEATURES The various steps in the solutions to examples are explained in detail. Many steps appear with annotations (highlighted in color) that involve the student in the solution. These comments explain how and why a solution is proceeding in a certain way.

- There are almost 2,800 homework exercises. Not only have the exercises been matched odd/even, but they have also been designed so that, in many situations, successive odd-numbered exercises compare and contrast subtle differences in applying the concepts covered in the section. Additionally, variety has been added to the exercise sets so that the student must be alert as to what the problem is asking.

  For example, the exercise sets in Sections 4.3 and 9.3, which deal primarily with adding rational expressions, also contain some exercises on multiplying and dividing rational expressions. The exercise set in Section 4.4 on solving fractional equations also asks the student to combine rational expressions. The exercise set in Section 11.1, which deals primarily with quadratic equations, contains some linear equations as well.

- One of the main sources of students' difficulties is that they do not know how to study algebra. In this regard we offer a totally unique feature. Each section in the first four chapters concludes with a **Study Skill**. This is a brief paragraph discussing some aspect of studying algebra, doing homework, or preparing for or taking exams. Our students who have used the previous editions of this book indicated that they found the Study Skills very helpful. The Algebra Study Skills sections in this text and in *Understanding Intermediate Algebra* are based on ideas in the book *Studying Mathematics* by Mary Catherine Hudspeth and Lewis R. Hirsch (1982, Kendall/Hunt Publishing Company, Dubuque, Iowa). For more information and ideas on improving mathematics learning, we direct you to that book.

- Almost every exercise set contains **Questions for Thought**, which offer the student an opportunity to *think* about various algebraic ideas. They may be asked to compare and contrast related ideas, or examine an incorrect solution and explain why the solution is wrong. The Questions for Thought are intended to be answered in complete sentences and in grammatically correct English. The Questions for Thought can also be used by instructors as a vehicle for having students write across the curriculum.

- Most sections contain a **Mini-Review**, which consists of exercises that allow students to periodically review important topics as well as help them prepare for the material to come. These Mini-Reviews afford the student additional opportunity to see new topics within the framework of what they have already learned.
- Each chapter contains a chapter summary describing the basic concepts in the chapter. Each point listed in the summary is accompanied by an example illustrating the concept or procedure.
- There are over 750 review exercises. Each chapter contains a set of chapter review exercises and a chapter practice test. Additionally, there are four cumulative review exercise sets and four cumulative practice tests following Chapters 3, 6, 9, and 11. These offer the student more opportunities to practice choosing the appropriate procedure in a variety of situations.
- The answer section contains answers to all the odd-numbered exercises, as well as to *all* the mini-reviews, chapter and cumulative review exercises, and practice test problems. The answer to each verbal problem contains a description of what the variable(s) represent and the equation (or system of equations) used to solve it. In addition, the answers to the cumulative review exercises and cumulative practice tests contain a reference to the section in which the relevant material is covered.

NEW TO THE THIRD EDITION Many of the comments and suggestions made by the users of the first two editions have been incorporated into the third edition. Among these are:

1. The material on graphing has been reorganized so that it begins earlier in the text. Chapter 5 now covers graphs and equations of straight lines. An entirely new section (6.1) on interpreting graphs has been added. This new section discusses how a graph can be used as a source of information about a relationship between two quantities.
2. The material on solving systems of linear equations in two variables has been placed earlier in the text (Chapter 6). This gives the student more opportunities to practice using both the one-variable and two-variable approaches to solving verbal problems.
3. A new section (11.7) on graphing parabolas has been added.
4. At the suggestion of some of the users of the second edition, we have expanded and modified many of the exercise sets to offer a broader selection of exercises ranging from the very basic to the more advanced.
5. Appendix A has been expanded to include a much more detailed review of the arithmetic of fractions, decimals, and percent for those students who need it.
6. Appendix B is devoted to a discussion of the use of calculators. This section should be valuable reading for many students even if they do not need the arithmetic review.

7. We have included more examples and exercises that illustrate the appropriate use of a calculator.
8. Additional applications have been integrated into the text wherever appropriate.
9. The text has gone to a four-color format. We have tried to use the enhanced color scheme to improve the pedagogical presentation, and also to make it easier to find material in the text. The additional colors have been used to highlight important ideas, definitions, and procedural outlines.

ANCILLARIES A student's study guide is available; it contains the worked-out solutions to many odd-numbered exercises, the answers to the Questions for Thought, two additional practice tests for each chapter, and four additional cumulative practice tests.

An instructor's manual contains the answers to *all* the exercises, answers to the Questions for Thought, five additional chapter tests for each chapter, two additional cumulative tests for every three chapters, and two final exams. The instructor's guide also contains suggestions on integrating and reinforcing the Study Skills within the course structure.

An extensive testing and software package is available. **WESTEST 3.0** is a computerized testing package that is available with the text. The software runs on Apple Macintoshes or IBM PCs and compatibles running DOS with Windows. WESTEST 3.0 allows the instructor to create and modify exams quickly and easily, using algorithms or questions from a prepared test bank.

WEST MATH TUTOR by Mathens Corp. is available for IBM PCs and compatibles and Apple Macintosh computers. This software provides algorithmically-based tutorials that are customized for each chapter of the text. These interactive tutorials give feedback and provide hints that allow the student to try the question again. The student is shown a complete solution after two incorrect attempts.

An entirely new set of instructional videotapes has been prepared for the third edition. Enhanced graphics provide an effective way to review important material. The videotapes are keyed to corresponding material in the text, making it easy for students to use the textbook in conjunction with the videotapes. There are accompanying exercise sheets that keep the student actively involved in the lesson.

An alternate version of this text, entitled *Understanding Elementary Algebra with Geometry*, is available. It contains an additional chapter on geometry that covers the basic vocabulary and important facts and formulas relating to parallel lines, angles, triangles, quadrilaterals, and circles.

ACKNOWLEDGMENTS The authors would like to acknowledge the many users, both instructors and students, of the previous editions whose invaluable comments and suggestions helped shape this third edition.

The authors would like to thank Martin Hoffman for his suggestion to ask students to create a verbal problem associated with a particular equation.

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Obviously, the production of a textbook is a collaborative effort, and we must thank our editor Ron Pullins for his constant support and encouragement, Susan Reiland for her expert supervision of the entire production, Luana Richards for her careful proofreading, and Nancy Little for checking the exercises. Of course, any errors that remain are the sole responsibility of the authors, and we would greatly appreciate their being called to our attention.

Finally, we would like to thank our wives, Sora and Cindy, and our families for their unwavering encouragement.

Preface to the Student

This text is designed to help you understand algebra. We are convinced that if you understand what you are doing and why, you will be a much better algebra student. (Our students who have used this book in its previous editions seem to agree with us.) This does not mean that after reading each section you will understand all the concepts clearly. Much of what you learn comes through the course of doing lots and lots of exercises and seeing for yourself exactly what is involved in completing an exercise. However, if you read the textbook carefully and take good notes in class, you will find algebra not quite so menacing.

Here are a few suggestions for using this textbook:

- Always read the textbook with a pencil and paper in hand. Reading mathematics is not like reading other subjects. *You* must be involved in the learning process. Work out the examples along with the textbook and *think* about what you are reading. Make sure you understand what is being done and why.
- You must work homework exercises on a daily basis. While attending class and listening to your instructor are important, do not mistake understanding someone else's work for the ability to do the work yourself. (Think about watching someone else driving a car, as opposed to driving yourself.) Make sure *you* know how to do the exercises.
- Read the Study Skills that appear at the end of each section in the first four chapters. They discuss the best ways to use the textbook and your notes. They also offer a variety of suggestions on how to study, do homework, and prepare for and take tests. If you want more information on improving your algebra study skills, we direct you to the book *Studying Mathematics*, by Mary Catherine Hudspeth and Lewis R. Hirsch (1982, Kendall/Hunt Publishing Company, Dubuque, Iowa).
- Do not get discouraged if you have difficulty with some topics. Certain topics may not be absolutely clear the first time you see them. Be persistent. We all need time to absorb new ideas and become familiar with them. What was initially difficult will become less so as you spend more time with a subject. Keep at it and you will see that you are making steady progress.

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The Integers

Study Skills

1.1

What Is Algebra? Introduction and Basic Notation

Ask most students, "What is algebra?" and *they* get a glassy-eyed look, but *you* will most likely get no response. It is not a very difficult question, and it has a very straightforward answer. Algebra is a language. It happens to be the language of mathematics.

We are going to learn this language in the same way we would learn any new language. We will begin by learning the alphabet (that is, the symbols) we will be using. For the most part our alphabet consists of letters and symbols we are already familiar with, such as the letters of the English alphabet, Arabic numerals, and the basic symbols of arithmetic. Next, we will learn the "grammar" of our new language, that is, the rules for putting the symbols together and manipulating them. After we learn the structure of the language, we can begin to actually use algebra to solve problems.

In some sense we can say that algebra is the generalization of arithmetic. We are going to let letters represent numbers and state our rules and our conclusions using letters, so that they will be valid for all numbers.

Sets

One concept that is used frequently is the idea of a set. The word *set* is used in mathematics in much the same way it is used in everyday life. A ***set*** is simply a well-defined collection of objects. The phrase *well-defined* means that there are clearly determined criteria for membership in the set. The criteria can be a list of those objects in the set, called the ***elements*** or ***members*** of the set, or it can be a description of those objects in the set.

For example, it is not sufficient to say "the set of all tall people in the class." *Tall* is a subjective criterion. It is possible to make the set well defined by saying "the set of all people in the class more than 6 feet tall."

One way to represent a set is to list the elements of the set, and enclose the list in "set braces" which look like { }.

We often designate sets by using capital letters such as A, B, C. For example:

$$A = \{3, 4, 5, 8\}$$
$$B = \{\text{a, e, i, o, u}\}$$
$$C = \{\text{red, white, blue}\}$$

are three sets.

The symbol we use to indicate that an object is a member of a particular set is $\in$. Thus, $x \in S$ is a symbolic way of writing that x is a member or an *element* of S. We use the symbol $\notin$ to indicate that an object is *not* an element of a set. [In general, when we put a "/" through a mathematical symbol it means *not*. Thus, "$\neq$" means *not equal*.] For example, using the sets A, B, and C listed above we have:

$5 \in A$ 5 is an element of A.

$\text{p} \notin B$ p is not an element of B.

In order to exhibit a set that contains many elements or a set that contains an infinite number of elements, we use a variation on the listing method. For example, the set of even numbers greater than 0 and less than or equal to 100 can be written as $\{2, 4, 6, \ldots, 100\}$. The three dots mean that the set continues according to the same pattern. The set $O = \{1, 3, 5, \ldots\}$ is the set of all odd numbers greater than 0. There is no number after the dots because this set is infinite; it has no last element.

Of course, this method of listing a set can be used only when the first few elements clearly show the pattern for *all* the elements in the set.

Certain frequently used sets of numbers are given special names.

The set of numbers we use for counting is called the set of ***natural numbers***, and is usually denoted with the letter N:

$$N = \{1, 2, 3, \ldots\}$$

If we add the number 0 to this set it is called the set of ***whole numbers*** and is denoted with the letter W:

$$W = \{0, 1, 2, 3, \ldots\}$$

Often when we describe a set we use the word "between," which can be ambiguous. When we say "the numbers between 5 and 10" do we mean to include or exclude 5 and 10? Let's agree that when we say "between" we mean "in between" and we do *not* include the first and last numbers.

EXAMPLE 1 List the elements of the following sets:

(a) The set A of whole numbers between 6 and 30

(b) The set B of odd numbers greater than 17

Solution

(a) The whole numbers are the same as the natural numbers, except that the whole numbers include 0. Note that 6 and 30 are not included.

$$\boxed{A = \{7, 8, 9, \ldots, 29\}} \ *$$

(b) Since no upper limit to this set is given, the answer is

$$\boxed{B = \{19, 21, 23, 25, \ldots\}}$$ *Note that 17 is not included.* ■

Sometimes we cannot *list* the elements of a set, but rather we must describe the set. When this is the case we use what is called *set-builder notation*. ***Set-builder notation*** consists of the set braces, a ***variable*** that acts as a place holder, a vertical bar (|) read "such that," and a sentence that describes what the variable can be. This last part is called the ***condition*** on the variable. For example:

$$\{ \underset{\text{Variable}}{x} \mid \underset{\text{Such that}}{} \ x \text{ is an even number greater than 0 and less than 10} \}$$

Condition on the variable

This is read "the set of all x such that x is an even number greater than 0 and less than 10," which is the set $\{2, 4, 6, 8\}$.

* Throughout the text we will use color boxes to indicate the final answer to an example.

EXAMPLE 2 List the elements of the following set:

$$\{x \mid x \text{ is a whole number divisible by } 3\}$$

Solution The number 0 is included because $0 \div 3 = 0$. Thus, our answer is

$$\boxed{\{0, 3, 6, 9, \ldots\}}$$ ■

It is possible to place a condition on a set which no elements satisfy, as, for example,

$$F = \{x \mid x \text{ is an odd number divisible by } 2\}$$

Since it is impossible for an odd number to be divisible by 2, the set F has no members. It is called the ***empty set*** or the ***null set*** and it is symbolized by $\varnothing$. Thus, we have $F = \varnothing$.

Before we can continue we must introduce some terminology and notation.

Sums, Terms, Products, and Factors

Most of the errors made by students in algebra are the result of confusing terms with factors and factors with terms. We can do some things with factors that we cannot do with terms, and vice versa. We will define them now for arithmetic expressions and point out the differences throughout the book.

Sum is the word we use for addition. In an expression involving a sum, the numbers to be added in the sum are called the ***terms***. The symbol used to indicate a sum is the familiar "+" sign.

Product is the word we use for multiplication. In an expression involving a product, the numbers being multiplied are called the ***factors***. Saying that "a is a ***multiple*** of b" is equivalent to saying that "b is a ***factor*** of a." For example,

20 is a multiple of 5 because 5 is a factor of 20.

48 is a multiple of 8 because 8 is a factor of 48.

Thus, a factor of n is a number that divides into n exactly, whereas a multiple of n is a number that is exactly divisible by n.

In algebra, we generally use the symbol "$\cdot$" to indicate multiplication. We do not use the "$\times$" to indicate multiplication because we very often use x in our work as a variable. Frequently we will also indicate a product simply by writing numbers or expressions next to each other with the appropriate "punctuation." For example, if we let x represent a number, then

Sums can be written as:

$3 + 4$ (the sum of 3 and 4)

$7 + x$ (the sum of 7 and x)

Note that in the sum $7 + x$, 7 and x are the terms.

Products can be written as:

$3 \cdot 4$ (the product of 3 and 4)

$7 \cdot x$ (the product of 7 and x)

$7x$ (also the product of 7 and x)

Note that in the product $7x$, 7 and x are the factors.