Guenther:

Concepts of Statistical Inference

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Concepts of Statistical Inference

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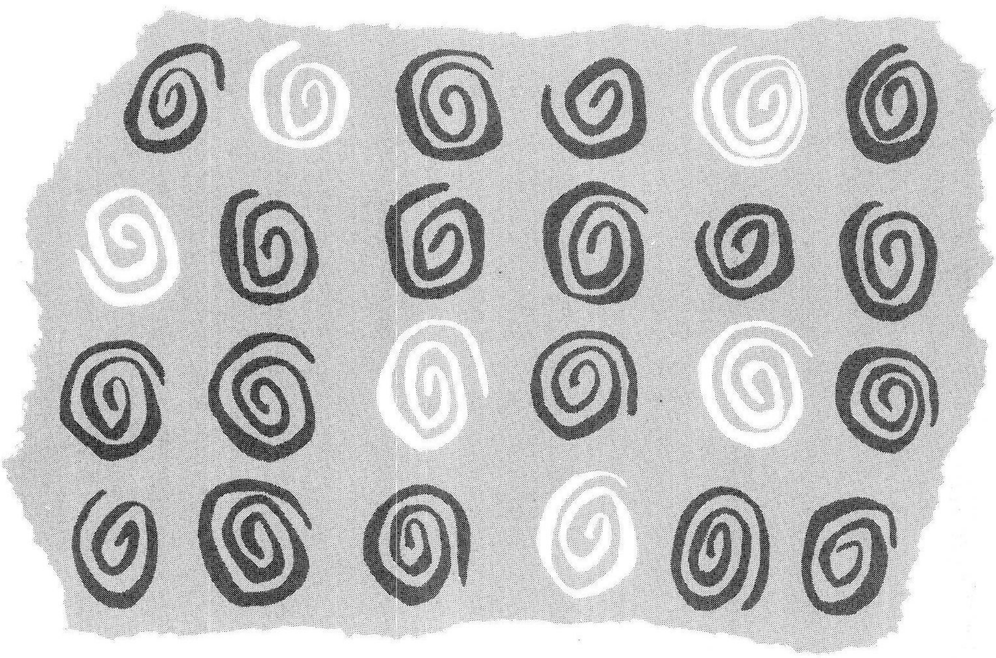

To Lee Ann

Preface

This book was written for a general one-semester (or one-quarter) introductory course in statistics. The primary objective has been to acquaint the student with the ideas and language of statistical inference without unduly slanting the examples and problems toward any particular field of application. Since so many new concepts have to be mastered, a minimum of emphasis has been placed upon computing, algebraic details, and complicated statistical methods.

The main features of the book are as follows:

1. Special emphasis is placed upon selecting the correct model and investigating the reasonableness of the conditions under which the model is derived.
2. A number of tables and graphs are presented; some of them appear for the first time in an elementary textbook. The state of statistical tables has been greatly improved over the past few years, and this book incorporates a number of the advances. Some calculations, such as those yielding power, were formerly very difficult but are now quite simple with the use of new graphs. Consequently, a more sophisticated approach to standard statistical problems can be taken.
3. Numerous worked examples are included which are written out in detail to show the student how to attack similar problems in the Exercises and in his practical application of statistical inference.

4 The Exercise sections contain problems pertaining to a wide variety of applications of statistical inference. Answers to the odd-numbered problems in Chapters 1 to 4 and selected sets of problems in Chapters 5 to 8 are given in the rear of the book, in greater detail than is usual where the problem seems to require this detail. Those Exercises for which answers are given are marked by a diamond symbol ◆ in the Exercise sections.

5 To assist the student with the organization and assimilation of Chapters 5 to 8, a summary has been provided at the end of each of these chapters.

No previous statistical experience has been assumed. A working knowledge of high-school algebra should provide sufficient mathematical background. It is, however, a good idea to precede a course taught from this book with a semester of college mathematics, mainly for the experience and maturity to be gained from it.

In a class meeting three times a week, it may be difficult to cover every section of the eight chapters. If material has to be omitted, it is recommended that the choice be some combination of the following: Section 6-6, Section 6-7, all or part of Chapter 7, all or part of Chapter 8. Chapters 7 and 8 are in no way dependent upon one another.

Logically this course can be followed by a number of special-topics or methods courses. Some of these include analysis of variance, regression and correlation, sampling, time series, nonparametric statistics, quality control, and methods applicable to some special field.

At the University of Wyoming this course has been taught with large lecture sections (preferably handled by an experienced staff member) meeting three times a week and small laboratory or problem sections (taught by graduate students) meeting twice a week. Although this system may not be ideal, it does provide a satisfactory method of handling a great many students, provided the laboratory sections are not too large.

I am indebted to Sir Ronald A. Fisher, F.R.S., Cambridge, and Dr. Frank Yates, F.R.S., Rothamsted, and to Messrs. Oliver and Boyd Ltd., Edinburgh, for permission to reprint parts of Table No. III from their book, "Statistical Tables for Biological, Agricultural, and Medical Research." Also, I want to acknowledge my appreciation to Prentice-Hall, Inc., for permission to reproduce Section 1-15 and Exercises 1.3, 1.5, 1.9, 1.12, 1.13, 1.14, 1.17, and 1.27 of my book, "Analysis of Variance," 1964, and to the other authors and publishers whose names appear with the tables and graphs of Appendix D.

WILLIAM C. GUENTHER

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Introduction

Many authors have defined statistics as a field of endeavor in which data are collected and analyzed for the purpose of drawing conclusions. The statement, although true, does not give the beginning student much appreciation for the subject. It is, however, very difficult to describe a field as comprehensive as statistics in a single sentence or paragraph.

The above definition as applied to the modern concept of statistics implies a number of things. First, the data that are collected are a sample or part of a much larger body of data (which it may or may not be possible to collect in its entirety) called the population. Second, the purpose of drawing the sample is to make some inference or draw some conclusion about the population. Third, inferences drawn are usually accompanied by statements which indicate the amount of confidence the statistician has in the results. To make such statements requires some knowledge of probability and what we shall call probability models. Fourth, the type of experiment which particularly lends itself to statistical analysis is one which yields different results when repeated under essentially the same conditions. Classic examples of such experiments are coin and dice throwing, but the results produced by subjecting human beings, animals, or other objects to the same environmental conditions yield this type of data.

As we have indicated, probability is a fundamental tool in statistical inference; consequently, our first objective will be to gain some understanding of that subject. With only an elementary knowledge of probability we can construct some highly useful probability models which will be used later to characterize the behavior of statistical experiments.

This is intended to be an introductory course in statistics. It is not expected that the student will be a statistician even if he receives a top grade. Some reasonable objectives for the course are:

1 To introduce students to the language and philosophy of statisticians.
2 To acquaint students with the types of problems that lend themselves to statistical solution.
3 To present enough basic statistical technique that the student can work some standard-type problems.
4 To enable the student to read and understand the summarized results of statistical experiments performed by others. He may never have to perform an experiment on his own, but he might have to read about the work of others in journals.
5 To interest some students in further study of statistics.

CHAPTER ONE

Probability

1-1 INTRODUCTION

Probability has become a very important field of study with many applications reaching outside the realm of statistics. Applied probability problems arise frequently in modern industries. As students of statistics, we are interested in probability primarily because it is essential that we have some understanding of the subject before we proceed to a discussion of statistical inference.

One might consider probability as a measure of likelihood so constructed that it ranges from 0 to 1. If an event is impossible, the weight or measure 0 is assigned to it. On the other hand, an event that is certain to happen is assigned weight 1. In between these extremes, weights are so chosen that the more likely an event, the higher its probability. The weights that are assigned may depend upon prior knowledge of the situation, experimental evidence, or just plain intuition. For example, if a coin is tossed a large number of times, it is reasonable to expect about the same number of heads and tails unless the coin is unbalanced or the tossing procedure is biased. It would seem sensible, therefore, to associate the weight $\frac{1}{2}$ with the head when discussing coin-tossing probabilities. We cannot prove that this is the correct number to use; nevertheless, it seems plausible.

In the coin example it is implied that the figure $\frac{1}{2}$ is associated with long-run behavior. Thus when we say that the probability of a head is $\frac{1}{2}$, we do not mean that 1 out of every 2 tosses results in a head; we mean, rather, that in the long run we expect heads to show about half the time.

1-2 DEFINITIONS OF PROBABILITY

Suppose that we have a well-balanced (or symmetric) die with six sides numbered from 1 to 6. We may feel that any one of the sides is as likely to show as any other when the die is rolled. If our assumption is true, then it is quite likely that we would associate a probability of $\frac{1}{6}$ with each of the six faces. Consideration of examples of this type led to this, the classical, definition of probability:

CLASSICAL DEFINITION *If an experiment can produce n different mutually exclusive results all of which are equally likely, and if f of these results are considered favorable, then the probability of a favorable result is f/n.* (1-1)

Two events are mutually exclusive if the occurrence of one prevents the occurrence of the other. The appearance of a 1 and a 2 on a single throw of die is impossible. A 1 can appear and a 2 can appear, but not both together. Thus the outcomes 1 and 2 are mutually exclusive events.

Let E stand for the event "getting a 4 or a 5 on the throw of a die." To compute the probability that E happens, which we shall denote by $\Pr(E)$, consider a 4 and 5 as favorable results. Thus $f = 2$, $n = 6$, and $\Pr(E) = \frac{2}{6}$ by the definition.

One objection to the classical definition is that it contains the phrase "equally likely," as yet undefined. Attempts to define it usually lead to the use of phrases such as "equally probable," and critics point out that this involves circular reasoning. The objection is not serious,