



# Scientific Computing with MATLAB®

Second Edition

Dingyü Xue  
YangQuan Chen



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# Scientific Computing with MATLAB®

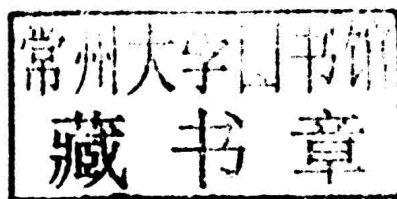
Second Edition

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# Scientific Computing with MATLAB®

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# Preface

Since the first edition of this book published in 2008, computing landscape changed radically, from cloud computing to big data science, from wearable computing to internet of things, from deep learning to driverless cars... yet in more general sense, computing happens in all walks of life, from falling rocks to withering leaves, from climate change to extreme weather, from gene editing to digital matter... But one thing not changed is the scientific computing fundamentals that cover all college mathematics. Busy students, engineers and scientists need “fast-food” ways to compute and get problems solved reliably.

Oliver Heaviside once said “Mathematics is of two kinds, Rigorous and Physical. The former is Narrow: the latter Bold and Broad. To have to stop to formulate rigorous demonstrations would put a stop to most physico-mathematical inquiries. Am I to refuse to eat because I do not fully understand the mechanism of digestion?”<sup>1</sup> Today, we can ask a similar question: Am I to refuse to compute because I do not fully understand the mechanism of numerics? As we discussed in the Preface of the first edition, we need a new way of learning scientific computing so that we can focus more on “computational thinking.”

With these goals in mind, this edition includes the following new features:

(1) A significant amount of new material is introduced, specifically: four-dimensional volume visualization, interval limit, infinite series convergence, numerical multiple integral, arbitrary matrix analysis, matrix power, difference equations, numerical integral transforms, Laurent series, matrix equation solutions, multi-objective optimizations, dynamic programming and shortest path problems, matrix differential equations, switching ODEs, delay ODEs, special functions, principal component analysis, Monte Carlo algorithm, outlier detection, radial basis network, particle swarm optimization, and a completely new section on fractional calculus.

(2) The three-phase solution procedure proposed by the authors has been followed throughout the book. Namely, to solve a problem, the physical explanation of the mathematical problem to be solved is given first, followed by the methodology of how to formulate the problem in MATLAB®-compatible framework, and finally, the third phase is to call MATLAB functions to solve the problem. The guideline is useful in real world problem solving with lots of illustrative examples.

(3) Mathematical branches are arranged more systematically. Using the traditional styles in mathematical presentation (as in typical mathematics courses), however, concentrations are made on how the problems are solved. If there are existing MATLAB functions, or third-party products, suggestions are made to use them directly. If there are not, or if existing ones are problematic, new MATLAB functions are written and easy-to-use calling syntaxes are designed and explained.

(4) Soon after the publication of the first edition, MATLAB R2008b was released, from which the symbolic engine is replaced, and some of the commands, especially those

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<sup>1</sup>Edge A. Oliver Heaviside (1850–1927) - Physical mathematician. *Teaching Mathematics and Its Applications* 2: 55–61, 1983.

involving overload functions and Maple internal functions, cannot be used for the symbolic computation problems. In the new edition, compatibility with the new versions of MATLAB are supported.

(5) Enhanced examples and exercises are included to support the materials throughout the new edition. A complete set of teaching materials, composed of about 1500 PPT slides and a solutions manual, is provided with the book. The relevant materials can be downloaded from the authors-maintained web-site at

<https://mechatronics.ucmerced.edu/Scientific-Computing-with-MATLAB-2ndEd>

Financial support from the National Natural Science Foundation of China under Grant 61174145 is acknowledged. Thanks also go to Drs. Yanliang Zhang and Lynn Crisanti for arranging the first author's visit to MathWorks, Natick, MA for discussing a possible MOOC project for the book. A MOOC in Chinese is just made ready and will be released soon, thanks to the support from Liaoning Provincial Education Bureau and Northeastern University, China. Classroom videos in English are scheduled. New information and links on the MOOC progress will be announced in the above web-site.

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This new edition was suggested and supported by Sunil Nair, Publisher, CRC Press, Taylor and Francis Group. We are thankful for Sunil's patience and constructive comments. We wish to specially thank Michael Davidson, Project Editor, Taylor and Francis Group, LLC for helping us with an excellent copy-editing service. We would like to extend our appreciation to Professor Jian-Qiao Sun of University of California, Merced for adopting the first edition of the book for his ME021 "Engineering Computing" course and for motivating a new edition with a bigger format size. The new materials of the book have been used in a course entitled "MATLAB and Scientific Computing" for two semesters in Northeastern University, China. All the students of Lang Shijun Automation Experimental Classes 1309 and 1410, especially Mr. Weiming Mi and Mr. Huajia Lin, are acknowledged for some new insights and the hard work of modifications of the PPT slides and solutions manual.

Last but not least, Dingyü Xue would like to thank his wife Jun Yang and his daughter Yang Xue; YangQuan Chen would like to thank his wife Huifang Dou and his sons Duyun, David and Daniel, for their patience, understanding and complete support throughout this book project.

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## *Preface of the First Edition*

*Computational Thinking*<sup>2</sup>, coined and promoted by Jeannette Wing of Carnegie Mellon University, is getting more and more attention. “It represents a universally applicable attitude and skill set everyone, not just computer scientists, would be eager to learn and use” as acknowledged by Dr. Wing, “Computational Thinking draws on math as its foundations.” The present book responds to “Computational Thinking” by offering the readers enhanced math problem solving ability and therefore, the readers can focus more on “Computational Thinking” instead of “Computational Doing.”

The breadth and depth of one’s mathematical knowledge might not match his or her ability to solve mathematical problems. In today’s applied science and applied engineering, one usually needs to get the mathematical problems at hand solved efficiently in a timely manner without complete understanding of the numerical techniques involved in the solution process. Therefore, today, arguably, it is a trend to focus more on how to formulate the problem in a form suitable for computer solution and on the interpretation of the results generated from the computer. We further argue that, even without a complete preparation of mathematics, it is possible to solve some advanced mathematical problems using a computer. We hope this book is useful for those who frequently feel that their level of math preparation is not high enough because they still can get their math problems at hand solved with the encouragement gained from reading this book.

Using computers to solve mathematical problems today is ubiquitous. MATLAB®/Simulink is considered as the dominant software platform for applied math related topics. Sometimes, one simply does not know one’s problem could be solved in a much simpler way in MATLAB or Simulink. From what Confucius wrote, “The craftsman who wishes to work well has first to sharpen his implements,”<sup>3</sup> it is clear that MATLAB is the right, already sharpened “implement.” However, a bothering practical problem is this: MATLAB documentation only shows “this function performs this,” and what a user with a mathematical problem at hand wants is, “Given this math problem, through what reformulation, and then, use of what functions will get the problem solved.” Frequently, it is very easy for one to get lost in thousands of functions offered in MATLAB plus the same amount, if not more, of functions contributed by the MATLAB users community. Therefore, the major contribution of this book is to bridge the gap between “problems” and “solutions” through well grouped topics and tightly yet smoothly glued MATLAB example scripts and reproducible MATLAB-generated plots.

A distinguishing feature of the book is the organization and presentation of the material. Based on our teaching, research and industrial experience, we have chosen to present the course materials following the sequence

- Computer Mathematics Languages — An Overview
- Fundamentals of MATLAB Programming
- Calculus Problems

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<sup>2</sup>[http://www.cs.cmu.edu/afs/cs/usr/wing/www/Computational\\_Thinking.pdf](http://www.cs.cmu.edu/afs/cs/usr/wing/www/Computational_Thinking.pdf)

<sup>3</sup>Confucius. <http://www.confucius.org/lunyu/ed1509.htm>.



- Linear Algebra Problems
- Integral Transforms and Complex Variable Functions
- Nonlinear Equations and Optimization Problems
- Differential Equations Problems
- Data Interpolation and Functional Approximation Problems
- Probability and Statistics Problems
- Nontraditional Methods

In particular, in the nontraditional mathematical problem solution methods, we choose to cover some interesting and practically important topics such as set theory and fuzzy inference system, neural networks, wavelet transform, evolutionary optimization methods including genetic algorithms and particle swarm optimization methods, rough set based data analysis problems, fractional-order calculus (derivative or integral of non-integer order) problems, etc., all with extensive problem solution examples. A dedicated CAI (computer aided instruction) kit including more than 1,300 interactive PowerPoint slides has been developed for this book for both instruction and self-learning purposes.

We hope that readers will enjoy playing with the scripts and changing them as they wish for a better understanding and deeper exploration with reduced efforts. Additionally, each chapter comes with a set of problems to strengthen the understanding of the chapter contents. It appears that the book is presenting in certain depth some mathematical problems. However, the ultimate objective of this book is to help the readers, after understanding *roughly* the mathematical background, to avoid the tedious and complex technical details of mathematics and find the reliable and accurate solutions to the interested mathematical problems with the use of MATLAB computer mathematics language. There is no doubt that the readers' ability to tackle mathematical problems can be significantly enhanced after reading this book.

This book can be used as a reference text for almost all college students, both undergraduates and graduates, in almost all disciplines which require certain levels of applied mathematics. The coverage of topics is practically broad yet with a balanced depth. The authors also believe that this book will be a good desktop reference for many who have graduated from college and are still involved in solving mathematical problems in their jobs.

Apart from the standard MATLAB, some of the commercial toolboxes may be needed. For instance, the Symbolic Math Toolbox is used throughout the book to provide alternative analytical solutions to certain problems. Optimization Toolbox, Partial Differential Equation Toolbox, Spline Toolbox, Statistics Toolbox, Fuzzy Logic Toolbox, Neural Network Toolbox, Wavelet Toolbox, and Genetic Algorithm and Direct Search Toolbox may be required in corresponding chapters or sections. A lot of MATLAB functions designed by the authors, plus some third-party free toolboxes, are also presented in the book. For more information on MATLAB and related products, please contact

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The writing of this book started more than 5 years ago, when a Chinese version<sup>4</sup> was

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<sup>4</sup>Xue Dingyü and Chen YangQuan, *Advanced applied mathematical problem solutions using MATLAB*, Beijing: Tsinghua University Press, 2004

published in 2004. Many researchers, professors and students have provided useful feedback comments and input for the newly extended English version. In particular, we thank the following professors: Xinhe Xu, Fuli Wang of Northeastern University; Hengjun Zhu of Beijing Jiaotong University; Igor Podlubny of Technical University of Kosice, Slovakia; Shuzhi Sam Ge of National University of Singapore, Wen Chen of Hohai University, China. The writing of some parts of this book has been helped by Drs. Feng Pan, Daoxiang Gao, Chunna Zhao and Dali Chen, and some of the materials are motivated by the talks with colleagues at Northeastern University, especially Drs. Xuefeng Zhang and Haibin Shi. The computer aided instruction kit and solution manual were developed by our graduate students Wenbin Dong, Jun Peng, Yingying Liu, Dazhi E, Lingmin Zhang and Ying Luo.

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