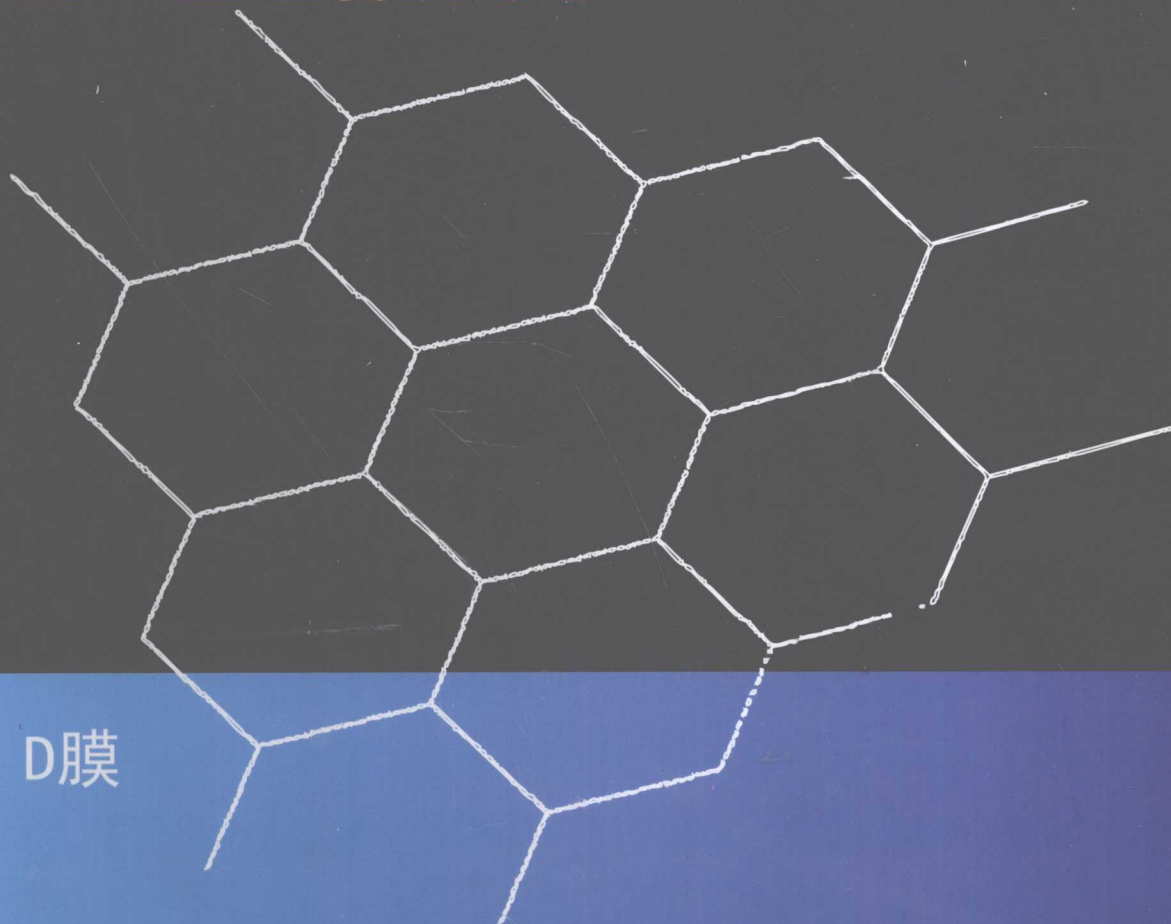


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Clifford. V. Johnson

D-Branes



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D-Branes

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Preface

In view of the exciting developments in our understanding of those particular aspects of fundamental physics that string theory seems to capture, it seems appropriate to collect together some of the key tools and ideas which helped move things forward. The developments included a true revolution, since the physical perspective changed so radically that it undermined the long-standing status of strings as the basic fundamental objects, and instead the idea has arisen that a string theory description is simply a special (albeit rather novel and beautiful) corner of a larger theory called ‘M-theory’. This book is not an attempt at a history of the revolution, as we are (arguably) still in the midst of it, especially since we are in the awkward position of not knowing even one satisfactory intrinsic definition of M-theory, and have implicit knowledge of it only through interconnections of its various limits.

All revolutions are supposed to have a collection of characters who played a crucial role in it, ‘heroes’ if you will. Hence, one would be expected to proceed to list here the names of various individuals. While I was lucky to be in a position to observe a lot of the activity at first hand and collect many wonderful anecdotes about how some things came to be, I will decline to start listing names at this juncture. It is too easy to yield to the temptation to emphasise a few personalities in a short space (such as this preface), and the result can sometimes be at the expense of others, a practice which happens all too often elsewhere. This seems to me to be especially inappropriate in a field where the most striking characteristic of the contributions has been the *collective* effort of hundreds of thinkers all over the planet, often linked by e-mail and the web, often never having met each other in person.

There were marvellous weeks, back in 1995 and 1996 especially, where there was one key paper after another, from all over the world, driven by

the fact that new ideas were pouring in from conversations everyone was excitedly having at blackboards, in the sand, over lunch, *via* e-mail, on the back of an envelope, *etc.* However, when one is speculating about aspects of fundamental physics which are not yet in the directly testable realm it should be noted that ideas – even radical ones – are cheap. Computational tools are needed to test them, and to provide access to the new regimes to which the ideas beckon. The collection of tools which filled this crucial role in this context was built around ‘D-branes’, and it was the change of perspective and computational power that they brought that unlocked that steady flow of marvellous papers. In my mind, they can indisputably be placed high on list of characters cast as heroes of the revolution. Indeed, many will speak of the feeling that often arose after working with them for a while in those exciting days, that the D-branes simply had a life and character of their own. They shaped the ideas and language of the field in a way that was directed by no single personality, and – most importantly – were a wonderful and sharp tool for investigating in detail the nature of the many bold conjectures which were made.

D-branes were discovered well before the revolution, of course, but in the Summer of 1995 it was shown by Joseph Polchinski that they were relevant to strongly coupled string theory. I arrived as a postdoctoral researcher at the Institute for Theoretical Physics (Santa Barbara, California) in the following Autumn, and by then it was already clear that there were many people, both young and old, who could benefit from a refresher course on issues outside the realm of heterotic string theory (on which much of the focus had been up to then, with an eye on phenomenology) and an introduction to D-branes. Furthermore, there was some need for an agreement about language and conventions, since there had not been much in the way of texts or other notes which focused on the relevant aspects. (Polchinski’s modern textbook¹ was still only partially written, and the manuscript had been seen only by a privileged few.)

Some of us begged Joe to give us some lectures at the ITP, and I (and probably others) quickly had the idea for a written set of notes that could be circulated to the world at large, as a basic toolbox. I suggested this to him, and he eventually agreed. During the lectures, I took such notes as I could and then together with Shyamoli Chaudhuri, we produced some notes with Joe, which we released² with his name listed as first author – breaking the strict alphabetical convention in this field – as it seemed to me highly inappropriate, given our roles as scribes, that his name might come last. Happily, the ‘D-notes’ (as I liked to call them) seemed to be well received by very many, and proved to be useful in forming a common point of departure for almost everyone working in the field.

I was fortunate enough to be asked to give introductory lectures on D-branes over the following months and years, and this led me to write more notes to embellish the D-notes, finding new ways of explaining things, sometimes making illustrative links between different aspects, depending on the theme of the lecture series in which I was participating.

This book grew out of such lecture notes^{3, 4}, and contains my own biased perspective on what aspects of D-branes ought to be included in an introductory text. Pressures of space mean that I have omitted a large number of remarkably interesting and useful material, and my choices will no doubt not suit everyone. I have made many efforts for it to be a stand alone handbook. It is intended that the person who knows little or no string theory (but with some background in quantum field theory and relativity) can open this book, and upon working through it, learn many things about string theory, and become adept at computing with D-branes, making no reference to another string theory text. Perhaps as a bonus, they will even learn various aspects of advanced topics in relativity, geometry and quantum gravity and quantum field theory since those are the meat and drink of D-brane physics. However, if they want a deeper knowledge of many aspects of string theory which are only sketched here due to lack of space, then they can consult the excellent text of Polchinski¹, and also that of Green, Schwarz and Witten⁵, which is still an excellent text for many aspects of the subject. There are also many other sources, on the web (*e.g.*, www.arXiv.org) and elsewhere, of detailed reviews of various specialised topics, even other string theory books⁶.

So, this is not intended to be a string theory textbook. It is instead a handbook or toolbox for concepts concerning branes in string theory, with emphasis on D-branes. However, since many of the applications are in what I like to call ‘extreme string theory’ – taking limits like strong coupling, low energy, large N , etc. – the reader will also learn important physics of those regimes and others, which are not covered in any other text at this time.

Over the years I have had the great benefit of lengthy conversations about string theory and D-branes with many people, out of which my intuition for these matters developed, and I would like to thank them all. Chief among these are Robert Myers, Joseph Polchinski, and Edward Witten, all of whose patience (and refreshing open-mindedness in the early days) is much appreciated. I also thank all of the people with whom I have collaborated in very many exciting research projects, and from whom I learned a great deal. Aspects of some of that work will appear in this text, and I would like it made clear that any inaccuracies in presenting the results are my own.

Parts of this book were written (or sometimes day-dreamt about) in many inspiring places, not all of which I can recall, but I should thank especially a number of institutions for providing facilities: The New York Public Library's Rose Reading Room, and Columbia University's Butler Library (New York, NY, USA), the Bodlean Library (Oxford, England), The Aspen Centre for Physics (Aspen, Co., USA), The Park City Mathematical Institute 2001 (Park City, Utah, USA), El Centro de Estudios Científicos (Valdivia, Chile), The Physics Department at Stellenbosch University, and the Stellenbosch Institute for Advanced Study (Stellenbosch, South Africa), The Perimeter Institute for Fundamental Research (Waterloo, Canada), The Village Vanguard (New York, USA), Broadway (New York, USA), and various United Airlines lounges worldwide.

Thanks to Ian Davies, James Gregory, Laur Järv, Ken Lovis, Rob Myers and David Page for reading and commenting on parts of the manuscript, and many people around the world for their useful remarks upon earlier notes which were absorbed into this book. Thanks also go to Jim Gates, Brian Greene, David Gross, Ted Jacobson and Lenny Susskind for their thoughts on a late title change, and on other important matters concerning the book, logistical and otherwise[†]. I'd like to thank all of my colleagues at the Department of Mathematical Sciences, University of Durham, for providing such a friendly and supportive working environment, and Carol, Delia and Robert Johnson for their encouragement. Thanks also to Elizabeth and Nich Butler for much appreciated culinary provision and other matters of hospitality over Christmas 2001.

I would especially like to thank Samantha Butler for her constant patience and support throughout this project, and beyond.



[†] A conversation with Brian led to a flirtation with the slightly irreverent idea of giving this book the simple title 'Volume III'. I abandoned this after a while, since it would produce confusion amongst those not aware of the affection held for (or existence of) the two-volume texts in references [1] and [5].

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