Ulrich Häussler-Combe

Computational Methods for Reinforced Concrete Structures

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Cover: The photo shows a part of the façade of the Pinakothek der Moderne, Munich. The grid indicates the subdivision of a complex structure into small simple elements or finite elements, respectively.

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Preface

This book grew out of lectures the author gives at the Technische Universität Dresden. These lectures are entitled "Computational Methods for Reinforced Concrete Structures" and "Design of Reinforced Concrete Structures." Reinforced concrete is a composite of concrete and reinforcement connected by bond. Bond is a key item for the behavior of the composite which utilizes compressive strength of concrete and tensile strength of reinforcement while leading to considerable multiple cracking. This makes reinforced concrete unique compared to other construction materials such as steel, wood, glass, masonry, plastic materials, fiber reinforced plastics, geomaterials, etc.

Numerical methods like the finite element method on the other hand disclose a way for a realistic computation of the behavior of structures. But the implementations generally present themselves as black boxes in the view of users. Input is fed in and the output has to be trusted. The assumptions and methods in between are not transparent. This book aims to establish transparency with special attention for the unique properties of reinforced concrete structures. Appropriate approaches will be discussed with their potentials and limitations while integrating them in the larger framework of computational mechancis and connecting aspects of numerical mathematics, mechanics, and reinforced concrete.

This is a wide field and the scope has to be limited. The focus will be on the behavior of whole structural elements and structures and not on local problems like tracking single cracks or mesoscale phenomena. Basics of multiaxial material laws for concrete will be treated but advanced theories for multiaxial concrete behavior are not a major subject of this book. Such theories are still a field of ongoing research which by far seems not to be exhausted up to date.

The book aims at advanced students of civil and mechanical engineering, academic teachers, designing and supervising engineers involved in complex problems of reinforced concrete, and researchers and software developers interested in the broad picture. Chapter 1 describes basics of modeling and discretization with finite element methods and solution methods for nonlinear problems insofar as is required for the particular methods applied to reinforced concrete structures. Chapter 2 treats uniaxial behavior of concrete and its combination with reinforcement while discussing mechanisms of bond and cracking. This leads to the model of the reinforced tension bar which provides the basic understanding of reinforced concrete mechanisms. Uniaxial behavior is also assumed for beams and frames under bending, normal forces and shear which is described in Chapter 3. Aspects of prestressing, dynamics and second-order effects are also treated in this chapter. Chapter 4 deals with strut-and-tie models whereby still a uniaxial material behavior is assumed. This chapter also refers to rigid plasticity and limit theorems.

Modeling of multiaxial material behavior within the framework of macroscopic continuum mechanics is treated in Chapter 5. The concepts of plasticity and damage are described with simple specifications for concrete. Multiaxial cracking is integrated within the model of continuous materials. Aspects of strain softening are treated leading to concepts of regularization to preserve the objectivity of discretizations. A bridge from microscopic behavior to macroscopic material modeling is given with a sketch of the microplane theory. Chapter 6 treats biaxial states of stress and strain as they arise with plates or deep beams. Reinforcement design is described based on linear elastic plate analysis and the lower bound limit theorem. While the former neglects kinematic compatibility, this is involved again with biaxial specifications of multiaxial stress–strain relations including crack modeling.

Slabs are described as the other type of plane surface structures in Chapter 7. But in contrast to plates their behavior is predominantly characterized by internal forces like bending moments. Thus, an adaption of reinforcement design based on linear elastic analysis and the lower bound limit theorem is developed. Kinematic compatibility is again brought into play with nonlinear moment–curvature relations. Shell structures are treated in Chapter 8. A continuum-based approach with kinematic constraints is followed to derive internal forces from multiaxial stress–strain relations suitable for reinforced cracked concrete. The analysis of surface structures is closed in this chapter with the plastic analysis of simple slabs based on the upper bound limit theorem. Chapter 9 gives an overview about uncertainty and in particular about the determination of the failure probability of structures and safety factor concepts. Finally, the appendix adds more details about particular items completing the core of numerical methods for reinforced concrete structures.

Most of the described methods are complemented with examples computed with a software package developed by the author and coworkers using the PYTHON programming language.

- Programs and example data should be available under `www.concrete-fem.com`. More details are given in Appendix F.

These programs exclusively use the methods described in this book. Programs and methods are open for discussion with the disclosure of the source code and should give a stimulation for alternatives and further developments.

Thanks are given to the publisher Ernst & Sohn, Berlin, and in particular to Mrs. Claudia Ozimek for the engagement in supporting this work. My education in civil engineering, and my professional and academic career were guided by my academic teacher Prof. Dr.-Ing. Dr.-Ing. E.h. Dr. techn. h.c. Josef Eibl, former head of the department of Concrete Structures at the Institute of Concrete Structures and Building Materials at the Technische Hochschule Karlsruhe (nowadays KIT – Karlsruhe Institute of Technology), to whom I express my gratitude. Further thanks are given to former or current coworkers Patrik Pröchtel, Jens Hartig, Mirko Kitzig, Tino Kühn, Joachim Finzel and Jörg Weselek for their specific contributions. I appreciate the inspiring and collaborative environment of the Institute of Concrete Structures at the Technische Unversität Dresden. It is my pleasure to teach and research at this institution. And I have to express my deep gratitude to my wife Caroline for her love and patience.

Ulrich Häussler-Combe — Dresden, in spring 2014

Notations

The same symbols may have different meanings in some cases. But the different meanings are used in different contexts and misunderstandings should not arise.

General

		firstly used
$\bullet^T$	transpose of vector or matrix $\bullet$	Eq. (1.5)
$\bullet^{-1}$	inverse of quadratic matrix $\bullet$	Eq. (1.13)
$\delta\bullet$	virtual variation of $\bullet$, test function	Eq. (1.5)
$\delta\bullet$	solution increment of $\bullet$ within an iteration of nonlinear equation solving	Eq. (1.70)
$\tilde{\bullet}$	$\bullet$ transformed in (local) coordinate system	Eq. (5.15)
$\dot{\bullet}$	time derivative of $\bullet$	Eq. (1.4)

Normal lowercase italics

a_s	reinforcement cross section per unit width	Eq. (7.70)
b	cross-section width	Section 3.1.2
b_w	crack-band width	Section 2.1
d	structural height	Section 7.6.2
e	element index	Section 1.3
f	strength condition	Eq. (5.42)
f_c	uniaxial compressive strength of concrete (unsigned)	Section 2.1
f_{ct}	uniaxial tensile strength of concrete	Section 2.1
f_t	uniaxial failure stress – reinforcement	Section 2.3
f_{yk}	uniaxial yield stress – reinforcement	Section 2.3
f_E	probability density function of random variable E	Eq. (9.2)
g_f	specific crack energy per volume	Section 2.1
h	cross-section height	Section 3.1.2
m_x, m_y, m_{xy}	moments per unit width	Eq. (7.8)
n	total number of degrees of freedom in a discretized system	Section 1.2
n_E	total number of elements	Section 3.3.1
n_i	order of Gauss integration	Section 1.6
n_N	total number of nodes	Section 3.3.1
n_x, n_y, n_{xy}	normal forces per unit width	Eq. (7.8)
p	pressure	Eq. (5.8)
p_F	failure probability	Eq. (9.18)
$\bar{p}_x, \bar{p}_z$	distributed beam loads	Eq. (3.58)
r	local coordinate	Section 1.3
s	local coordinate	Section 1.3

s_{bf}	slip at residual bond strength	Section 2.4
$s_{b\,\mathrm{max}}$	slip at bond strength	Section 2.4
t	local coordinate	Section 1.3
t	time	Section 1.2
t_x, t_y, t_{xy}	couple force resultants per unit width	Eq. (7.67)
u	specific internal energy	Eq. (5.12)
v_x, v_y	shear forces per unit width	Eq. (7.8)
w	deflection	Eq. (1.56)
w	fictitious crack width	Eq. (2.4)
w_{cr}	critical crack width	Section 5.7.1
z	internal lever arm	Section 3.5.4

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$\mathbf{b}$	body forces	Section 1.2
$\mathbf{f}$	internal nodal forces	Section 1.2
$\mathbf{p}$	external nodal forces	Section 1.2
$\mathbf{n}$	normal vector	Eq. (5.5)
$\mathbf{t}$	surface traction	Section 1.2
$\mathbf{t}_c$	crack traction	Eq. (5.123)
$\mathbf{u}$	displacement field	Section 1.2
$\boldsymbol{v}$	nodal displacements	Section 1.2
$\mathbf{w}_c$	fictitious crack width vector	Eq. (5.122)

Normal uppercase italics

A	surface	Section 1.2, Eq. (1.5)
A	cross-sectional area of a bar or beam	Eq. (1.54)
A_s	cross-sectional area reinforcement	Example 2.4
A_t	surface with prescribed tractions	Section 1.2, Eq. (1.5)
A_u	surface with prescribed displacements	Eq. (1.53)
C	material stiffness coefficient	Eq. (2.32)
C_T	tangential material stiffness coefficient	Eq. (2.34)
D	scalar damage variable	Eq. (5.106)
D_T	tangential material compliance coefficient	Eq. (5.160)
D_{cT}	tangential compliance coefficient of cracked element	Eq. (5.132)
D_{cLT}	tangential compliance coefficient of crack band	Eq. (5.132)
E	Young's modulus	Eq. (1.43)
E_0	initial value of Young's modulus	Eq. (2.13)
E_c	initial value of Young's modulus of concrete	Section 2.1
E_s	initial Young's modulus of steel	Section 2.3
E_T	tangential modulus	Eq. (2.2)
F	yield function	Eq. (5.64)
F_E	distribution function of random variable E	Eq. (9.1)
G	shear modulus	Eq. (3.8)

G	flow function	Eq. (5.63)
G_f	specific crack energy per surface	Eq. (2.7)
I_1	first invariant of stress	Eq. (5.20)
J	determinant of Jacobian	Eq. (1.67)
J_2, J_3	second, third invariant of stress deviator	Eq. (5.20)
L_c	characteristic length of an element	Eq. (6.32)
L_e	length of bar or beam element	Section 1.3
M	bending moment	Section 3.1.2
N	normal force	Section 3.1.2
P	probability	Eq. (9.1)
T	natural period	Eq. (3.211)
V	shear force	Section 3.1.2
V	volume	Section 1.2, Eq. (1.5)

Bold uppercase roman

$\mathbf{B}$	matrix of spatial derivatives of shape functions	Section 1.2, Eq. (1.2)
$\mathbf{C}$	material stiffness matrix	Eq. (1.47)
$\mathbf{C}_T$	tangential material stiffness matrix	Eq. (1.50)
$\mathbf{D}$	material compliance matrix	Eq. (1.51)
$\mathbf{D}_T$	tangential material compliance matrix	Eq. (1.51)
$\mathbf{E}$	coordinate independent strain tensor	Eq. (8.15)
$\mathbf{G}_1, \mathbf{G}_2, \mathbf{G}_3$	unit vectors of covariant system	Eq. (8.16)
$\mathbf{G}^1, \mathbf{G}^2, \mathbf{G}^3$	unit vectors of contravariant system	Eq. (8.17)
$\mathbf{I}$	unit matrix	Eq. (1.85)
$\mathbf{J}$	Jacobian	Eq. (1.20)
$\mathbf{K}$	stiffness matrix	Eq. (1.11)
$\mathbf{K}_e$	element stiffness matrix	Eq. (1.61)
$\mathbf{K}_T$	tangential stiffness matrix	Eq. (1.66)
$\mathbf{K}_{Te}$	tangential element stiffness matrix	Eq. (1.65)
$\mathbf{M}$	mass matrix	Eq. (1.60)
$\mathbf{M}_e$	element mass matrix	Eq. (1.58)
$\mathbf{N}$	matrix of shape functions	Section 1.2, Eq. (1.1)
$\mathbf{Q}$	vector/tensor rotation matrix	Eq. (5.15)
$\mathbf{S}$	coordinate independent stress tensor	Eq. (8.24)
$\mathbf{T}$	element rotation matrix	Eq. (3.109)
$\mathbf{V}_n$	shell director	Section 8.1
$\mathbf{V}_\alpha, \mathbf{V}_\beta$	unit vectors of local shell system	Eq. (8.2)

Normal lowercase Greek

α	tie inclination	Eq. (3.157)
α_E, α_R	sensitivity parameters	Eq. (9.13)
α	coefficient for several other purposes	
β	shear retention factor	Eq. (5.137)
β	reliability index	Eq. (9.12)

β_t	tension stiffening coefficient	Section 2.7
ϵ	uniaxial strain	Section 1.4, Eq. (1.43)
ϵ	strain of a beam reference axis	Section 3.1.1, Eq. (3.4)
$\epsilon_1, \epsilon_2, \epsilon_3$	principal strains	Section 5.2.3
ϵ_{ct}	concrete strain at uniaxial tensile strength	Section 2.1
ϵ_{cu}	concrete failure strain at uniaxial tension	Eq. (5.152)
ϵ_{c1}	concrete strain at uniaxial compressive strength (signed)	Section 2.1
ϵ_{cu1}	concrete failure strain at uniaxial compression (signed)	Section 2.1
ϵ_I	imposed uniaxial strain	Section 2.2
ϵ_V	volumetric strain	Eq. (5.102)
ϕ	cross-section rotation	Eq. (3.1)
ϕ	angle of external friction	Eq. (5.91)
φ	angle of orientation	Section 6.1, Eq. (6.5)
φ	creep coefficient	Eq. (2.26)
φ_c	creep coefficient of concrete	Eq. (3.119)
γ	shear angle	Eq. (3.1)
γ_E, γ_R	partial safety factors	Eq. (9.44)
κ	curvature of a beam reference axis	Section 3.1.1, Eq. (3.4)
κ_p	state variable for plasticity	Section 5.5.1
κ_d	state variable for damage	Section 5.6
μ_E	mean of random variable E	Section 9.1
ν	Poisson's ratio	Eq. (1.44)
ν	coefficient of variation	Eq. (9.46)
θ	strut inclination	Eq. (3.148)
θ	deviatoric angle	Eq. (5.46)
ϑ	angle of internal friction	Eq. (5.89)
ρ	deviatoric length	Eq. (5.45)
ρ_s	reinforcement ratio	Eq. (6.8)
ϱ_s	specific mass	Eq. (1.52)
σ	uniaxial stress	Section 1.4, Eq. (1.43)
$\sigma_1, \sigma_2, \sigma_3$	principal stresses	Section 5.2.3
σ_E	standard deviation of random variable E	Section 9.1
τ	bond stress	Section 2.4, Eq. (2.44)
τ	time variable in time history	Section 2.2
τ_{bf}	residual bond strength	Section 2.4
$\tau_{b\max}$	bond strength	Section 2.4
ω	circular natural frequency	Eq. (3.211)
ξ	hydrostatic length	Eq. (5.44)

Bold lowercase Greek

$\boldsymbol{\epsilon}$	small strain	Section 1.2
$\boldsymbol{\epsilon}$	generalized strain	Eq. (1.33)
$\boldsymbol{\epsilon}_p$	plastic small strain	Eq. (5.61)

$\boldsymbol{\kappa}$	vector of internal state variables	Eq. (5.39)
$\boldsymbol{\sigma}$	Cauchy stress	Section 1.2
σ	generalized stress	Eq. (1.34)
$\boldsymbol{\sigma}'$	deviatoric part of Cauchy stress	Section 5.2.2

Normal uppercase Greek

Φ	standardized normal distribution function	Eq. (9.19)

Bold uppercase Greek

$\boldsymbol{\Sigma}$	viscous stress surplus	Eq. (1.76)

Contents

Chapter 1

Finite Elements Overview

1.1 Modeling Basics

"There are no exact answers. Just bad ones, good ones and better ones. Engineering is the art of approximation." Approximation is performed with models. We consider a reality of interest, e.g., a concrete beam. In a first view, it has *properties* such as dimensions, color, surface texture. From a view of structural analysis the latter ones are irrelevant. A more detailed inspection reveals a lot of more properties: composition, weight, strength, stiffness, temperatures, conductivities, capacities, and so on. From a structural point of view some of them are essential. We combine those essential properties to form a *conceptual model*. Whether a property is essential is obvious for some, but the valuation of others might be doubtful. We have to choose. By choosing properties our model becomes approximate compared to reality. Approximations are more or less accurate.

On one hand, we should reduce the number of properties of a model. Any reduction of properties will make a model less accurate. Nevertheless, it might remain a good model. On the other hand, an over-reduction of properties will make a model inaccurate and therefore useless. Maybe also properties are introduced which have no counterparts in the reality of interest. Conceptual modeling is the art of choosing properties. As all other arts it cannot be performed guided by strict rules.

The chosen properties have to be related to each other in quantitative manner. This leads to a *mathematical model*. In many cases, we have systems of differential equations relating variable properties or simply *variables*. After prescribing appropriate boundary and initial conditions an exact, unique solution should exist for variables depending on spatial coordinates and time. Thus, a particular variable forms a field. Such fields of variables are infinite as space and time are infinite.

As analytical solutions are not available in many cases, a discretization is performed to obtain approximate numerical solutions. *Discretization* reduces underlying infinite space and time into a finite number of supporting points in space and time and maps differential equations into algebraic equations relating a finite number of variables. This leads to a *numerical model*.

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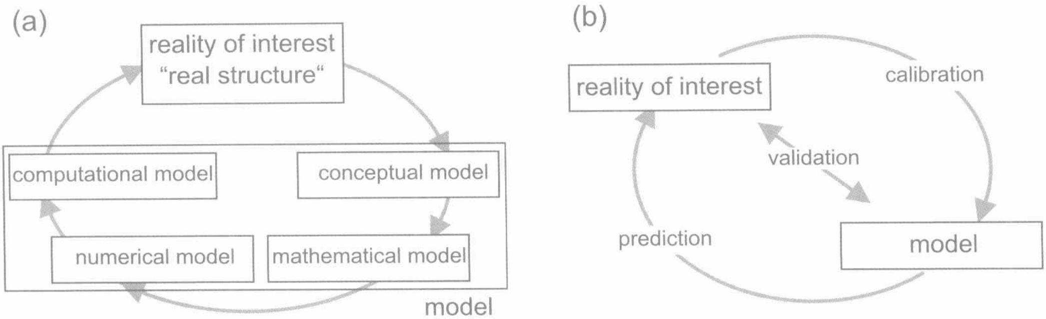

Figure 1.1: Modeling (a) Type of models following [83]. (b) Relations between model and reality.

A numerical model needs some completion as it has to be described by means of programming to form a *computational model*. Finally, programs yield solutions through processing by computers. The whole cycle is shown in Fig. 1.1. Sometimes it is appropriate to merge the sophisticated sequence of models into the *model*.

A final solution provided after computer processing is approximate compared to the exact solution of the underlying mathematical model. This is caused by discretization and round-off errors. Let us assume that we can minimize this mathematical approximation error in some sense and consider the final solution as a *model solution*. Nevertheless, the relation between the model solution and the underlying reality of interest is basically an issue. Both – model and reality of interest – share the same properties by definition or conceptual modeling, respectively. Let us also assume that the real data of properties can be objectively determined, e.g., by measurements.

Thus, real data of properties should be properly approximated by their computed model counterparts for a problem under consideration. The difference between model solution data and real data yields a *modeling error*. In order to distinguish between bad (inaccurate), good (accurate), and better model solutions, we have to choose a reference for the modeling error. This choice has to be done within a larger context, allows for discretion and again is not guided by strict rules like other arts. Furthermore, the reference may shift while getting better model solutions during testing.

A bad model solution may be caused by a bad model – bad choice of properties, poor relations of properties, insufficient discretization, programming errors – or by incorrect model parameters. *Parameters* are those properties which are assumed to be known in advance for a particular problem and are not object to a computation. Under the assumption of a good model, the model parameters can be corrected by a *calibration*. This is based upon appropriate problems from the reality of interest with the known real data. On one hand calibration minimizes the modeling error by adjusting of parameters. On the other hand, *validation* chooses other problems with known real data and assesses the modeling error without adjusting of parameters. Hopefully model solutions are still good.

Regarding reinforced concrete structures, calibrations usually involve the adaption of material parameters like strength and stiffness as part of *material models*. These parameters are chosen such that the behavior of material specimen observed in experiments is reproduced. A validation is usually performed with structural elements such as bars, beams, plates, and slabs. Computational results of *structural models* are compared with the corresponding experimental data.

This leads to basic peculiarities. Reproducible experiments performed with structural elements are of a small simplified format compared with complex unique buildings. Furthermore, repeated experimental tests with the same nominal parameters exhibit scattering results. Standardized *benchmark tests* carving out different aspects of reinforced concrete behavior are required. Actually a common agreement about such benchmark tests exists only in the first attempts. Regarding a particular problem a corresponding model has to be validated on a case-by-case strategy using adequate experimental investigations. Their choice again has no strict rules as the preceding arts.

Complex proceedings have been sketched hitherto outlining a model of modeling. Some benefit is desirable finally. Thus, a model which passed validations is usable for *predictions*. Structures created along such predictions hopefully prove their worth in the reality of interest.

This textbook covers the range of conceptual models, mathematical models, and numerical models with special attention to reinforced concrete structures. Notes regarding the computational model including available programs and example data are given in Appendix F. A major aspect of the following is modeling of *ultimate limit states*: states with maximum bearable loading or acceptable deformations and displacements in relation to failure. Another aspect is given with *serviceability*: Deformations and in some cases oscillations of structures have to be limited to allow their proper usage and fulfillment of intended services. *Durability* is a third important aspect for building structures: deterioration of materials through, e.g., corrosion, has to be controlled. This is strongly connected to cracking and crack width in the case of reinforced concrete structures. Both topics are also treated in the following.

1.2 Discretization Outline

The finite element method (FEM) is a predominant method to derive numerical models from mathematical models. Its basic theory is described in the remaining sections of this chapter insofar as it is needed for its application to different types of structures with reinforced concrete in the following chapters.

The underlying mathematical model is defined in one-, two-, or three-dimensional fields of space related to a *body* and one-dimensional space of time. A body undergoes deformations during time due to loading. We consider a simple example with a plate defined in 2D space, see Fig. 1.2. Loading is generally defined depending on time whereby time may be replaced by a loading factor in the case of quasistatic problems. Field variables depending on spatial coordinates and time are, e.g., given by the displacements.

- Such fields are discretized by dividing space into *elements* which are connected by *nodes*, see Fig. 1.3a. Elements adjoin but do not overlap and fill out the space of the body under consideration.

- Discretization basically means *interpolation*,, i.e., displacements within an element are interpolated using the values at nodes belonging to the particular element.