



国家出版基金项目
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中外物理学精品书系

引进系列 · 56

Fundamentals and New Frontiers of Bose-Einstein Condensation

玻色-爱因斯坦凝聚的基础与前沿

(影印版)

〔日〕上田正仁 (M. Ueda) 著



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著作权合同登记号 图字:01-2014-5532

图书在版编目(CIP)数据

玻色-爱因斯坦凝聚的基础与前沿 = Fundamentals and new frontiers of Bose-Einstein condensation: 英文/(日)上田正仁著. —影印本. —北京:北京大学出版社, 2014. 12

(中外物理学精品书系)

ISBN 978-7-301-25173-7

I. ①玻… II. ①上… III. ①玻色凝聚—英文 IV. ①O414.2

中国版本图书馆 CIP 数据核字(2014)第 278933 号

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Reprint arranged with World Scientific Co. Pte. Ltd., Singapore.

书 名: **Fundamentals and New Frontiers of Bose-Einstein Condensation(玻色-爱因斯坦凝聚的基础与前沿)**(影印版)

著作责任者:〔日〕上田正仁(M. Ueda) 著

责任编辑:刘 啸

标准书号:ISBN 978-7-301-25173-7/O·1049

出版发行:北京大学出版社

地 址:北京市海淀区成府路 205 号 100871

网 址: <http://www.pup.cn>

新浪微博: @北京大学出版社

电子信箱: zpup@pup.cn

电 话: 邮购部 62752015 发行部 62750672 编辑部 62752038 出版部 62754962

印刷者:北京中科印刷有限公司

经 销 者:新华书店

730 毫米×980 毫米 16 开本 23.25 印张 371 千字

2014 年 12 月第 1 版 2014 年 12 月第 1 次印刷

定 价: 63.00 元

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序 言

物理学是研究物质、能量以及它们之间相互作用的科学。她不仅是化学、生命、材料、信息、能源和环境等相关学科的基础,同时还是许多新兴学科和交叉学科的前沿。在科技发展日新月异和国际竞争日趋激烈的今天,物理学不仅囿于基础科学和技术应用研究的范畴,而且在社会发展与人类进步的历史进程中发挥着越来越关键的作用。

我们欣喜地看到,改革开放三十多年来,随着中国政治、经济、教育、文化等领域各项事业的持续稳定发展,我国物理学取得了跨越式的进步,做出了很多为世界瞩目的研究成果。今日的中国物理正在经历一个历史上少有的黄金时代。

在我国物理学科快速发展的背景下,近年来物理学相关书籍也呈现百花齐放的良好态势,在知识传承、学术交流、人才培养等方面发挥着无可替代的作用。从另一方面看,尽管国内各出版社相继推出了一些质量很高的物理教材和图书,但系统总结物理学各门类知识和发展,深入浅出地介绍其与现代科学技术之间的渊源,并针对不同层次的读者提供有价值的教材和研究参考,仍是我国科学传播与出版界面临的一个极富挑战性的课题。

为有力推动我国物理学研究、加快相关学科的建设与发展,特别是展现近年来中国物理学者的研究水平和成果,北京大学出版社在国家出版基金的支持下推出了“中外物理学精品书系”,试图对以上难题进行大胆的尝试和探索。该书系编委会集结了数十位来自内地和香港顶尖高校及科研院所的知名专家学者。他们都是目前该领域十分活跃的专家,确保了整套丛书的权威性和前瞻性。

这套书系内容丰富,涵盖面广,可读性强,其中既有对我国传统物理学发展的梳理和总结,也有对正在蓬勃发展的物理学前沿的全面展示;既引进和介绍了世界物理学研究的发展动态,也面向国际主流领域传播中国物理的优秀专著。可以说,“中外物理学精品书系”力图完整呈现近现代世界和中国物理

科学发展的全貌,是一部目前国内为数不多的兼具学术价值和阅读乐趣的经典物理丛书。

“中外物理学精品书系”另一个突出特点是,在把西方物理的精华要义“请进来”的同时,也将我国近现代物理的优秀成果“送出去”。物理学科在世界范围内的重要性不言而喻,引进和翻译世界物理的经典著作和前沿动态,可以满足当前国内物理教学和科研工作的迫切需求。另一方面,改革开放几十年来,我国的物理学研究取得了长足发展,一大批具有较高学术价值的著作相继问世。这套丛书首次将一些中国物理学者的优秀论著以英文版的形式直接推向国际相关研究的主流领域,使世界对中国物理学的过去和现状有更多的深入了解,不仅充分展示出中国物理学研究和积累的“硬实力”,也向世界主动传播我国科技文化领域不断创新的“软实力”,对全面提升中国科学、教育和文化领域的国际形象起到重要的促进作用。

值得一提的是,“中外物理学精品书系”还对中国近现代物理学科的经典著作进行了全面收录。20世纪以来,中国物理界诞生了很多经典作品,但当时大都分散出版,如今很多代表性的作品已经淹没在浩瀚的图书海洋中,读者们对这些论著也都是“只闻其声,未见其真”。该书系的编者们在这方面下了很大工夫,对中国物理学科不同时期、不同分支的经典著作进行了系统的整理和收录。这项工作具有非常重要的学术意义和社会价值,不仅可以很好地保护和传承我国物理学的经典文献,充分发挥其应有的传世育人的作用,更能使广大物理学人和青年学子亲身体会我国物理学研究的发展脉络和优良传统,真正领悟到老一辈科学家严谨求实、追求卓越、博大精深的治学之美。

温家宝总理在2006年中国科学技术大会上指出,“加强基础研究是提升国家创新能力、积累智力资本的重要途径,是我国跻身世界科技强国的必要条件”。中国的发展在于创新,而基础研究正是一切创新的根本和源泉。我相信,这套“中外物理学精品书系”的出版,不仅可以使所有热爱和研究物理学的人们从中获取思维的启迪、智力的挑战和阅读的乐趣,也将进一步推动其他相关基础科学更好更快地发展,为我国今后的科技创新和社会进步做出应有的贡献。

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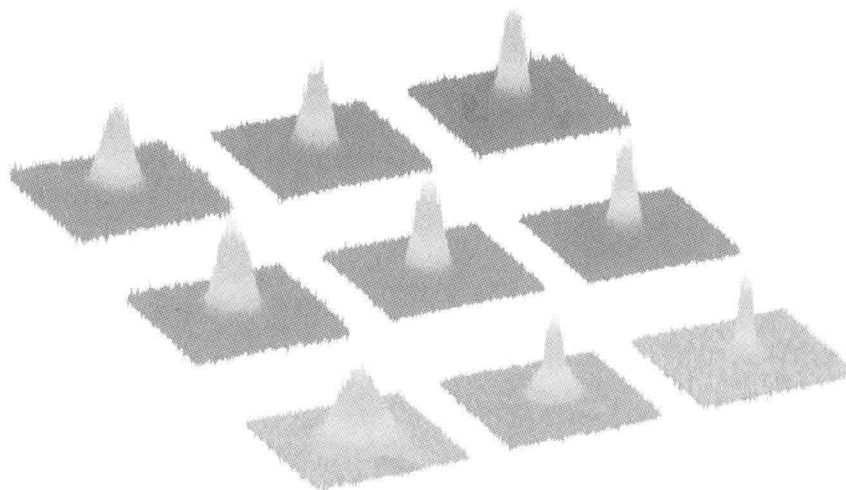
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2010年5月于燕园

FUNDAMENTALS AND NEW FRONTIERS OF BOSE-EINSTEIN CONDENSATION

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Preface

Experimental realization of Bose–Einstein condensation (BEC) of dilute atomic gases [Anderson, *et al.* (1995); Davis, *et al.* (1995); Bradley, *et al.* (1995, 1997)] has ignited a virtual explosion of research. The unique feature of the atomic gas BEC is its unprecedented controllability, which makes the previously unthinkable possible. Almost all parameters of the system such as the temperature, number of atoms, and even strength and sign (attractive or repulsive) of interaction can be varied by several orders of magnitude. The interaction between atoms is usually considered to be an immutable, inherent property of individual atomic species. In alkali and some other Bose–Einstein condensates, we can not only control the strength of interaction but also switch the sign of interaction from repulsive to attractive and vice versa [Inouye, *et al.* (1998); Cornish (2000)]. The atomic-gas BEC may thus be regarded as an artificial macroscopic matter wave that act as an ideal testing ground for the investigation of quantum many-body physics. The atomic-gas BEC may also be regarded as an atom laser because the condensate provides a phase-coherent, intense atomic source with potential applications for precision measurement, lithography, and quantum computation. Fermionic species may also undergo BEC by forming molecules or Cooper pairs. Both molecular condensates [Greiner, *et al.* (2003); Zwierlein, *et al.* (2003)] and Bardeen–Cooper–Schrieffer-type resonant superfluids [Regal, *et al.* (2004); Zwierlein, *et al.* (2004)] have been realized using alkali fermions, opening up the new research field of strongly correlated gaseous superfluidity. This book is intended as an introduction to this rapidly developing, interdisciplinary field of research.

Most phase transitions occur due to interactions between constituent particles. For example, superconductivity occurs due to effective interac-

tions between electrons, and ferromagnetism is caused by the exchange interaction between spins. In contrast, BEC is a genuinely quantum-statistical phase transition in that it occurs without the help of interaction (Einstein called it “condensation without interaction” [Einstein (1925)]). The fundamentals of noninteracting BECs are reviewed in Chapter 1.

In a real BEC system, interactions between atoms play a crucial role in determining the basic properties of the system. Neutral atoms have a hard core that is short-ranged ($\sim 1 \text{ \AA}$) and strongly repulsive. At a longer distance ($\sim 100 \text{ \AA}$), the atoms are attracted to each other because of the van der Waals force. When two atoms collide, they experience both these forces, and the net interaction can be either repulsive or attractive depending on the hyperfine and translational states of the colliding atoms. Under normal conditions, a dilute-gas BEC system can be treated as a weakly interacting Bose gas. The Bogoliubov theory of a weakly interacting Bose gas and related topics are described in Chapter 2.

One of the remarkable aspects of a dilute gas BEC system is the great success of the mean-field theory governed by the Gross–Pitaevskii (GP) equation [Gross (1961); Pitaevskii (1961)]. The GP equation describes the mean-field ground state as well as the linear and nonlinear response of the system. Various nonlinear matter-wave phenomena including four-wave mixing [Deng, *et al.* (1999); Rolston and Phillips (2002)] and topological excitations such as solitons [Denschlag, *et al.* (2000)] and vortices [Matthews, *et al.* (1999); Madison, *et al.* (2000)], have been successfully described by the GP equation. This remarkable success of the mean-field theory is due to the high ($> 99\%$) degree of condensation of bosons into a single-particle state, which in turn originates in an extremely low density ($\sim 10^{11} - 10^{15} \text{ cm}^{-3}$) of the system operating at ultralow temperatures ($\lesssim 10^{-6} \text{ K}$). The Gross–Pitaevskii theory together with its various applications is discussed in Chapter 3.

The linear response theory provides a general theoretical framework to investigate collective modes of Bose–Einstein condensates and superfluids. A sum-rule approach is also very useful for this purpose because the ground state for a dilute-gas Bose–Einstein condensate can be obtained very accurately. These subjects are discussed in Chapter 4.

Superfluidity manifests itself as a response of the system to its moving container. A statistical-mechanical theory to tackle such problems and some basic properties of superfluidity are described in Chapter 5.

Alkali atoms have both electronic spin \mathbf{s} and nuclear spin \mathbf{i} , and these two spins interact with each other via the hyperfine interaction. When the

energy of the hyperfine coupling exceeds the electronic and nuclear Zeeman energies as well as the thermal energy, the total spin $\mathbf{f} = \mathbf{s} + \mathbf{i}$, which is called the hyperfine spin, is a conserved quantum number. When atoms are confined in a magnetic potential, the spin of each atom points in the direction of an external magnetic field. The spin degrees of freedom are therefore frozen and the mean-field properties of the system are described by a scalar order parameter. When the system is confined in an optical trap, the frozen degrees of freedom are liberated, yielding a rich variety of phenomena arising from the magnetic moment of the atom. Since the magnetic moments of alkali atoms originate primarily from the electronic spin, this system's response to an external magnetic field is much greater than that of superfluid helium-3. We can expect interesting interplay between superfluidity and magnetism with the possibility of new ground states, spin domains, and vortex structures. Spinor condensates are discussed in Chapter 6.

When the rotational speed of the container of the system is faster than the critical frequency, vortices enter the system and form a vortex lattice. The direct observation of vortex lattice formation [Madison, *et al.* (2000); Abo-Shaer, *et al.* (2001)] has attracted considerable interest in the equilibrium and nonequilibrium dynamics of condensates. The effect of rotation on neutral particles is equivalent to that of a magnetic field on charged particles. Therefore, the properties of a vortex lattice of neutral particles are similar to those of superconductors. Furthermore, it is pointed out that in systems containing neutral bosons that are subject to very fast rotation, the vortex lattice melts, and a new vortex liquid state similar to the Laughlin state in the fractional quantum Hall system may be realized. A brief overview of these subjects is presented in Chapter 7.

Almost every bosonic atom has its fermionic counterpart. Fermions and bosons of the same species exhibit the same properties at high temperature, but they exhibit remarkably different behavior when quantum degeneracy sets in. Bosons undergo BEC below the transition temperature; in contrast, fermions become degenerate below the Fermi temperature, where almost every quantum state below the Fermi energy is occupied by one fermion and most quantum states above the Fermi energy are empty. At even lower temperatures, fermionic systems may exhibit superfluidity by forming Cooper pairs via the Bardeen–Cooper–Schrieffer transition. This is a rapidly developing field that has relevance to high-temperature superconductivity. We describe the basics and some of the recent developments of ultracold fermionic systems in Chapter 8.

It is known that BEC does not occur at finite temperature in one- or two-dimensional infinite systems because thermal fluctuations destroy the off-diagonal long-range order (ODLRO). In one-dimensional systems, BEC does not occur even at absolute zero because quantum fluctuations wash out the ODLRO. However, confined low dimensional systems can exhibit BEC because long-wavelength fluctuations are cut off by confinement. We may thus investigate interesting phenomena associated with low-dimensional BEC, such as solitons and the Berezinskii-Kosterlitz-Thouless transition. These subjects are discussed in Chapter 9.

Atoms with magnetic moments and polar molecules undergo dipole-dipole interactions, which are long-ranged and anisotropic and yield a wealth of novel phenomena. The magnetic dipole-dipole interaction is by far the weakest of the relevant interactions in cold atom systems; yet it plays a dominant role in forming spin textures and magnetic ordering and produces a spectacular effect in the course of the collapsing dynamics. The electric dipole-dipole interaction between polar molecules, in contrast, is very strong and may cause instabilities of the system; at the same time, it has the potential to yield several exotic phases and for use in quantum information processing. Some basic properties of the dipolar condensates are reviewed in Chapter 10.

An optical lattice is a periodic potential created by interference between two counterpropagating laser beams. Atoms in an optical lattice behave like electrons in a crystal. An optical lattice can host bosons as well as fermions, and it offers an ideal testbed to simulate quantum many-body physics and quantum information processing. Chapter 11 provides a brief overview of some basic properties of this artificial condensed matter system.

Superfluids host a rich variety of topological defects such as vortices, monopoles, and skyrmions. Those topological excitations are best described by the homotopy theory. Chapter 12 is devoted to an introduction of the homotopy theory, classification of topological excitations, and an account of how to calculate various topological charges.

Fifteen years after its first experimental realization, the field of ultracold atomic gases is still growing at a remarkable speed, such that coverage of every topic of importance far exceeds the range of this or perhaps any book. Rather, I have chosen a small number of important issues and tried to discuss their physical aspects as engagingly as possible. Many of the phenomena that have been observed in the past decade and those that will possibly be observed in the near future are of fundamental importance because of the very fact that they are being “seen” on a macroscopic scale.

If this book succeeds in conveying even a portion of the fascination inherent in this field, it will have well served its intended purpose.

This book derives from a set of lecture notes delivered at several universities over the past decade or so. I have benefited greatly from students and colleagues who actively participated in the class and collaboration. Special thanks are due to Rina Kanamoto, Yuki Kawaguchi, Michikazu Kobayashi, Tony Leggett, Hiroki Saito, and Masaki Tezuka. I would like to thank all of them for their questions, comments, and criticisms that helped me clarify my thoughts and improve the presentation of the material in this book. I am grateful to A. Koda, Y. Ookawara, and A. Yoshida for their efficient editing and preparation of the figures.

March 2010
Tokyo
Masahito Ueda

Revisions and corrections will be posted on:
http://cat.phys.s.u-tokyo.ac.jp/~ueda/E_kyokasyo.html/

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