

# MODELING, ANALYSIS AND DESIGN OF CONTROL SYSTEMS IN MATLAB AND SIMULINK

Dingyü Xue · YangQuan Chen



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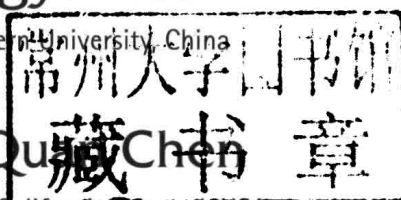
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MODELING, ANALYSIS  
AND DESIGN OF  
CONTROL SYSTEMS IN  
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# Foreword

Most people are familiar with the “mad scientist,” usually a brilliant, sometimes comical figure found in books and movies. Years ago I developed the motif of the “MAD control engineer” as a way to inspire a bit of passion and even brilliance in the students taking my feedback control classes. As such, I am pleased to see the new edition of this book, *Modeling, Analysis and Design of Control Systems in MATLAB and Simulink*, by Professors Xue and Chen, which follows the same motif.

“MAD” is an acronym that stands for Modeling, Analysis, and Design. My particular perspective is that these are the three essential activities required to design a control system. Specifically, given a physical system that we want to control, along with a desired behavior or performance for the controlled system, we determine a control law that will cause the closed-loop system to exhibit the desired behavior by:

- (1) **M**odeling (mathematically) the system, based on measurement of essential system characteristics.
- (2) **A**nalysis of the model to determine the properties of the system.
- (3) **D**esign of the controller which, when coupled with the model of the system, produces the desired closed-loop behavior. This will involve development of
  - (a) Control law algorithms.
  - (b) Measurement and testing techniques for the specific physical system.
  - (c) Signal processing and signal conditioning algorithms necessary for interfacing the sensor and controller to the physical system and to each other.
  - (d) Simulation studies of the individual components of the control system as well as simulation of the closed-loop system in which all the components are interconnected. Simulation studies are an essential part of the design and development process and are highly dependent on the models obtained from the measurement process.

Wrapped around these three activities are two other key parts of the controller development process:

- (1) **Development of performance specifications** that define the objective of

the control design.

- (2) **Implementation of the controller** through software and hardware realizations of the control law, including complete specification of the sensor, signal processing, and control elements, and final assembly, testing and validation, delivery, and operation of the control system.

The five activities described above are summarized in Fig. 0-1, which shows an overall conceptual flowchart of the control system design process. As shown in the figure, starting with the original system we wish to control (defined as including the plant, sensors, and actuators), we proceed with two tasks in parallel: defining the required performance specifications and developing a model of the process. The modeling activity will often include some form of measurement to determine key system properties. Note that mathematical modeling is a particularly important part of the process of control system development. By having a framework for describing the system in a precise way, it is possible to develop rigorous techniques for analyzing and designing systems. Once a math model is available and we have decided the goal of the design, it is possible to proceed with the analysis of the model and design of the control law. Finally, once the control law is finalized the implemented controller is combined with the physical system to collectively act as a new system - one that meets the desired performance.

Central to the process shown in Fig. 0-1 is the iteration of simulation, modeling, and design. Indeed, in today's world simulation cannot be separated from analysis and design. Further, the process of arrive at a math model of the controller is itself a feedback process. Once a controller model is defined it is necessary to evaluate its effectiveness in combination with the math model of the process (via simulation of the complete control system) before proceeding to implementation. We also note that today the process of going from a completed math model of the controller to its implementation is typically highly coupled to the same software environment used for simulation, using a hardware-in-the-loop approach to rapid prototyping. One such environment is the MATLAB/Simulink package.

This discussion then brings me back to the present book by Professors Xue and Chen. My pleasure in the new edition comes from seeing how first they have developed an exposition that embodies the perspective of the MAD Control Engineering and second they have integrated the real-world practices of simulation as part of analysis and design process together with controller implementation using rapid prototyping tools, using MATLAB/Simulink. I believe students who study and follow this text will be well-equipped to "hit the ground running!"

*Kevin L. Moore, Colorado School of Mines, Golden, Colorado, United States*

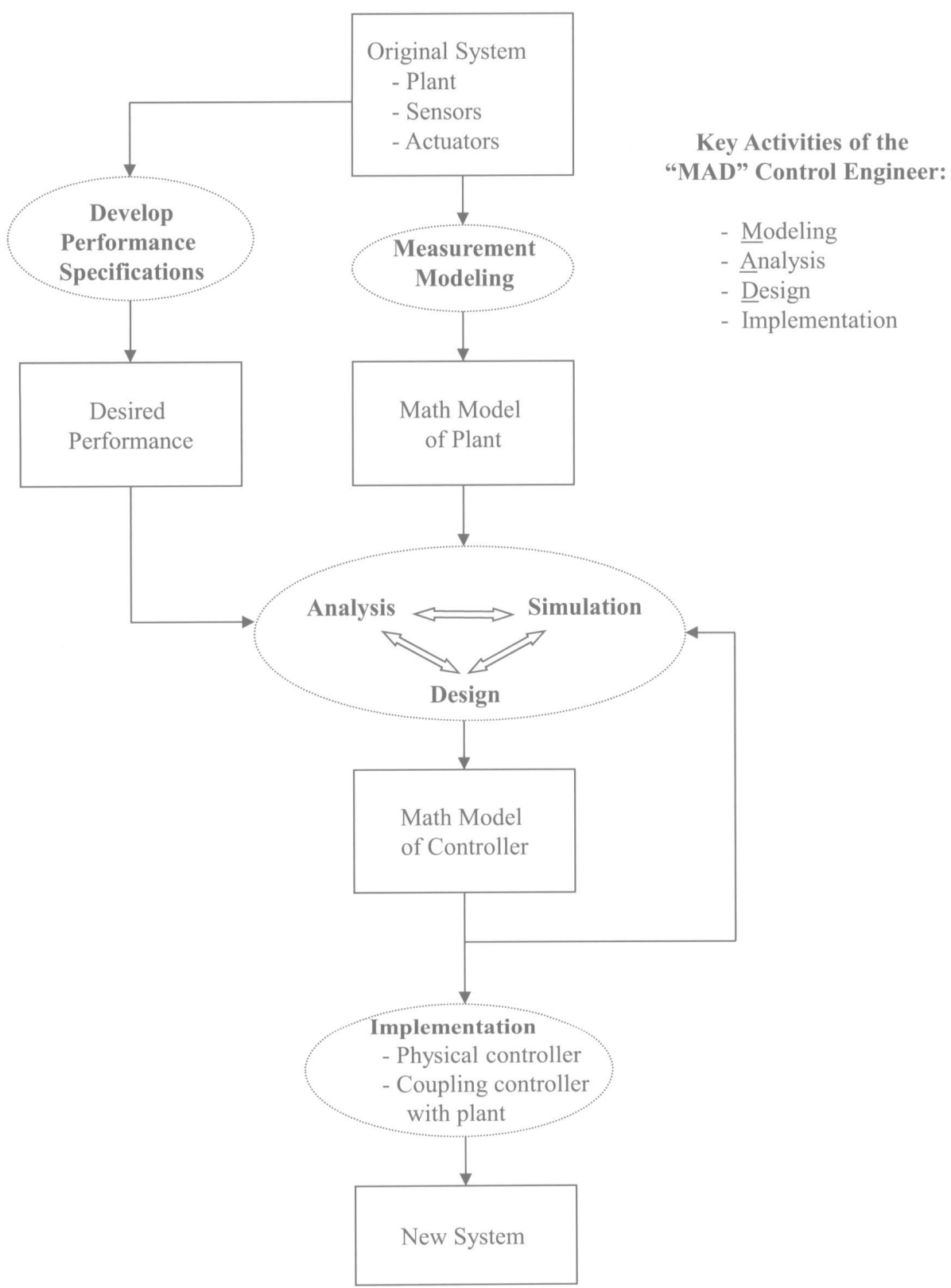


Fig. 0-1 Flowchart of the control system design process.





# Preface

Although the benefits from the wise use of control engineering such as improved product/life quality, minimized waste materials, reduced pollution, increased safety, reduced energy consumption etc. are widely recognized, as per Professor Karl J. Åström, “Control is a hidden technology.” To promote the appreciation of control subject, an interesting book “*Feedback and Control for Everyone*” (2010, Pedro Albertos and Iven Mareels, Springer) serves the purpose. Conventional control systems are mostly signal-based control systems which are the foundation to our modern life. While moving from signal-based control to information-based control systems, the basic ideas of control systems are ubiquitous even in this increasingly information-rich world.

This textbook is about signal-based control systems following the so-called “MAD (modeling, analysis and design) notion” suggested by Professor Kevin L. Moore of Colorado School of Mines in 1990s. To have a good control system, one needs to be asked on “What do you have?” and “What do you want?” Many iterations may be needed to design a controller with knowledge about plant model (“What do you have?”) and performance and constraints (“What do you want?”). Clearly, a computer-aided design (CAD) platform must be used. MATLAB and Simulink are considered as the dominant software platform for control system modeling, analysis and design, with numerous off-the-shelf toolboxes and blocksets dedicated to control systems and related topics. The major objective of this book is to provide first-hand information on how MATLAB/Simulink can be used in control system modeling, analysis, design as well as hardware-in-the-loop rapid prototyping. The main structure of the book is outlined as follows:

Foundation: Chapters 1, 2 and 3;

Modeling: Chapters 4, 6, 11

Analysis: Chapters 5, 6, 11

Design: Chapters 7, 8, 9, 10, 11

Rapid Prototyping: Chapter 12

The inclusion of Chapter 12 is on purpose since in industry, MATLAB/Simulink is widely used for control system MAD and even deployment via the hardware-in-the-loop real time simulation and targeting onto various microprocessors. Another

distinguishing feature is the inclusion of a dedicated PID control design chapter (Chapter 8) with various tuning methods introduced with the dedicated handy tools (e.g., OCD, PID\_Tune and OptimPID) developed for this book. Chapters 10 and 11 are both interesting and useful with emerging topics for potential further research and development. Chapter 10 presented concise introduction to some major adaptive and intelligent control system design methods with illustrative design examples. Chapter 11 serves as a survival guide to the analysis and design of fractional-order systems governed by fractional-order differential equations with differentiation or integration of non-integer orders. Extensive illustrative examples are presented in cartoon style so the readers can reproduce the results and gain hands-on working experience on fractional-order control systems analysis and design. This is a nice feature of this book consistently seen in each section with smooth mixture of MATLAB scripts and figures. These scripts are carefully designed so the readers can mimic and even reuse the codes in their own future work.

This book can be used as a reference text in the introductory control course for undergraduates in all engineering schools. The coverage of topics is broad, yet balanced, and should provide a solid foundation for the subsequent control engineering practice in both industry and research institutes. For graduates and researchers not majoring in control, this textbook is useful for knowledge enhancement. The authors also believe that this book will be a good desktop reference for control engineers and many codes and tools in this book may be directly applicable in real world problem solving.

The first version of this textbook, entitled “*Computer-aided design of control systems — MATLAB and its Applications*”, was published by the first author in Chinese by Tsinghua University Press, Beijing, China in 1996. It was the earliest textbook on CADCS (computer aided design of control systems) in MATLAB in China and together with its several later editions, it has been among the most popular textbooks on control systems in China with more than 60,000 copies sold. This new English edition has leveraged all welcoming aspects of the past Chinese editions in terms of the presentation style that has been optimized for self-learning as well as classroom teaching. Most importantly, we followed the MADCS (modeling, analysis and design of control systems) notion and organized the contents in the MADCS way as outlined above. The enhancement in contents includes the intelligent control, fractional-order control as well as rapid prototyping of real-time control systems etc.

This textbook has a book companion website (<http://mechatronics.ucmerced.edu/MADbook>) which contains downloadable resources such as teaching slide set with over 1,000 PPTs, solution manual (for instructors only), all codes/scripts for reproducing the figure/results in this textbook, as well as several useful MATLAB tools developed exclusively for this textbook.

We would like to thank Professor Kevin L. Moore for preparing a foreword for

this book sharing his further insights in MAD.

During the evolution of this book, many researchers, professors, and students have provided useful feedback, comments, and inputs. In particular, we thank the following professors: Xinhe Xu, Xingquan Ren, Derek P. Atherton, Yuanwei Jing, Feng Pan, Dali Chen, Igor Podlubny, Blas M. Vinagre. The writing of the materials on fractional-order systems is partly supported by the National Natural Science Foundation of China under grant number 61174145. The “Book Program” from MathWorks Inc., is acknowledged for the latest MATLAB software access.

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