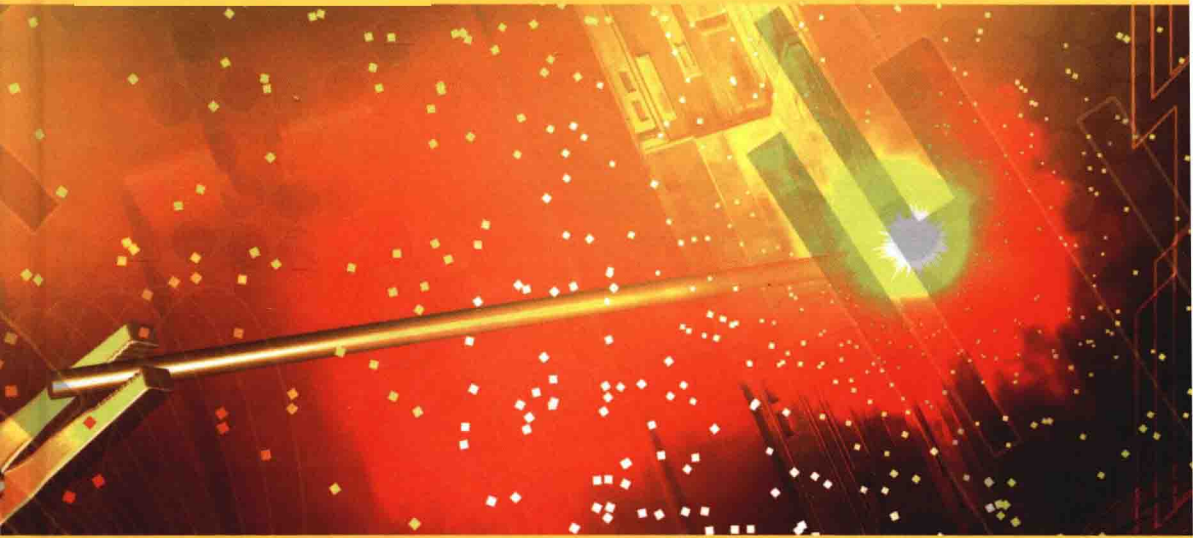


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# **Fatigue Limit in Metals**

**Claude Bathias**

**ISTE**

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Claude Bathias

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**WILEY**

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## Fatigue Limit in Metals



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# Introduction on Very High Cycle Fatigue

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This chapter is a summary of several decades of research on gigacycle fatigue of metals. For more detail please see references [BAT 04] and [BAT 10].

## 1.1. Fatigue limit, endurance limit and fatigue strength

Fatigue limit, endurance limit and fatigue strength are all expressions used to describe a property of materials under cyclic loading: the amplitude (or range) of *cyclic stress* that can be applied to the material without causing *fatigue failure*. In these cases, a number of cycles (usually  $10^7$ ) are chosen to represent the fatigue life of the material.

According to the American Society for Testing and Materials (ASTM) Standard E 1150, the definition of *fatigue* is summarized as follows: “The process of progressive localized permanent structural damage occurring in a material subjected to conditions that produce fluctuating stresses and strains at some point or points and that may culminate in cracks or complete fracture after a sufficient number of fluctuations”. The plastic strain resulting from cyclic stress initiates the crack; the tensile stress promotes crack growth propagation. Microscopic plastic strains also can be present at low levels of stress where the strain might otherwise appear to be totally elastic. The ASTM defines *fatigue strength*,  $S_{N_f}$ , as the value of stress at which failure occurs after  $N_f$  cycles,

and *fatigue limit*,  $S_f$ , as the limiting value of stress at which failure occurs as  $N_f$  becomes very large. The ASTM does not define *endurance limit*, the stress value below which the material will withstand many load cycles, but implies that it is similar to fatigue limit.

Some authors use *endurance limit* for the stress below which failure never occurs, even for an indefinitely large number of loading cycles, as in the case of steel, and *fatigue limit* or *fatigue strength* for the stress at which failure occurs after a specified number of loading cycles, such as 500 million, as in the case of aluminum. Other authors do not differentiate between the expressions even if they do differentiate between the face center cubic (FCC) metals and the base center cubic (BCC) metals [BAT 10].

Since the word “fatigue” was used by Braithwaite, A. Wöhler established the first basic approach to the fatigue life of metals, in the mid-1800s, when the main industrial applications were railcar axles and steam engines for railways and boats [BAT 10]. The slow rotation of a steam engine was about 50 cycles per minute, more or less. Thus, the fatigue limit was defined by Wöhler to be between  $10^6$  and  $10^7$  cycles, but it seems that the quasi-hyperbolic stress number of cycle (SN) curve was suggested by Basquin [BAS 10]. Today, the fatigue life of a high-speed train ranges in the gigacycle,  $10^9$ , regime and for an aircraft turbine it is of the order of  $10^{10}$  cycles, according to the rotation speed of several thousand turns per minute.

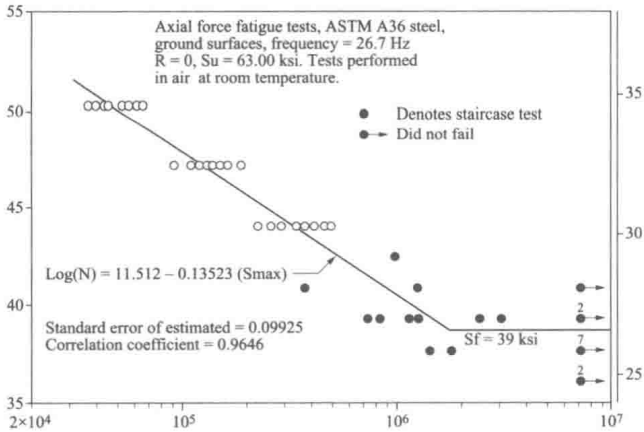


Figure 1.1. International standard for SN curve and fatigue limit

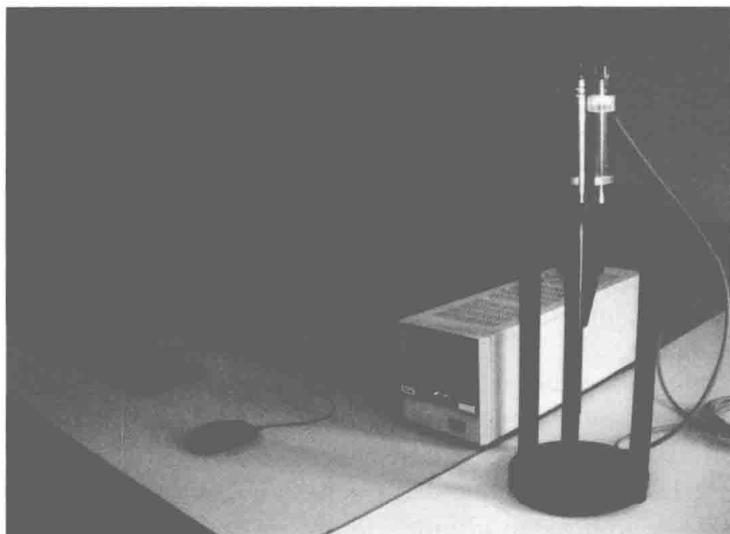
The fatigue curve or SN curve is usually defined in reference to carbon steel. The SN curve is generally limited to  $10^7$  cycles and it is acknowledged, according to the standard, that a horizontal asymptote allows us to determine a fatigue limit value for an alternating stress between  $10^6$  and  $10^7$  cycles. Beyond  $10^7$  cycles, the standard considers that the fatigue life is infinite. For other alloys, it is assumed that the asymptote of the SN curve is not horizontal.

A few results for fatigue limit based on  $10^9$  cycles can be found in the literature [BAT 10]. Using standard practice, the shape of the SN curve beyond  $10^7$  cycles is predicted using the probabilistic method, and this is also true for the fatigue limit. In principle, the fatigue limit is given for a number of cycles to failure (Figure 1.1). Using, for example, the staircase method, the fatigue limit is given by the average alternating stress  $\sigma_D$  and the probability of fracture is given by the standard deviation of the scatter ( $s$ ). The classical way to determine the infinite fatigue life is to use a Gaussian function. Roughly speaking, it is said that  $\sigma_D$  minus  $3s$  gives a probability of fracture close to zero. Assuming  $s$  is equal to 10 MPa, the true infinite fatigue limit should be  $\sigma_D - 30$  MPa. However, experiments show that between  $\sigma_D$  for  $10^6$  and  $\sigma_D$  for  $10^9$ , the difference is greater than 30 MPa for many alloys.

The so-called standard deviation (SD) approach to the average fatigue limit is certainly not the best way to reduce the risk of rupture in fatigue. When one is conscious that it is the last resort, only experience can remove this ambiguity by appealing to some tests of accelerated fatigue. Today some piezoelectric fatigue machines are very reliable, capable of producing  $10^{10}$  cycles in less than one week, whereas the conventional systems require more than 3 years of tests for only one sample.

To summarize the present situation, it is acknowledged that the concept of a fatigue limit is bound to the hypothesis of the existence of a horizontal asymptote on the SN curve between  $10^6$  and  $10^7$  cycles (Figure 1.1). Thus, a sample that reaches  $10^7$  cycles and is not broken is considered to have an infinite life; that is, in fact, a convenient and economical approximation but not a rigorous approach. It is important to understand that if the staircase method is popular today to

determine the fatigue limit, this is because of the convenience of this approximation. A fatigue limit determined by this method to  $10^7$  cycles requires 30 h of tests to get only one sample with a machine working at 100 Hz. To reach  $10^8$  cycles, 300 h of tests would be required, which is expensive. Using a 20 kHz piezoelectric fatigue machine, it takes around 14 h to obtain  $10^9$  cycles, 6 days for  $10^{10}$  cycles and 58 days for  $10^{11}$  cycles. The basic design of the piezoelectric fatigue machine is the same at 30 kHz as a 20 kHz piezoelectric fatigue machine, where the vibration of the specimen is induced by a piezoceramic converter, which generates acoustic waves in the specimen through a power concentrator (horn) in order to obtain desired displacement and an amplification of the stress [WU 93]. The resonant specimen dimension and stress concentration factor were calculated by the Finite Element Method (FEM) subject to 20 and 30 kHz [WU 93]. Such computer-controlled piezoelectric fatigue machines are able to work in tension-compression, tension-tension-tension, bending and torsion loading (Figure 1.2). It is of importance to note that the temperature of the specimen and the amplitude of the stress must stay constant during a standard test at 20 kHz to keep the comparison with low-frequency testing. A complete description of the procedure is given in [BAT 04].



**Figure 1.2.** *Experimental system for ultrasonic fatigue at 20 kHz*

## 1.2. Absence of an asymptote on the SN curve

Generally speaking, it is assumed that the steel SN curves are different from the others. To get an overview of the gigacycle behavior, many alloys, including steel, are considered in this chapter. For results of fatigue SN curves based on  $10^9$  cycles, a few results are available in the literature. Many of those results come from our laboratory [BAT 04]. The other results come from Japanese researchers such as Naito [NAI 84], Kanazawa [NIS 97], Murakami [MUR 99] and Sakai [SAK 07]. They are limited to  $10^8$  cycles. Also, some SN curves for light alloys come from the laboratory of S. Stanzl-Tschegg and H.R. Mayer [STA 99]. They are limited to  $10^9$  cycles.

Safe-life design based on the infinite life criteria was initially developed from the Wöhler approach, which is the stress-life or SN curve related to the asymptotic behavior of steel. Some materials display a fatigue limit or an “endurance” limit at a high number of cycles (typically  $>10^6$ ). Most other materials do not exhibit this response; instead, they display a continuously decreasing stress-life response, even at a large number of cycles ( $10^6$ – $10^9$ ), which is more correctly described by fatigue strength at a given number of cycles.

The actual shape of the SN curve between  $10^6$  and  $10^{10}$  cycles is a better way to help the prediction of risk in fatigue cracking (Figure 1.3).

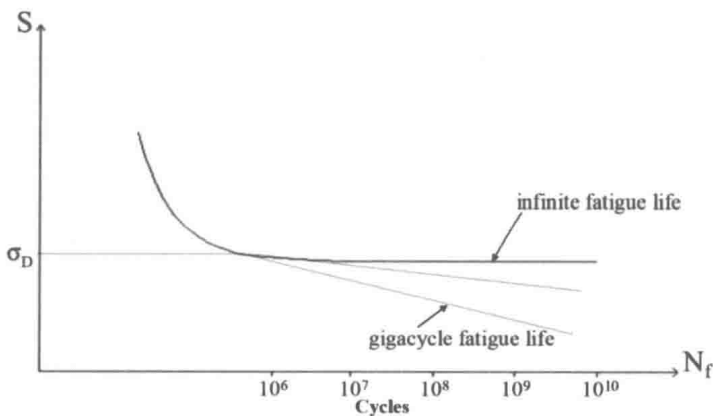


Figure 1.3. The concept of gigacycle fatigue SN curve

Since Wöhler, the standard has been to represent the SN curve by a hyperbole more or less modified as indicated below.

Hyperbole  $\text{Ln } N_f = \text{Log } a - \text{Ln } \sigma^a$ , while other methods may be listed as:

- Wöhler  $\text{Ln } N_f = a - b \sigma^a$ ;
- Basquin  $\text{Ln } N_f = a - b \text{Ln } \sigma^a$ ;
- Stromeier  $\text{Ln } N_f = a - b \text{Ln } (\sigma^a - c)$ ;

Only the exploration of the life range between  $10^6$  and  $10^{10}$  cycles will create a safer approach to modeling.

### 1.3. Initiation and propagation

It is of great importance to understand and predict a fatigue life in terms of crack initiation and small crack propagation. It has been generally accepted that at high stress levels, fatigue life is determined primarily by crack growth, while at low stress levels, most of the life is consumed by the process of crack initiation. In low cycle fatigue, it is generally understood that about 50% of the life is devoted to initiation of the micro crack. But many authors demonstrated that the portion of life attributed to crack nucleation is the upper 90% in the high cycle regime ( $10^6$ – $10^7$  cycles) for steel, aluminum, titanium and nickel alloys. In the case for which the crack nucleates from a defect, such as an inclusion or pore, it is said that a relation must exist between the fatigue limit and the crack growth threshold.

However, the relation between the crack growth and initiation is not obvious for many reasons. First, it is not certain that a fatigue crack grows immediately at the first cycle from a sharp defect. Second, when a defect is small, a short crack does not grow as a long crack. In particular, the effect of ratio  $R$  or the closure effect depends

on the crack length. Thus, the relationship between  $\Delta K_{th}$  and  $\sigma_D$  is still to be discussed (BAT 00).

Another important aspect is the concept of infinite fatigue life. It is understood that below  $\Delta K_{th}$  and below  $\sigma_D$  the fatigue life is infinite. In fact, the fatigue limit  $\sigma_D$  is usually determined for  $N_f = 10^7$  cycles. Since the fatigue failure can appear up to or beyond  $10^9$  cycles, the fatigue strength difference at  $10^7$  and  $10^9$  cycles could be more than 100 MPa. This means that the relationship between  $\sigma_D$  and  $\Delta K_{th}$  must be established in the gigacycle regime if any relation exists.

#### 1.4. Fatigue limit or fatigue strength

How can we model the fatigue limit or the gigacycle fatigue strength of industrial alloys?

The procedure is given below.

First, a new SN curve must be determined up to  $10^{10}$  cycles, which is, in fact, more than the fatigue life of most technological machines.

Second, new fatigue strength at  $10^9$  cycles has to be predicted using regular statistical method.

In more detail, the prediction of gigacycle fatigue is based on two different mechanisms:

- Initiation is related to flaws (inclusions, defects, pores): prediction is derived from stress concentration, fracture mechanics or short crack approaches.

- Initiation is not related to defect: in this case, microstructure is a key parameter, such as grain size, interface, load transfer and microplasticity.

Thus, the discussion of gigacycle fatigue prediction is split into two parts. The first part is devoted to alloys with flaws.



1.5. SN curves up to  $10^9$  cycles

In specialized literature, few results were given on this topic until “Euromech 382” was held in Paris in June 1998. Typical gigacycle SN curves are given below as examples.

The experimental results (Figures 1.4–1.6) show that specimens can fail up to  $10^9$  and beyond. It means the SN curve is not an asymptotic curve. Thus, the concept of infinite life fatigue is not correct and the definition of a fatigue limit at  $10^6$ – $10^7$  cycles is not conservative [BAT 99 ]. In Figures 1.4–1.6, it is shown that fatigue failure can occur after  $10^{10}$  cycles in cast aluminum, in SG cast iron and in bearing steel. Depending on the alloy, the difference between the fatigue strength at  $10^6$  and  $10^9$  cycles can range from 50 to 200 MPa. From the practical point of view, the gigacycle fatigue strength becomes the most realistic property for predicting very long life [BAT 10].

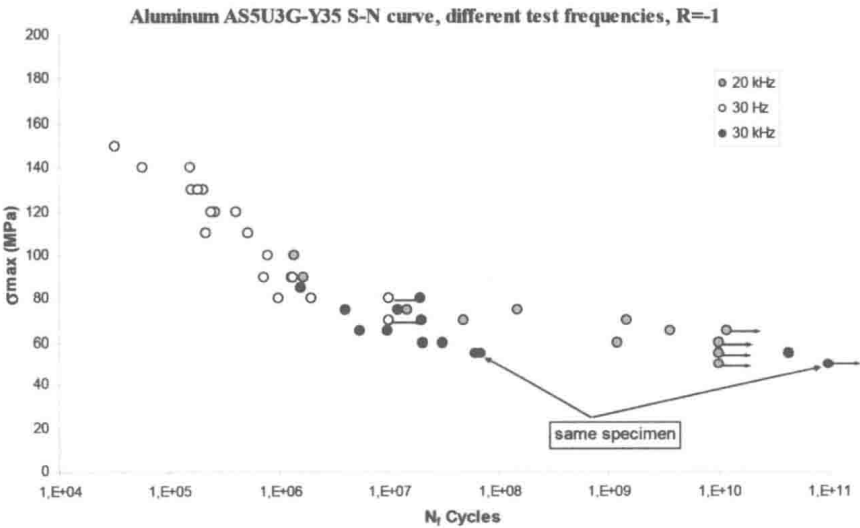


Figure 1.4. Gigacycle SN curve for a cast aluminum alloy