Linear System Theory and Design

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This text is a major revision of *Introduction to Linear System Theory* by Chi-Tsong Chen, originally published in 1970. © 1970 by Holt, Rinehart and Winston

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Preface

This text is intended for use at the senior-graduate level in university courses on linear systems and multivariable system design. It may also be used for independent study and reference by engineers and applied mathematicians. The mathematical background assumed for this book is a working knowledge of matrix manipulation and an elementary knowledge of differential equations. The unstarred sections of this book have been used, for over a decade, in the first graduate course on linear system theory at the State University of New York at Stony Brook. The majority of the starred sections were developed during the last three years for a second course on linear systems, mainly on multivariable systems, at Stony Brook and have been classroom tested at a number of universities.

With the advancement of technology, engineers have become interested in designing systems that are not merely workable but also the best possible. Consequently, it is important to study the limitations of a system; otherwise, one might unknowingly try to design an impossible system. Thus, a thorough investigation of all the properties of a system is essential. In fact, many design procedures have evolved from such investigations. This text is devoted to this study and the design procedures developed thereof. This is, however, not a control text per se, because performance criteria, physical constraints, cost, optimization, and sensitivity problems are not considered.

This text is a revised and expanded edition of *Introduction to Linear System Theory* which discussed mostly the state variable approach and was published in 1970. Since then, several important developments have been made in linear system theory. Among them, the geometric approach and the transfer-function matrices in fractional forms, called the matrix-fraction description, are most pertinent to the original text. The geometric approach is well covered in W. M. Wonham's *Linear Multivariable Control: A Geometric Approach*, 2d ed., Springer-Verlag, New York, 1979 and is outside the scope of this text. Hence the new material of this edition is mainly in the transfer-function matrix in fractional form. Because of this addition, we are able to redevelop, probably more simply in concepts and computations, the results of the state variable approach and establish a fairly complete link between the state-variable approach and the transfer-function approach.

We aim to achieve two objectives in the presentation. The first one is to develop major results and design procedures using simple and efficient methods. Thus the presentation is not exhaustive; only those concepts which are essential in the development are introduced. For example, the Smith-McMillan form is not used in the text and is not discussed. The second objective is to enable the reader to employ the results developed in the text. Consequently, most results are developed in a manner suitable for numerical computation and for digital computer programming. We believe that solving one or two problems of each topic by hand will enhance the understanding of the topic and give confidence in the use of digital computers. With the introduction of the row searching algorithm (Appendix A), which has been classroom tested, this is possible even for multivariable systems, as long as their degrees are sufficiently small.

The level of mathematics used in this edition is about the same as that of the original edition. If concepts and results in modern algebra more extensive than those in Chapter 2 are introduced, some results in the text can be developed more elegantly and extended to more general settings. For example, the Jordan form can be established concisely but abstractly by using the concepts of invariance subspaces and direct sum. Its discussion can be found in a large number of mathematical texts and will not be repeated here. In view of our objectives, we discuss the computation of the required basis and then develop the Jordan form. By using some concepts in abstract algebra, such as ring, principal ideal domain, and module, the realization problem (Chapter 6) can be developed more naturally and some results in this text can be extended to delay differential equations, linear distributed systems, and multidimensional systems. These are extensively studied in *Algebraic System Theory*, which was initiated by R. E. Kalman in the late 1960s and has extended in recent years most of the results in this text to linear systems over rings. The concepts used in algebraic system theory are less familiar to engineering students and require more mathematical sophistication and will not be discussed. All the results and design procedures in this text are developed by using only elementary concepts and results in linear algebra.

The results in this text may eventually be implemented on digital computers. Because of the finite word length, the sensitivity of problems and the stability of algorithms become important on computer computations. These problems are complex and extensively discussed in texts on numerical analysis. In our development, we will take note of these problems and remark briefly wherever appropriate.

The arrangement of the topics in this text was not reached without any difficulty. For example, the concepts of poles and zeros seem to be best intro-

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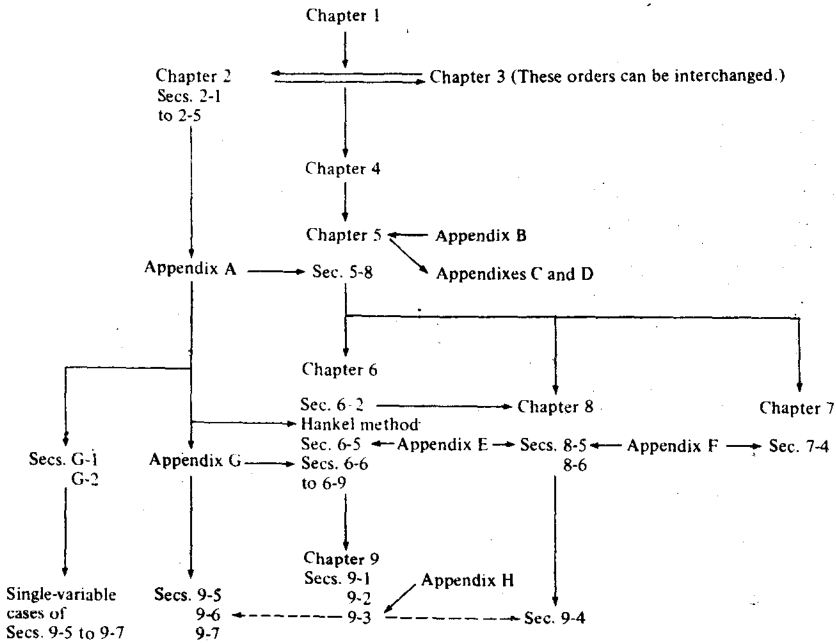
duced in Chapter 4. However, their complete treatments require irreducible realizations (Chapter 6) and coprime fractions of transfer-function matrices (Appendix G). Moreover, the concept of zeros is used only in Section 9-6. Hence it was decided to create an appendix for the topic. The coprimeness of polynomials and polynomial matrices might be inserted in the main text. This, however, will digress too much from the state-variable approach; thus the topic was grouped in an appendix.

The logical sequences of various chapters and appendixes are as follows:



The material connected by a broken line is not essential in the development. The logical dependencies among Chapters 6, 7, 8, and 9 are loose, and their various combinations can be adopted in one- or two-semester courses. When I teach a one-semester course at Stony Brook, the *unstarred sections* of the following chapters are covered:

Chapter 1
Chapter 2
Chapter 3 (Skip Theorem 3-1 and its corollary.)
Chapter 4 (Skip Theorem 4-11.)
Chapter 5 (Emphasize the time-invariant part by skipping Theorems 5-2, 5-5, and 5-11.)
Chapter 6
Chapter 7
Chapter 8

We emphasize the exact meanings of theorems and their implications; hence the proofs of a number of theorems are skipped. For example, we prove only Theorems 2-1 and 2-2 in Chapter 2. We skip the proofs of Theorems 4-1, 4-2, and others. In the second course, we cover the following:

Appendix A
Section 5-8, controllability and observability indices
Hankel method (Section 6-4 and method II of Section 6-5)
Appendix E
Singular value decomposition method (Method I of Section 6-5)
Appendix G
Sections 6-6 to 6-9
Starred sections of Chapter 7
Appendix H
Chapter 9

Those who are interested in quick access to the design methods using the transfer-function matrix in fractional form may proceed from Sections 2-1 to 2-5, Appendixes A and G, and then to Sections 9-5 to 9-7, or only their single-variable cases.

The problem sets form an integral part of the book. They are designed to help the reader understand and utilize the concepts and results covered. In order to retain the continuity of the main text, some important results are stated in the problem sets. A solutions manual is available from the publisher.

The literature on linear system theory is very extensive. The length of this text, however, is limited. Hence the omission of some significant results is inevitable and I would like to apologize for it. I am indebted to many people in writing this book. Kalman's work and Zadeh and Desoer's book *Linear System Theory* form the foundation of the original edition of this book. Rosenbrock's and Wolovich's works are essential in developing the present edition. I have benefited immensely in my learning from Professor C. A. Desoer. Even to this date, I can always go to him whenever I have questions. For this, I can never express enough of my gratitude. To Professors B. J. Leon, E. J. Craig, I. B. Rhodes, P. E. Barry (first edition) and to Professors M. E. Van Valkenburg, W. R. Perkins, D. Z. Zheng (present edition), I wish to express my appreciation for their reviews and valuable suggestions. I would like to thank President F. Zhang and Professor K. W. You of Chengdu University of Science and Technology, Professor S. B. Park of Korea Advanced Institute of Science and Technology, Professor T. S. Kuo of National Taiwan University, and Professor S. K. Chow of National Sun Yat-Sen University, Taiwan, for providing opportunities for me to lecture on an earlier draft of Chapter 9 and Appendix G. I especially appreciate the opportunity at Chengdo University to interact with several faculty members, especially Professor L. S. Zhang, from various universities in China; their suggestions have improved considerably the presentation of the text. I am grateful to many of my graduate students, specially C. Waters, C. H. Hsu (the first edition), I. S. Krishnarao, Y. S. Lai, C. C. Tsui and S. Y. Zhang (the present edition) whose assistance in the form of dissertations and discussions has clarified considerably my understanding of the subject matter. I am grateful to Mrs. V. Donahue, C. LaGuardis, T. Marasco, and F. Trace for typing various drafts of this text, to Mr. P. Becker of Holt, Rinehart and Winston and the staff of Cobb/Dunlop for their assistance in the production and to Professors S. H. Wang, K. W. You, and D. Z. Zheng, visiting scholars from the People's Republic of China, for their proofreading. My special thanks go to my wife Beatrice and my children Janet, Pauline, and Stanley for their support during the writing of this text.

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Glossary of Symbols

Q.E.D.	End of the proof of a theorem.
▮	This symbol denotes the end of a statement or an example.
A, **B**, **P**, . . .	Capital boldface letters denote matrices.
u, **y**, **α**, . . .	Lowercase boldface letters denote vectors.
$u, y, \alpha, \ldots$	Lowercase italic and Greek type denote scalar-valued functions or scalars. Capital italic letters are also used in Chapter 9 and Appendix G to denote scalars.
$\mathscr{L}$	Laplace transform.
$\hat{\mathbf{u}}(s)$, $\hat{\mathbf{y}}(s)$, $\hat{\mathbf{G}}(s)$, $C(s)$	If a letter is used in both time and frequency domains, circumflex will be used to denote the Laplace transform such as $\hat{\mathbf{u}}(s) = \mathscr{L}[\mathbf{u}(t)]$ and $\hat{\mathbf{G}}(s) = \mathscr{L}[\mathbf{G}(t)]$. If a letter is used in only one domain, no circumflex will be used, for example, $C(s)$.
$\nu(\mathbf{A}), \ldots$	The nullity of the constant matrix **A**.
$\delta(\hat{\mathbf{G}}(s))$, deg $\hat{\mathbf{G}}(s)$	The degree of the rational matrix $\hat{\mathbf{G}}(s)$.
$\mathbf{A}', \mathbf{x}', \ldots$	The transpose of the matrix **A** and the vector **x**.
$\mathbf{A}^*, \mathbf{x}^*, \ldots$	The complex-conjugate transpose of the matrix **A** and the vector **x**.
det **A**, . . .	The determinant of **A**.
$\mathbb{C}$	The field of complex numbers.
$\mathbb{R}$	The field of real numbers.
$\mathbb{R}(s)$	The field of rational functions of s with coefficients in $\mathbb{R}$.
$\mathbb{R}[s]$	The set of polynomials of s with coefficients in $\mathbb{R}$.
$\rho(\mathbf{A})$, rank **A**	The rank of **A**. If **A** is a constant matrix, the rank is

defined over $\mathbb{C}$ or $\mathbb{R}$. If **A** is a rational or polynomial matrix, the rank is defined over $\mathbb{R}(s)$.

deg det $\mathbf{A}(s)$ — The degree of the determinant of $\mathbf{A}(s)$.

diag $\{\mathbf{A}, \mathbf{B}, \mathbf{C}\}$ — A diagonal matrix with **A**, **B**, and **C** as block diagonal elements as

$$\begin{bmatrix} \mathbf{A} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{B} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{C} \end{bmatrix}$$

where **A**, **B**, and **C** are matrices, not necessarily square and of the same order.

$\triangleq$ — Equals by definition.

$\frac{d}{dt}\mathbf{A} \triangleq \left(\frac{d}{dt}a_{ij}\right)$,

$\mathscr{L}[\mathbf{A}] \triangleq (\mathscr{L}[a_{ij}]), \ldots$ — When an operator is applied to a matrix or a vector, it means that the operator is applied to every entry of the matrix or the vector.

$\dot{\mathbf{x}} \triangleq \frac{d}{dt}\mathbf{x}$

Contents

*May be omitted without loss of continuity.

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