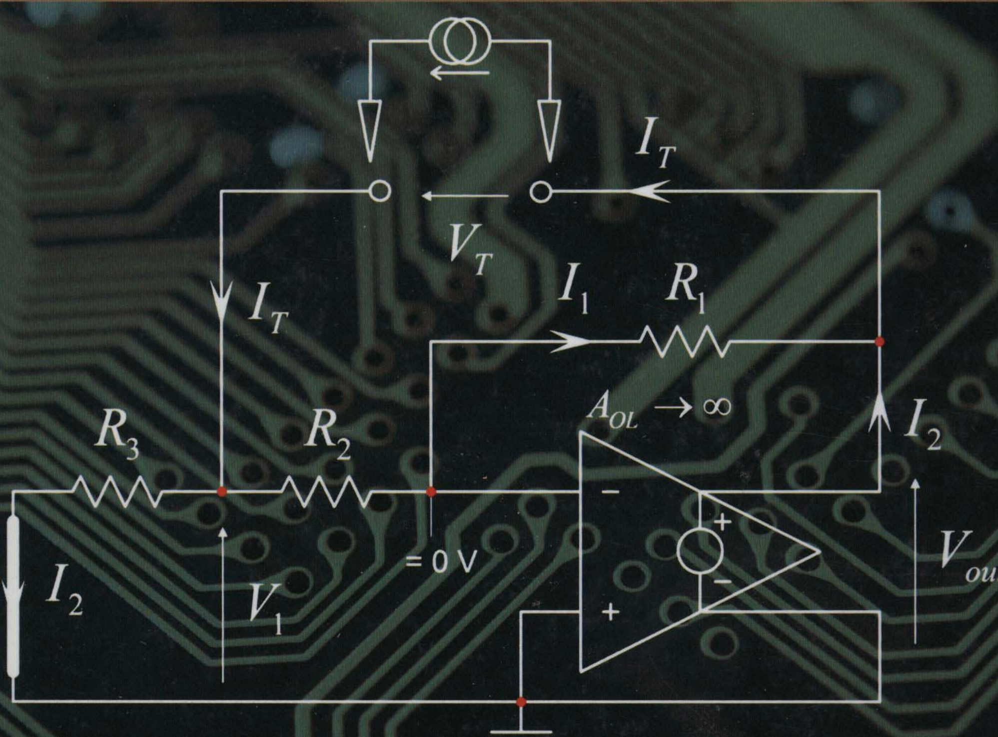
LINEAR CIRCUIT TRANSFER FUNCTIONS

AN INTRODUCTION TO FAST ANALYTICAL TECHNIQUES



CHRISTOPHE P. BASSO

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About the Author

Christophe Basso is a Technical Fellow at ON Semiconductor in Toulouse, France, where he leads an application team dedicated to developing new offline PWM controller's specifications. He has originated numerous integrated circuits among which the NCP120X series has set new standards for low standby power converters.

Further to his 2008 book *Switch-Mode Power Supplies: SPICE Simulations and Practical Designs*, published by McGraw-Hill, he released a new title in 2012 with Artech House, *Designing Control Loops for Linear and Switching Power Supplies: a Tutorial Guide*. He holds 17 patents on power conversion and often publishes papers in conferences and trade magazines including How2Power and PET.

Christophe has over 20 years of power supply industry experience. Prior to joining ON Semiconductor in 1999, Christophe was an application engineer at Motorola Semiconductor in Toulouse. Before 1997, he worked at the European Synchrotron Radiation Facility in Grenoble, France, for 10 years. He holds a BSEE equivalent from the Montpellier University (France) and a MSEE from the Institut National Polytechnique of Toulouse (France). He is an IEEE Senior member.

When he is not writing, Christophe enjoys snowshoeing in the Pyrenees.

Preface

First as a student and later as an engineer, I have always been involved in the calculation of transfer functions. When designing power electronics circuits and switch mode power supplies, I had to apply my analytical skills on passive filters. I also had to linearize active networks when I needed the control-to-output dynamic response of my converter. Methods to determine transfer functions abounded and there are numerous textbooks on the subject. I started in college with mesh-node analysis, and at some point ended up using state variables. If all paths led to the correct result, I often struggled rearranging equations to make them fit a friendly format. Matrices were useful for immediate numerical results but, when trying to extract a meaningful symbolic transfer function, I was often stuck with an intractable result. What matters with a transfer function formula is that you can immediately distinguish poles, zeros and gains without having to rework the expression. This is the idea behind the term *low-entropy*, a concept forged by Dr. Middlebrook.

Simulation gives you an idea where poles and zeros hide by interpreting the phase and magnitude plots with minimum-phase functions. However, inferring which terms really affect a pole or a zero position from a Bode plot is a different story. Fortunately, if the transfer function is written the right way, then you can immediately identify which elements contribute to the roots and assess how they impact the dynamic response. As some of these parasitics vary in production or drift with temperature, you have to counteract their effects so that reliability is preserved during the circuit's life. The typical example is when you are asked to assess the impact of a parasitic term variation on a product you have designed: if a new capacitor or a less expensive inductor is selected by the buyers, will production be affected? Is there a chance that stability will be jeopardized in some operating conditions? Implementing the classical analysis method will surely deliver a result describing the considered circuit, but extracting the information you need from the final expression is unlikely to happen if the equations you have are disorganized or in a *high-entropy* form.

This is where Fast Analytical Circuit Techniques (FACTs) come into play. The acronym was formed by Dr. Vatché Vorpérian, who formalized the technique you are about to discover here. Before him, Dr. Middlebrook published numerous papers and lectured on his Extra-Element Theorem (EET), later generalized to the *N* extra-element theorem by one of his alumni. Since Hendrik Bode in the 40's, authors have come up with techniques aiming to simplify linear circuit analysis through various approaches. All of them were geared towards determining the transfer function at a pace quicker than what traditional methods could provide. Unfortunately, while traveling and visiting customers world-wide, I have found that, despite all the available documentation, FACTs were rarely adopted by engineers or students. When describing examples in my seminars and showing the method at work in small-signal analysis, I could sense interest from the audience through questions and comments. However, during the discussions I had later on with some of the engineers or students, they confessed that they tried to acquire the skill but gave up because of the intimidating mathematical formalism and the complexity of the examples. If one needs to be rigorous when tackling electrical analysis, perhaps a different approach and pace could make people feel at ease when learning the method. This is what I strived to do with this new book, modestly shedding a different light on the subject by progressing with simple-to-understand examples and clear explanations. As a student, I too struggled to apply these fast analytical circuits techniques to real-world problems; as such, I identified the obstacles and worked around them with success. Thus, the seeds for this book were sown.

This book consists of five chapters. The first chapter is a general introduction to the technique, explaining what transfer functions are and how time constants characterize a circuit. The second chapter digs into transfer function definitions and polynomial forms, introducing the low-Q approximation, and how to organize 2^{nd} and 3^{rd}-order denominators or numerators. The third chapter uses the superposition theorem to gently introduce the extra-element theorem. Numerous examples are given to illustrate its usage in different 1^{st}-order configurations. The fourth chapter deals with the 2-extra element theorem, generalized and applied to 2^{nd}-order networks. Numerous examples illustrated with Mathcad® and SPICE punctuate the explanations. Finally, the fifth chapter tackles 3^{rd}- and 4^{th}-order circuits, all illustrated with examples. Each chapter ends with 10 fully documented problems. There is no secret; mastering a technique requires patience and practice, and I encourage you to test what you have learned after each chapter through these problems.

I have adopted the same casual writing style already used in my previous books, as readers' comments show that the way I present things better explains complex matters. Please let me know if my approach still applies here and if you enjoy reading this new book. As usual, feel free to send me your comments or any typos you may find at cbasso@wanadoo.fr. I will maintain an errata list in my personal webpage as I did for the previous books (http://cbasso.pagesperso-orange.fr/Spice.htm). Thank you, and have fun determining transfer functions!

Christophe Basso
May 2015

Acknowledgement

A book like this one could not have been written and published without the help of many contributing friends. My warmest thanks and love first go to my sweet wife Anne who endured my ups and downs when determining some of the book transfer functions: equations time is over and we can now enjoy the long and warm evenings of summer to come!

I was fortunate to share my work with my ON Semiconductor colleagues and friends who played a crucial role in reviewing my pages and challenging the method. Stéphanie Cannenterre reviewed and practiced numerous book exercises. She now masters the method: well done! Dr. José Capilla raced with me several times to determine a transfer function with his Driving Point Impedance method and I recognize his skills in doing so. Special thanks go to my friend Joël Turchi with whom I spent endless hours debating the method or discussing the validity of an equation. Merci Joël for your kindness and invaluable support for this book!

Two people did also accompany me from the beginning of the writing process. Mon ami Canadien Alain Laprade from ON Semiconductor in East Greenwich who developed an addicted relationship to the FACTs and kindly reviewed all my work. Monsieur Feucht from Innovatia did also a tremendous work in correcting my pages but also kindly polished my English. I am not exactly a novelist and cannot hide my French origins Dennis!

I want to warmly thank the following reviewers for their kind help in reading my pages during the 2015 summer: Frank Wiedmann (Rhode & Schwarz), Thierry Bordignon, Doug Osterhout (both are with ON Semiconductor), Tomas Gubek – děkuji! (FEI), Didier Balocco (Fairchild), Jochen Verbrugghe, Bart Moeneclaey (both are with Ghent University), Bruno Allard (INSA Lyon), Vatché Vorpérian (JPL), Luc Lasne (Bordeaux University) and Garrett Neaves (Freescale Semiconductor).

Last but not least, I would like to thank Peter Mitchell at Wiley & Sons UK for giving me the opportunity to publish my work.

Contents

1

Electrical Analysis – Terminology and Theorems

This first chapter is an introduction to some of the basic definitions and terms you must understand in order to perform electrical analysis with efficiency and speed. By electrical analysis, I imply finding the various relationships that characterize a particular electrical network. To excel in this field, as in any job, you need to master a few tools. Obviously, they are innumerable and I am sure you have learned a plethora of theorems during your student life. Some names now seem distant simply because you never had a chance to exercise them. Or you actually did but implementation was so obscure and complex that you left quite a few of them aside. This situation often happens in an engineer's life where real-case experience helps clean up what you have learned at school to only retain techniques that worked well for you. Sometimes, when what you know fails to deliver the result, it is a good opportunity to learn a new procedure, better suited to solve your current case. In this chapter, I will review some of the founding theorems that I extensively use in the examples throughout this book. However, before tackling definitions and examples, let us first understand what the term *transfer function* designates.

1.1 Transfer Functions, an Informal Approach

Assume you are in the laboratory testing a circuit encapsulated in a box featuring two connectors: one for the input, the second for the output. You do not know what is inside the box, despite the transparent case in the picture! You now inject a signal with a function generator to the input connector and observe the output waveform with an oscilloscope. Using the right terminology, you *drive* the circuit input and observe its *response* to the stimulus. The input waveform represents the *excitation* denoted u and it generates a *response* denoted y. In other words, the excitation variable propagates through the box, undergoes changes in phase, amplitude, perhaps induces distortion etc. and the oscilloscope reproduces the response on its screen.

The waveform displayed by the oscilloscope is a *time-domain* graph in which the horizontal axis x is graduated in seconds while the vertical axis y indicates the signal *amplitude* (positive or negative). Its dimension depends on the observed variable (volts, amperes and so on). The input waveform is denoted in lower case as it is an *instantaneous* signal, observed at a time – the *instant* t – $u(t)$. A similar notation applies to the output signal, $y(t)$. In Figure 1.1, you see a low duty ratio square-wave injected in the box engendering a rather distorted waveform on the output.

Linear Circuit Transfer Functions: An Introduction to Fast Analytical Techniques, First Edition. Christophe P. Basso.

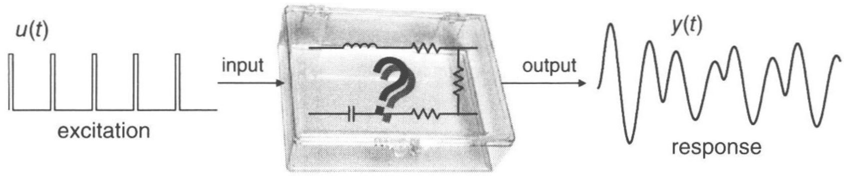

Figure 1.1 A black box featuring an input and an output signal. What is the relationship linking output and input waveforms?

This ringing signal tells us that the box could associate resonant elements, probably capacitors and inductors but not much more than that. If we change the excitation, what type of shape will we obtain? Knowing what is inside the box will let us predict its response to various types of excitation signals.

There are several available ways to characterize an electrical linear circuit. One of them is called *harmonic analysis*. The input signal is replaced by a sinusoidal waveform and you observe how the stimulus propagates through the box to form the response. This is shown in Figure 1.2:

The excitation level must be of reasonable amplitude – understand *small* – so that the response signal is not distorted. The input signal dc bias must also be set accounting for the physical constraints imposed by the active circuit so that upper- or lower-rail saturation is avoided. In other words, the box internal circuitry is not *overdriven* and remains *linear* during the analysis. Linearity is confirmed if the output signal is sinusoidal with the same frequency as the input sine and only varies in amplitude and phase while you ac-sweep the network. This is a so-called *small-signal* analysis. In the Laplace domain, you perform such harmonic analysis when you set $s = j\omega$ in which $\omega = 2\pi f$ represents the angular frequency expressed in radians per seconds (rads/s). Laplace analysis with $s = j\omega$ applies to linear circuits only.

Should you increase the input signal amplitude or change the operating bias point, slewing or clipping may happen. In this case, you explore the box *large-signal* or *nonlinear* response. This is a characterization different than the small-signal approach and it offers another insight into the circuit operation. Let us keep linear and once the right input amplitude is found, i.e. a signal of comfortable amplitude is observed on the oscilloscope screen, the frequency is varied step by step while output amplitude/phase couples are recorded in an array. At each frequency point f, we store the ratio of the response amplitude $Y(f)$ in volts to the excitation amplitude $U(f)$ in volts also. At each frequency point f, we save the phase information linking both input and output waveforms. As U and Y are complex variables affected by a magnitude and a phase, we can write:

$$A_v(s) = \frac{Y(s)}{U(s)} \tag{1.1}$$

A_v represents a *transfer function*, a mathematical relationship linking a response signal Y to an excitation signal U. Please note that the excitation signal U resides in the transfer function

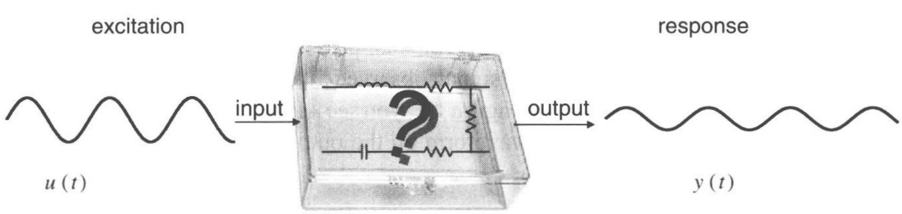

Figure 1.2 The black box is now driven by a sinusoidal stimulus for a small-signal analysis.