CALCULUS AND ITS APPLICATIONS

Larry J. Goldstein ▲ David C. Lay ▲ David I. Schneider

NINTH EDITION

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Calculus and Its Applications

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Preface

We have been very pleased with the enthusiastic response to the first eight editions of *Calculus and Its Applications* by teachers and students alike. The present work incorporates many of the suggestions they have put forward.

Although there are many changes, we have preserved the approach and the flavor. Our goals remain the same: to begin the calculus as soon as possible; to present calculus in an intuitive yet intellectually satisfying way; and to illustrate the many applications of calculus to the biological, social, and management sciences.

The distinctive order of topics has proven over the years to be successful—easier for students to learn, and more interesting because students see significant applications early. For instance, the derivative is explained geometrically before the analytic material on limits is presented. This approach gives the students an understanding of the derivative at least as strong as that obtained from the traditional approach. To reach the applications in Chapter 2 quickly, we present only the differentiation rules and the curve sketching needed for those applications. Advanced topics come later when they are needed. Other aspects of this student-oriented approach follow below.

Applications

We provide realistic applications that illustrate the uses of calculus in other disciplines. See the Index of Applications on the inside cover. Wherever possible, we have attempted to use applications to motivate the mathematics.

Examples

The text includes many more worked examples than is customary. Furthermore, we have included computational details to enhance readability by students whose basic skills are weak.

Exercises

The exercises comprise about one-quarter of the text—the most important part of the text in our opinion. The exercises at the ends of the sections are usually arranged in the order in which the text proceeds, so that the homework assignments may easily be made after only part of a section is discussed. Interesting applications and more challenging problems tend to be located near the ends of the exercise sets. Supplementary exercises at the end of each chapter expand the other exercise sets and include problems that require skills from earlier chapters.

Practice Problems

The practice problems have proven to be a popular and useful feature. Practice Problems are carefully selected questions located at the end of each section, just before the exercise set. Complete solutions are given following the exercise set. The practice problems often focus on points that are potentially confusing or are likely to be overlooked. We recommend that the reader seriously attempt the practice problems and study their solutions before moving on to the exercises. In effect, the practice problems constitute a built-in workbook.

Minimal Prerequisites

In Chapter 0, we review those concepts that the reader needs to study calculus. Some important topics, such as the laws of exponents, are reviewed again when they are used in a later chapter. Section 0.6 prepares students for applied problems that appear throughout the text. A reader familiar with the content of Chapter 0 should begin with Chapter 1 and use Chapter 0 as a reference, whenever needed.

New in this Edition

Among the many changes in this edition, the following are the most significant:

1. *Delta Notation* We introduce delta notation in Chapter 0 and use it in our discussion of the derivative. As in previous editions, we have tried to minimize the use of complicated notation, preferring instead verbal descriptions. However, in the case of the delta notation, we feel that the clarity achieved is worth the extra notation.

2. *Derivative as a Rate of Change* We preview the derivative as a rate of change at the beginning of Chapter 1, anticipating the more detailed discussion in Section 1.8. Since students have difficulty interpreting the derivative as a rate of change, we felt it prudent to allow them to practice repeatedly with the concept.

3. *Analysis of Data* We added a broad theme that might best be described as "calculus for functions defined by data." Throughout the book, we include discussions about real-life applications whose underlying functions are defined by tables of data.

4. *More on Regression (optional)* We added the optional Section 7.6 on multiple and nonlinear regression analysis. The goal in this section is to provide a taste of what a business student will encounter in a course in regression analysis. Our emphasis is on using technology, especially spreadsheets, to do the computations for various flavors of regression (multiple-linear, quadratic, exponential, etc.).

5. *Additional Technology (optional)* The new technology appendix to Chapter 0 includes the graphing calculator material previously found within the chapter, a discussion of calculus and spreadsheets, and a new exercise set testing student technology skills.

6. *Real-Life Data* We have collected spreadsheets containing real-life statistical data and made them available to students and faculty on the Web site `www.prenhall.com/goldstein`.

7. *Projects* Each chapter now includes a project, designed to provide more open-ended problem solving, critical thinking, verbal expression, and integration of mathematical techniques, both manual and technological.

8. *Other Changes* We made improvements throughout the text based on suggestions from students, teachers, reviewers, and editors. Our thanks to all who assisted us with their valuable suggestions.

This edition contains more material than can be covered in most two-semester courses. Optional sections are starred in the table of contents. In addition, the level of theoretical material may be adjusted to the needs of the students. For instance, only the first two pages of Section 1.4 are required in order to introduce the limit notation.

A ***Study Guide*** for students containing detailed explanations and solutions for every sixth exercise is available. The *Study Guide* also includes helpful hints and strategies for studying that will help students improve their performance in the course. In addition, the *Study Guide* contains a copy of *Visual Calculus*, the popular, easy-to-use software for IBM compatible computers. *Visual Calculus* contains over 20 routines that provide additional insights into the topics discussed in the text. Also, instructors find the software valuable for constructing graphs for exams.

An *Instructor's Solutions Manual* contains worked solutions to every exercise.

TestGen EQ provides nearly 1000 suggested test questions, keyed to chapter and section. *TestGen EQ* is a text-specific testing program networkable for administering tests and capturing grades online. Edit and add your own questions, or use the new "Function Plotter" to create a nearly unlimited number of tests and drill worksheets.

Designed to complement and expand upon the text, the *text Web site* offers a variety of interactive teaching and learning tools. Since many of the text projects use real-life data, we made the data easier to use by making it available in Excel spreadsheets on the Web site. The Web site also includes links to related Web sites, quizzes, Syllabus Builder, and more. For more information, visit `www.prenhall.com/goldstein` or contact your local Prentice Hall representative.

Acknowledgments

The following is a list of reviewers from this and previous editions. We apologize for any omissions. While writing this book, we have received assistance from many persons. And our heartfelt thanks goes out to them all. Especially, we would like to thank the following reviewers, who took the time and energy to share their ideas, preferences, and often their enthusiasm with us.

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Larry J. Goldstein
David C. Lay
David I. Schneider

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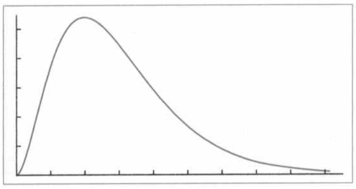

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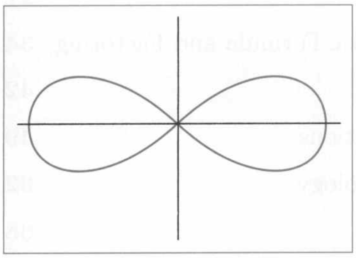

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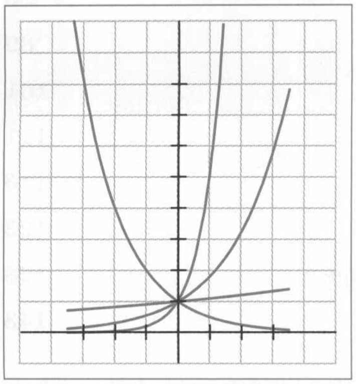

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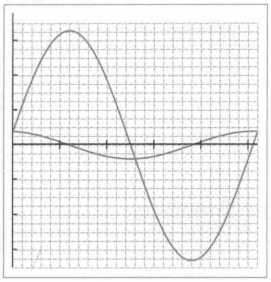

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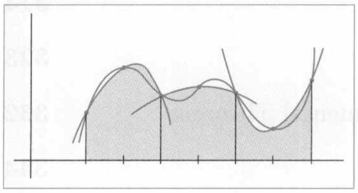

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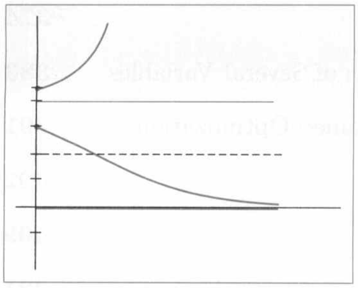

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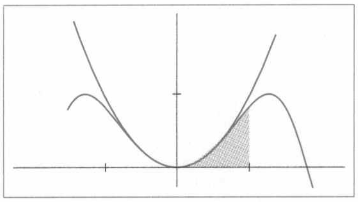

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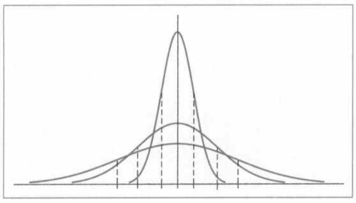

Introduction

Often it is possible to give a succinct and revealing description of a situation by drawing a graph. For example, Fig. 1 describes the amount of money in a bank account drawing 5% interest, compounded daily. The graph shows that as time passes, the amount of money in the account grows. In Fig. 2 we have drawn a graph that depicts the weekly sales of a breakfast cereal at various times after advertising has ceased. The graph shows that the longer the time since the last advertisement, the fewer the sales. Figure 3 shows the size of a bacteria culture at various times. The culture grows larger as time passes. But there is a maximum size that the culture cannot exceed. This maximum size reflects the restrictions imposed by food supply, space, and similar factors. The graph in Fig. 4 describes the decay of the radioactive isotope iodine 131. As time passes, less and less of the original radioactive iodine remains.

Each of the graphs in Figs. 1 to 4 describes a change that is taking place. The amount of money in the bank is changing as are the sales of cereal, the size of the bacteria culture, and the amount of the iodine. Calculus provides mathematical tools to study each of these changes in a quantitative way.

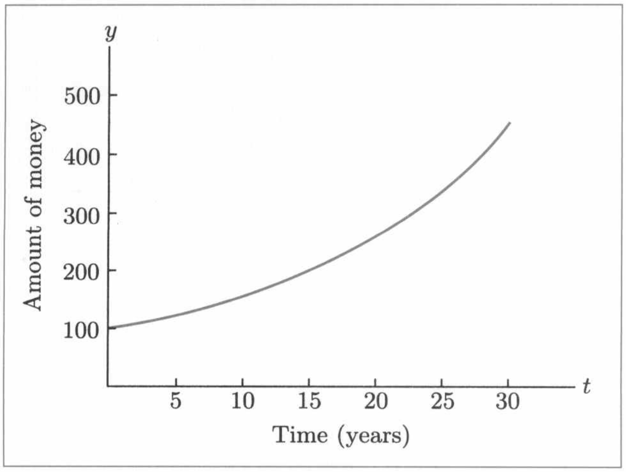

Figure 1. Growth of money in a savings account.

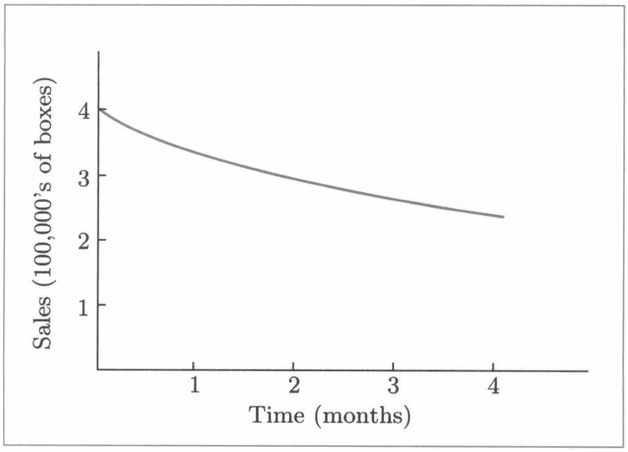

Figure 2. Decrease in sales of breakfast cereal.

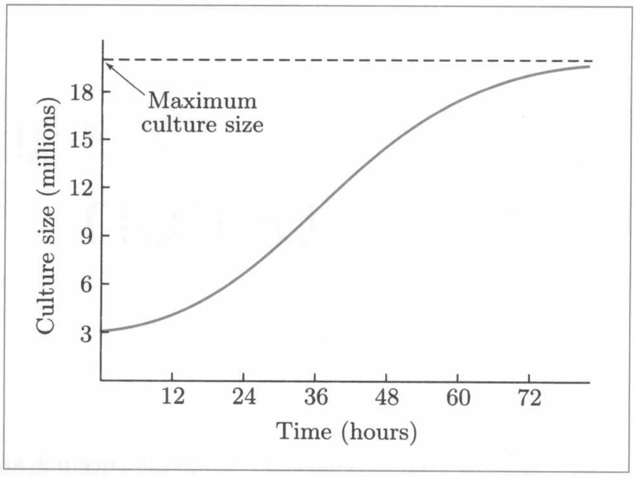

Figure 3. Growth of a bacteria culture.

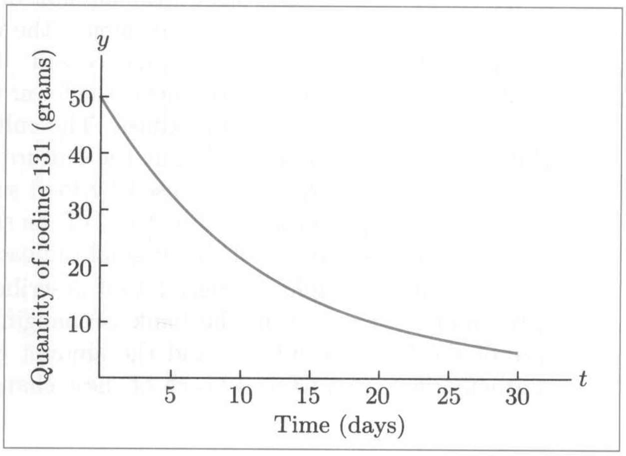

Figure 4. Decay of radioactive iodine.

CHAPTER

0 Functions

Each of the graphs in Figs. 1 to 4 of the Introduction depicts a relationship between two quantities. For example, Fig. 4 illustrates the relationship between the quantity of iodine (measured in grams) and time (measured in days). The basic quantitative tool for describing such relationships is a *function*. In this preliminary chapter, we develop the concept of a function and review important algebraic operations on functions used later in the text.

0.1 Functions and Their Graphs

Real Numbers

Most applications of mathematics use real numbers. For purposes of such applications (and the discussions in this text), it suffices to think of a real number as a decimal. A *rational* number is one that may be written as a finite or infinite repeating decimal, such as

$$-\frac{5}{2} = -2.5, \qquad 1, \qquad \frac{13}{3} = 4.333\ldots \qquad \text{(rational numbers)}.$$

An *irrational* number has an infinite decimal representation whose digits form no repeating pattern, such as

$$-\sqrt{2} = -1.414214\ldots, \qquad \pi = 3.14159\ldots \qquad \text{(irrational numbers)}.$$

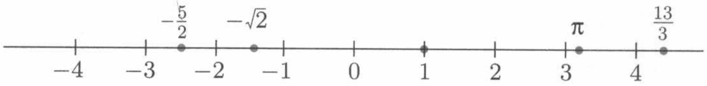

Figure 1. The real number line.

The real numbers are described geometrically by a *number line*, as in Fig. 1. Each number corresponds to one point on the line, and each point determines one real number.

We use four types of inequalities to compare real numbers.

$x < y$	x is less than y
$x \leq y$	x is less than or equal to y
$x > y$	x is greater than y
$x \geq y$	x is greater than or equal to y

The double inequality $a < b < c$ is shorthand for the pair of inequalities $a < b$ and $b < c$. Similar meanings are assigned to other double inequalities, such as $a \leq b < c$. Three numbers in a double inequality, such as $1 < 3 < 4$ or $4 > 3 > 1$, should have the same relative positions on the number line as in the inequality (when read left to right or right to left). Thus $3 < 4 > 1$ is never written because the numbers are "out of order."

Geometrically, the inequality $x \leq b$ means that either x equals b or x lies to the left of b on the number line. The set of real numbers x that satisfy the double inequality $a \leq x \leq b$ corresponds to the line segment between a and b, including the endpoints. This set is sometimes denoted by $[a, b]$ and is called the *closed interval* from a to b. If a and b are removed from the set, the set is written as (a, b) and is called the *open interval* from a to b. The notation for various line segments is listed in Table 1.

The symbols ∞ ("infinity") and $-\infty$ ("minus infinity") do not represent actual real numbers. Rather, they indicate that the corresponding line segment extends infinitely far to the right or left. An inequality that describes such an infinite interval may be written in two ways. For instance, $a \leq x$ is equivalent to $x \geq a$.

◗ Example 1 Describe each of the following intervals both graphically and in terms of inequalities.

(a) $(-1, 2)$ (b) $[-2, \pi]$ (c) $(2, \infty)$ (d) $(-\infty, \sqrt{2}]$

Solution The line segments corresponding to the intervals are shown in Fig. 2(a)–(d). Note that an interval endpoint that is included (e.g., both endpoints of $[a, b]$) is drawn

Table 1 Intervals on the Number Line

Inequality	Geometric Description	Interval Notation
$a \leq x \leq b$		$[a, b]$
$a < x < b$		(a, b)
$a \leq x < b$		$[a, b)$
$a < x \leq b$		$(a, b]$
$a \leq x$		$[a, \infty)$
$a < x$		(a, ∞)
$x \leq b$		$(-\infty, b]$
$x < b$		$(-\infty, b)$

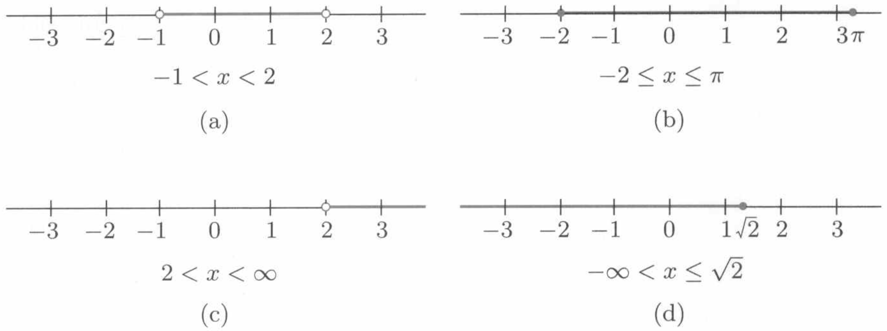

Figure 2. Line segments.

as a solid circle, whereas an endpoint not included (e.g., the endpoint a in $(a, b]$) is drawn as an unfilled circle. ◆

◗ **Example 2** The variable x describes the profit that a company is anticipated to earn in the current fiscal year. The business plan calls for a profit of at least 5 million dollars. Describe this aspect of the business plan in the language of intervals.

Solution The phrase "at least" means "greater than or equal to." The business plan requires that $x \ge 5$ (where the units are millions of dollars). This is equivalent to saying that x lies in the infinite interval $[5, \infty)$. ◆

Functions A *function* of a variable x is a *rule* f that assigns to each value of x a unique number $f(x)$, called *the value of the function at* x. [We read "$f(x)$" as "f of x."] The variable x is called the *independent variable.* The set of values that the independent variable is allowed to assume is called the *domain* of the function. The domain of a function may be explicitly specified as part of the definition of a function or it may be understood from context. (See the following discussion.) The *range* of a function is the set of values that the function assumes.

The functions we shall meet in this book will usually be defined by algebraic formulas. For example, the domain of the function

$$f(x) = 3x - 1$$

consists of all real numbers x. This function is the rule that takes a number, multiplies it by 3, and then subtracts 1. If we specify a value of x, say $x = 2$, then we find the value of the function at 2 by substituting 2 for x in the formula:

$$f(2) = 3(2) - 1 = 5.$$

◗ **Example 3** Let f be the function with domain all real numbers x and defined by the formula

$$f(x) = 3x^3 - 4x^2 - 3x + 7.$$

Find $f(2)$ and $f(-2)$.

Solution To find $f(2)$ we substitute 2 for every occurrence of x in the formula for $f(x)$:

$$\begin{aligned} f(2) &= 3(2)^3 - 4(2)^2 - 3(2) + 7 \\ &= 3(8) - 4(4) - 3(2) + 7 \\ &= 24 - 16 - 6 + 7 \\ &= 9. \end{aligned}$$