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mathematical theory of production planning

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Mathematical Theory of Production Planning

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EXTENSIVE DESCRIPTION OF THE BOOK

This book proposes a unified mathematical treatment of production planning and production smoothing problems, in the framework of optimal control theory. General concave and convex cost models which relate most closely to real life applications are considered. Planning horizon results are always central to the discussion and developments. They allow to consider only finite horizon problems, and guarantee that the production plan implemented over the first periods is optimal with regard to any demand pattern beyond the planning horizon. Algorithms are proposed to compute the optimal production policy, together with the corresponding softwares designed to be implemented on any microcomputer.

The book is organized in seven chapters and one mathematical appendix. The following topics are covered.

Demand is deterministic in the first three chapters and stochastic in chapter 4. The formulation is in discrete time except in chapter 3 where time is continuous. In chapter I we consider production planning with concave production and inventory costs. Chapter II considers the case of convex cost production planning and production smoothing models. In chapter III we extend the models developed in chapters I and II to a continuous time formulation. Two alternative techniques are used : continuous control and impulse control. In chapter IV we address the problem of production planning and production smoothing in a stochastic environment. In chapters V to VII we propose different softwares corresponding to the previous theoretical developments.

Finally a mathematical appendix provides the reader with all the necessary background in order to make the book self-content.

The last section of the first four chapters comments our results, and relates them to the relevant literature. They can be read as an initial motivation, independently of the mathematical derivations.

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INTRODUCTION

PRODUCTION PLANNING IN PERSPECTIVE

Production planning is primarily concerned with the adaptation, or more exactly the tuning of the firm's industrial resources, in order to meet demand for its final products. The production manager has the responsibility to implement the production plan, once it has been agreed upon by the "planning committee". The production manager must then take all decisions to make sure that the necessary capacity and qualified workforce, the required materials and components will be available in right quantity, at the right place, at the right time ; the objective is to fulfil a marketing plan at minimum overall cost.

Production planning becomes a challenging problem for at least three important reasons :

1 - Demand and costs vary over time, usually according to a seasonal pattern. This calls for some adjustment in the production capacity and the use of it. Moreover, it is difficult to precisely forecast demand at the most detailed level of the end products. At regular intervals the marketing plan must be revised, and as a consequence the production plan also.

2 - There is in the firm less and less flexibility to modify the operating conditions. Rigidities are at a peak with regard to labor management. At least in some countries, any change in the size of the labor force, resort to overtime and temporary manpower must be negociated with the unions, and the (regional bureau of the) labor department. Moreover, too many frequent changes in operating conditions may deteriorate the workers' morale and affect productivity.

3 - There are long and uncertain technical delays in obtaining industrial resources. These are : delays to install new machines, to train new workers, to negociate subcontracting capacity, to deliver materials and components from suppliers. These long delays are often the consequence of the rigidities borne by the other firms. In addition, suppliers are ready to allow clients substantial rebates if they are able to sign yearly blanket orders with some indication of demand distribution, for some families of products. Usually, detailed supply schedules are only notified on short notice. A longer forecast horizon gives the suppliers the opportunity to plan more efficiently their production, and to pass on their clients part of the cost reduction so obtained.

There is therefore an apparent contradiction between :

- the need for more flexibility and the necessity to quickly adjust the production plan with respect to changes which have occured in the commercial objectives,

- and, the relatively fixed production capacity available in the short run, with strong pressure for long term commitments concerning material procurement, work force and subcontracting.

The solution to overcome this conflict is more planning with an adequate planning horizon which, in any rate, must be greater than the seasonal cycle and the production lead time. This medium range horizon is usually one year, but may be longer up to two years for heavy industries where the production lead time is in the order of one year. However, it is practically impossible on this medium range horizon to forecast demand at the most detailed level, neither to plan precisely the industrial resources.

As a consequence, firms must adopt a two stage production planning process : production planning is first conducted at the aggregate level over the medium range horizon,and mainly concerns capacity and procurement planning in the aggregate ; then, in the short run detailed plans, named production schedules, are released. Only these detailed plans can be implemented. They are derived from short run forecasts and must satisfy the constraints imposed by the aggregate plans. We shall elaborate on the production planning process in the first section.

Obviously, in a steady state environment where not only demand, but also the cost structure would not change with time, the aggregate planning stage could be entirely bypassed . Capacity and procurement planning would be made once for all, and the only problem left to the production manager would be the implementation of the detailed production schedules, and short run capacity adjustments due to day to day operational aleas like machine break-downs, late deliveries of materials, absenteeism , ... Real life industrial environments are unfortunately more intricate than this ideal situation.

Over the medium range horizon,capital resources like expensive machines, which determine the base plant size, are assumed to be fixed. The delay in installing new pieces of equipment is generally longer than this horizon. Paradoxically this does not imply that production capacity is fixed. There is still the possibility to regulate, or smooth, capacity through the combined change of flexible resources which are complementary to capital equipments. In practice the capacity slack is quite important and production planning encompasses a wide range of decisions. The following represents the main alternatives opened to a production manager to smooth capacity (see Buffa and Taubert [1972]) :

- change the size of the work force through hiring and firing of workers ;
- use overtime in peak demand periods and idle time when demand is low, to vary the production rate while maintaining the work force level constant ;
- absorb excess demand through outside subcontracting ;
- build seasonal inventories during periods of slack activity in anticipation of higher demand in a near future ;
- resort either to planned backlogs in peak demand periods whenever customers are willing to accept a longer delivery lead time, or to lost sales otherwise.

The next two decisions are still relevant to production smoothing but relate to a higher level of management :

- adopt a pricing policy combined with advertising campains in order to induce customers to shift their buying decision from peak to off season ;
- adopt a mix of product lines which use the same resources (machines

and labor skills) but with counter seasonal demand cycles.

However, internal and environmental constraints may limit, or even forbid the use of some of these practices. For a more elaborate discussion of these smoothing techniques and examples, see Crouhy [1983].

The optimal combination of these smoothing techniques involves the search for the minimum cost tradeoff inside the following cost structure :

- Regular time costs associated with operations under normal conditions and which include direct and overhead costs. We can further split direct costs into fixed (set-up costs for example) and variable costs.

- Overtime and undertime penalty costs which consists of a wage rate premium for overtime work, and costs associated with maintaining an idle work force in the other case.

- Subcontracting costs which correspond to a premium over production costs in regular time, the expenses associated with monitoring subcontracted production, and the cost of duplicating some tools.

- Costs of changing the production rate which are associated with hiring, training or laying off workers. There are also costs of reorganizing production in the workshops like new line balancing of assembly lines, machine set-up, ... and the opportunity losses during the transition period due to quality problems and adjustment to new operating conditions.

- Inventory costs which include :

. holding costs : financing, storage, insurance, depreciation,...
. shortage costs due to backlogged demand and lost sales, which include extra cost of expediting late orders, loss of consumer goodwill, lost profits, ...

A detailed discussion of the nature and the structure of these costs may be found in McGarrah [1963] ; see also Holt et al. [1960, Chapters 2 and 3] for quadratic approximations of these cost elements.

I - THE PRODUCTION PLANNING PROCESS

From our introductory comments, it is now clear that a production plan cannot be implemented efficiently with a short sight view. It forces the production manager to plan ahead of time simply because manufacturing resources are not fully flexible, and there is an incompressible procurement and manufacturing lead time.

Moreover,the optimal economic tradeoff among the smoothing techniques can only be reached over a minimum length period, given the seasonal demand pattern for the final products. It is called the medium range planning horizon, and as we mentioned ealier it varies between one and two years. Usually it is one year which corresponds to the budget cycle.Since the plans are reviewed periodically, say monthly on a rolling horizon basis, it is important to avoid discountinuities in the planned production decisions for the last months of the horizon, at each update. Therefore demand is forecasted over a forecast horizon, longer than the planning horizon, usually 1.5 times the length of the planning horizon.

Over this forecast horizon it is practically impossible to forecast demand at the level of the detailed end product references, which may vary from one another just by the size, color, packaging, or some optional accessories.

For this basic reason a production plan can only be elaborated at the <u>aggregate</u>

level of product families, where a family is a group of items which share common manufacturing resources (machines and labor skills). Each item corresponds to the most detailed end product reference. Moreover demand forecasted at the family level should be a good approximation of the sum of the ex-post realized demands at the item level. Experience shows that aggregate forecasts are rather accurate.

Only a small number of aggregate plans need to be made for a wide range of items. In some industries like hosiery,farming equipments, electronic ... the aggregation ratio is in the order of one family for one hundred items.

With a medium range view of the firm's activity,not only item demand must be aggregated in family demand, but also machines in load centers, labor skills in manpower classes, materials and components in procurement groups, and so on. The aggregation process may differ from one industry to another and should always be ad hoc to the precise environment of the firm.

Hax and Meal [1975] consider a third level of product aggregation which they call "type",where each type includes families of items which have the same seasonal demand pattern and the same production rate. In such instance aggregate plans are developed at the type level. However, in most industrial cases two levels of aggregation are enough to solve the aggregate planning problem.

Detailed production plans at the item level are provided only in the short run,up to an horizon of one to six months, for which detailed forecasts are made possible. Detailed schedules may proceed from individual forecasts at the item level but they must fit in the aggregate constraints which concern production rates, workforce levels, overtime and idle time, supply of componentsand materials. Rationing or inventory building may occur depending whether the sum of the detailed requirements is higher or smaller than the aggregate forecast. Figure 1 summarizes the articulation between both planning levels (see also Meal [1978]).

Both aggregate and detailed plans are revised on a rolling horizon basis, monthly for the first ones, weakly for the second ones, from updated information on demand, capacity, costs,suppliers lead time, ... Because of the uncertainties on these data,only the first periods (frozen horizon) of the plans are implemented. The length of the frozen horizon is dictated by the procurement and manufacturing lead time.

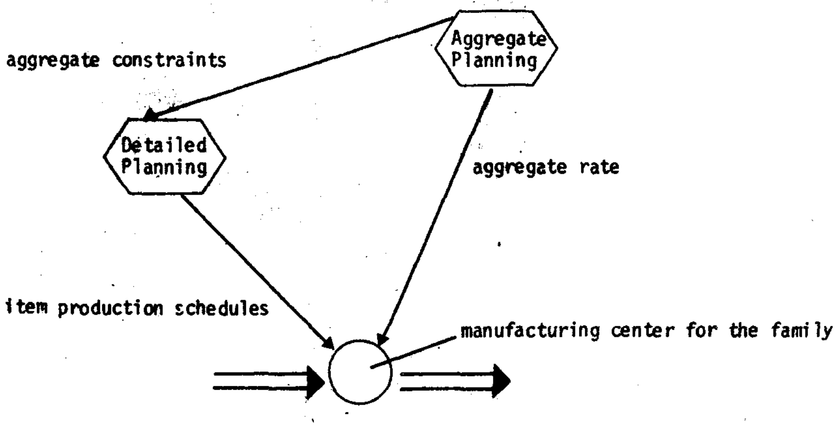

FIGURE 1 : Consistency between aggregate and detailed planning

To sum up, aggregate planning must be viewed as capacity and procurement planning in the aggregate. Total capacity is allocated among all the product families. Smoothing techniques are used to find the minimum total production cost at the aggregate level. Aggregate plans determine for each product family and each month over the entire planning horizon :

- the production rates,
- the work force levels,
- the number of worked hours (idle time and overtime),
- seasonal inventories and stockouts,
- the quantities to be delivered of materials and components by supply groups.

On the other hand, these aggregate outputs are aggregate constraints which must be satisfied by the short run detailed production schedules.

In addition, aggregate planning corresponds to the operational phase of the budgeting process, which deals with physical quantities. Budgets are obtained simply by direct valuation of aggregate plans.

II - MAIN SCOPE OF THE BOOK

There is an extensive literature on aggregate production planning models which starts in the mid-1950's with the pioneering works of Holt, Modigliani, Muth and Simon (HMMS) [1960].

Roughly there are two broad classes of models depending on the nature of the cost structure involved :

1 - Production planning models which include only two cost elements : a production cost $c_t(v_t)$ where v_t denotes the production level in period t, and an inventory cost $f_t(y_t)$ where y_t denotes the inventory level at the beginning of period t ; $f_t(y_t)$ refers to a holding cost or a shortage cost whether y_t is positive or negative. Obviously production planning models are just lot sizing models considered in a production instead of a distribution (or procurement) environment.

Analytical techniques and properties of the optimal production policy differ whether the cost structure is concave or convex, i.e. describes a production process with economies or diseconomies of scale. Concave cost functions allow for set-up costs which are ubiquitous in production, and constitute key elements in the short run production scheduling decision ; however,they are often neglected at the aggregate planning level when we deal with product families. The concave cost litterature relates to the seminal work by Wagner and Within [1959] , while the convext cost literature follows the initial contribution of Modigliani and Hohn [1955].

Surveys of the major contributions and expositions of the techniques in inventory theory are proposed by Scarf [1963] and Veinott [1966]. See also Johnson and Montgomery [1974].

2 - Production smoothing models which in addition to production and inventory costs consider adjustment (or smoothing) costs usually associated with a change in the production level from one period to the next : $g_t(v_{t-1},v_t)$. They may possibly be directly related to changes in the work force level,when the number of workers is an explicit decision variable in the model formulation. These adjustment costs are mostly hiring and training expenses, set-up charges for additional equipment when production is increased ; firing costs, and overheads for equipments used below normal capacity, when production is reduced.

General reviews of the contributions in this area are due to Silver [1967], Buffa and Taubert [1972], Johnson and Montgomery [1974], Peterson and Silver [1979] and Hax [1979], the latter being the most comprehensive.

Table 1 drawn from Kleindorfer et al. [1975] sketches the major classes of assumptions and the most common alternatives incorporated in production planning and production smoothing models. Mutually exclusive alternatives are bracketed within each class.

The purpose of this book is to extend the single product models, and to consider the most general versions workable in the framework of optimal control theory. Only general concave and convex cost models, which relate most closely to real life applications, are analyzed. Ad hoc formulations which, for computational purpose, have adopted linear costs or even quadratic costs will not be discussed. They are simply special cases of more general formulations for which we are able to propose algorithms to compute the optimal production policy. We also provide the reader with the corresponding softwares which are easy to implement on any minicomputer.

Planning horizon results will be always central to our discussion and developments. Roughly H is said to be a "planning horizon" if for any problem horizon greater than H, the period 1 to H portion of the optimal production plan stays unaffected. Rigorous definitions will be given in Chapter I, section III.4. These results are of great value for practical purpose, since they allow to consider only finite horizon problems and guarantee the production manager, that the production plan implemented over the first periods (frozen horizon) is optimal with regard to any demand pattern beyond the planning horizon.

Since we are mainly interested in analytical techniques, we shall not comment on the various heuristic decision rules which have proliferated in the literature.

The book is organized in seven chapters and one mathematical appendix. The following topics are covered.

Demand is deterministic in the first three chapters and stochastic in chapter IV. The formulation is in discrete time except in chapter III where time is continuous. In chapter I we consider production planning with concave production and inventory costs. Chapter II considers the case of convex cost production planning and production smoothing models. In chapter III we extend the models developed in chapter I and II to a continuous time formulation. Two alternative techniques will be used : continuous control and impulse control. In chapter IV we address the problem of production planning and production smoothing in a stochastic environment. In chapters V to VII we propose different softwares corresponding to the previous theoretical developments. Finally a mathematical appendix provides the reader with all the necessary background in order to make the book self-content.

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