

FRANK S. BUDNICK

APPLIED MATHEMATICS

FOR BUSINESS, ECONOMICS, AND THE SOCIAL SCIENCES

APPLIED MATHEMATICS

for Business,
Economics,
and the
Social Sciences

Second Edition

FRANK S. BUDNICK

University of Rhode Island

McGraw-Hill Book Company

New York | St. Louis | San Francisco | Auckland | Bogotá
• Hamburg | Johannesburg | London | Madrid | Mexico
Montreal | New Delhi | Panama | Paris | São Paulo
Singapore | Sydney | Tokyo | Toronto

CHAPTER-OPENING PHOTO CREDITS

1: Dianne Arndt; 2: Bruce Davidson/Magnum; 3: Barrie Rokeach; 4: Jean Gaumy/Magnum; 5: Barrie Rokeach; 6: Barrie Rokeach; 7: Sculpture by Kenneth Snelson; 8: Geisco; 9: J. Kalvar/Magnum; 10: Burk Vzzle/Magnum; 11: Burk Vzzle/Magnum; 12: Reflejo/Nancy Palmer; 13: American Museum of Natural History; 14: Barrie Rokeach; 15: David A. Rahm/McGraw-Hill; 16: Barrie Rokeach; 17: Barrie Rokeach; 18: Barrie Rokeach

APPLIED MATHEMATICS FOR BUSINESS, ECONOMICS, AND THE SOCIAL SCIENCES

Copyright © 1983, 1979 by McGraw-Hill, Inc.

All rights reserved.

Printed in the United States of America.

Except as permitted under the United States Copyright Act of 1976,
no part of this publication may be reproduced or distributed
in any form or by any means, or stored
in a data base or retrieval system, without
the prior written permission of the publisher.

1 2 3 4 5 6 7 8 9 0 VNHVNH 8 9 8 7 6 5 4 3

ISBN 0-07-008858-6

This book was set in Palatino by Progressive Typographers.
The editors were Donald G. Mason and Jonathan Palace;
the designer was Nicholas Krenitsky;
the production supervisor was Dominick Petrelese.
New drawings were done by Fine Line Illustrations, Inc.
Von Hoffmann Press, Inc., was printer and binder.

Library of Congress Cataloging in Publication Data

Budnick, Frank S.

Applied mathematics for business, economics, and
the social sciences.

Includes index.

1. Mathematics—1961-

I. Title.

QA37.2.B83 1983

510

82-21684

ISBN 0-07-008858-6

PREFACE

INTRODUCTION

Mathematics is an integral part of the education of students in business, economics, and the social sciences. There is increasingly a desire to improve the level of quantitative sophistication possessed by graduates in these types of programs. The objective is not to make mathematicians of these students, but to make them as comfortable as possible in an environment which increasingly makes use of quantitative analysis and the computer. Students are discovering that they must integrate mathematics, statistical analysis, and the computer in both required and elective courses within their programs. Furthermore, organizations are becoming more effective users of quantitative tools and the computer. Decision makers will be better equipped to operate within this type of environment if they are familiar with the more commonly used types of quantitative analyses and the technology of the computer. Such familiarity can assist them in being better "critics" and "users" of these tools, and hopefully, better decision makers.

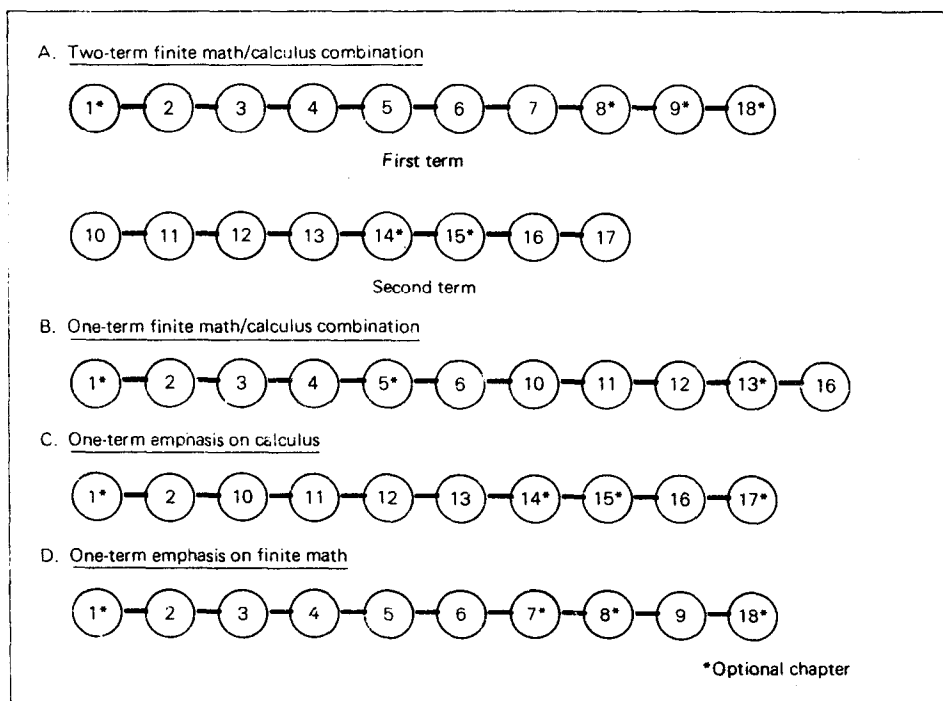
DESIGN OF BOOK

This book is an applied mathematics book for students in business, economics, and the social sciences. It provides a comprehensive treatment of selected topics in both finite mathematics and calculus. Although intended principally for students in business and economics, the book is appropriate for students in the social sciences. Designed primarily for a two-term course, the book can be adapted easily for a one-term course. It is appropriate for use in both two-year schools and four-year schools, as well as at the "foundation" level for graduate programs which require some mathematics background. M.B.A. and M.P.A. programs are typical graduate programs having this type of requirement.

The figure on the following page illustrates some *suggested* ways in which this text might be used.

Specific features of this book include:

- 1 A level of presentation which carefully develops and reinforces topics.
- 2 A style which appeals to the intuition of students and provides a great deal of visual reinforcement (over 300 figures).
- 3 An applied orientation which motivates students and provides a sense of purpose for studying mathematics.
- 4 An approach which first develops the mathematical concept and then reinforces with applications.
- 5 An approach which minimizes the use of rigorous mathematical proofs. Proofs are included at the end of selected chapters for interested persons.
- 6 Special aids which address the most universal shortcoming of students entering this type of course: weak algebra skills. These aids include a review of key algebra principles in Chapter 1. A chapter pretest allows the student and



instructor to identify areas requiring special attention. In addition, “Algebra Flashbacks” are used throughout the book to assist the student in the recall of key rules or concepts. The flashback usually consists of a restatement of a rule or concept with a reference to the appropriate section in Chapter 1.

7 Notes to students which provide them with special insights.

8 “Points for Thought and Discussion” which allow students to pause for a moment and reconsider a concept or example from a different perspective. Their purpose is to reinforce and extend the student’s understanding.

9 “Minicases” at the end of many chapters which provide challenging applications.

10 A multitude of other learning aids, including almost 450 solved examples, a wealth of exercises (over 2,200, most of which are new) chapter tests, chapter objectives, lists of key terms and concepts, and summary lists of important formulas.

11 An instructors manual which contains answers for all exercises and tests, suggestions for different course structures, prototype examples for new applications, transparency masters for selected figures, and a bank of questions for constructing quizzes and tests.

Although applications are presented throughout the book, Chapters 5 and 13 are devoted entirely to applications. The intent is that instructors cover as many applications in these chapters as they feel appropriate for their students.

Some exercises in the book are considered to be of a higher level of difficulty than most others. These are preceded by an asterisk (*).

SUMMARY OF NEW FEATURES AND CHANGES FOR THIS EDITION

In preparing this revision I surveyed those who taught from the first edition as well as those who decided not to adopt the first edition. The comments of those surveyed proved extremely valuable. The redesign of the book has resulted in the elimination or reduced coverage of some material and the expansion of other material, as well as some rearrangement of subject matter. I hope that those who continue to use the book find it more satisfactory for their needs; to those who are adopting for the first time, I hope that you and your students will be pleased.

Highlights of the significant changes follow:

- In this edition, the treatment of *algebra* and *set theory* has been consolidated in one chapter. The algebra treatment is a bit more abbreviated but continues to be as inclusive as in the first edition. The set theory portion of this chapter is also shortened from the chapter status it enjoyed in the first edition.
- In Chapter 2, the discussion of cartesian products has been eliminated. A new section provides a brief overview of different classes of mathematical functions.
- In Chapter 5, the material on break-even models has been expanded to include examples of *three-alternative analysis* and *multiproduct break-even analysis*.
- Chapter 6 represents a repositioning of the material on matrix algebra. Aside from a new subsection on the properties of determinants, the most significant change is an *expanded section on applications*.
- Chapter 7 represents a major expansion of the material on linear programming. More attention is given to problem formulation, and a new section discusses *computer solution methods*. A sample LP package is illustrated. *Shadow price* and *sensitivity analysis* concepts are introduced and illustrated using the LP package.
- Chapter 8 is an expanded treatment of the original simplex chapter. A new section discusses *alternative optimal solutions*, *no feasible solution*, and *unbounded solutions*. Another new section introduces the *dual problem*.
- Chapter 9 represents a repositioning of the material on probability theory.
- The material on exponential and logarithmic functions has been separated from other functional forms. Chapter 10 introduces nonlinear functions by discussing quadratic and other polynomial functions and their application. Chapter 14 consolidates the presentation of exponential and logarithmic functions and their application. New sections discuss the *conversion to base-e-functions* and *solving logarithmic and exponential equations*. This chapter also contains an *expanded set of applications*.
- Chapter 15 represents a significant expansion in the treatment of the calculus of functions of several variables. New (optional) sections examine *n-variable optimization* and *optimization subject to constraints*. This chapter also illustrates *three-dimensional graphics capabilities of computers*.

- The material on integral calculus has been expanded slightly and repackaged in two chapters. Chapter 16 introduces integral calculus and its methodology. The focus is upon the techniques of integration. A new section discusses *differential equations*. Chapter 17 focuses upon applications of integral calculus.
- Chapter 18 updates the material on the mathematics of finance by expanding the interest table ranges. A new extension of this material is a section which discusses *cost-benefit analysis*.
- Another new feature in this edition is the inclusion of *minicases* at the ends of 11 chapters. These provide a challenge to students which goes beyond the regular exercises in the text.

ACKNOWLEDGMENTS

I wish to express my sincere appreciation to those persons who have contributed either directly or indirectly to this project. I wish to thank Professor Emily Bronstein, Prince George's Community College; Professor John Dettman, Oakland University; Professor Benjamin Eichhorn, Temple University; Professor Grace Greenberg, Loyola College; Professor D. Kannan, University of Georgia; Professor Dean C. Morrow, Robert Morris College; and Professor Mustafa R. Yilmaz, Northeastern University. Their comments proved extremely helpful in rewriting.

A very special thanks goes to Professor Robert A. Moreland, Texas Tech University. His detailed comments on the entire manuscript were particularly helpful. Also, special thanks go to Dr. Janet McLeavey, who provided an important final review of the manuscript.

I want to thank the people at McGraw-Hill with whom I worked directly. These persons include Donald Mason, Jon Palace, and Nick Krenitsky. They each provided the kind of support that an author truly appreciates.

I also want to thank Nancy Nakamoto, Sue Rose, and Joe Daly for their assistance in developing problems and solution sets; and Ede Williams and Charlotte Manni for their superb work in typing the manuscript and its revisions. Special thanks also go to my students who served as "guinea pigs" for debugging new exercise sets.

I also wish to thank my parents, Mr. and Mrs. Willard L. Budnick, for their continued support, encouragement, and love during this and all other endeavors.

Finally, I want to thank my precious family—Jane, Chris, Scott, and Kerry—for their patience, understanding, encouragement, and love.

Frank S. Budnick

CONTENTS

PREFACE	xv
CHAPTER 1	
A REVIEW OF ALGEBRA AND SET THEORY (OPTIONAL)	3
1.1 The Real Number System	4
Real Numbers / Absolute Value	
1.2 Polynomials	6
Positive Integer Exponents / Polynomial Expressions / Addition and Subtraction of Polynomials / Multiplication of Polynomials / Division of Polynomials	
1.3 Factoring	12
Monomial Factors / Quadratic Polynomials / Other Special Forms	
1.4 Fractions	16
Addition and Subtraction of Fractions / Multiplication and Division	
1.5 Exponents and Radicals	20
Fractional Exponents / Radicals	
1.6 Equations	23
Equations and Their Properties / Solving First-Degree Equations / Solving Second-Degree Equations / Solving Inequalities	
1.7 Sets Defined	28
Sets / Special Sets / Venn Diagram Representation	
1.8 Set Operations	32
Set Equality / Union of Sets / Intersection of Sets / Other Properties	
1.9 Sample Applications	35
CHAPTER 2	
MATHEMATICAL FUNCTIONS	49
2.1 Functions	49
Functions Defined / The Nature and Notation of Functions / Domain and Range Considerations / Restricted Domain and Range / Multivariate Functions	
2.2 Types of Functions	57
Constant Functions / Linear Functions / Quadratic Functions / Cubic Functions / Polynomial Functions / Rational Functions / Exponential Functions / Logarithmic Functions / Combinations of Functions / Composite Functions / Implicit Functions	
2.3 Graphical Representation of Functions	65
Rectangular Coordinate System / Graphing Functions in Two Dimensions	

CHAPTER 3	
LINEAR EQUATIONS	79
3.1 Characteristics of Linear Equations	80
General Form / Representation Using Linear Equations /	
Generalizing for n -Variable Linear Equations	
3.2 Graphical Characteristics	86
Graphing Two-Variable Equations / Intercepts /	
The Equation $x = k$ / The Equation $y = k$ / Slope	
3.3 Slope-Intercept Form	95
From a Different Vantage Point / Interpreting the Slope	
and y Intercept	
3.4 Determining the Equation of a Straight Line	98
Slope and Intercept / Slope and One Point / Two Points	
3.5 Linear Equations Involving More than Two Variables	104
Three-Dimensional Coordinate Systems / Equations Involving Three	
Variables / Equations Involving More than Three Variables	
3.6 Additional Applications	110
 CHAPTER 4	
SYSTEMS OF LINEAR EQUATIONS	119
4.1 Introduction	119
Systems of Equations / Solution Sets	
4.2 Two-Variable Systems of Equations	120
Graphical Analysis / Graphical Solutions / The Elimination	
Procedure / $(m \times 2)$ Systems	
4.3 Three-Variable Systems	129
Graphical Analysis / Elimination Procedure for (3×3) Systems /	
Fewer than Three Equations / More than Three Equations /	
n -Variable Systems	
4.4 Gauss-Jordan Procedure	139
The General Idea / The Method	
4.5 Summary	145
<i>Minicase A: XYZ Manufacturing Company</i>	148
<i>Minicase B: Gauss-Jordan Procedure: Computerized Version</i>	149
 CHAPTER 5	
APPLICATIONS OF LINEAR FUNCTIONS	
AND SYSTEMS OF EQUATIONS	151
5.1 Linear Functions	151
General Form and Assumptions / Linear Cost Functions /	
Linear Revenue Functions / Linear Profit Functions	
5.2 Other Examples of Linear Functions	158
5.3 Break-Even Models	165
Assumptions / Break-Even Analysis	

5.4 Other Applications	175
Equilibrium between Supply and Demand / Defining Mathematical Functions / Product-Mix Problem / Portfolio Model / Blending Model	
<i>Minicase: Automobile Replacement Decision</i>	189
 CHAPTER 6	
MATRIX ALGEBRA	191
6.1 Introduction to Matrices	191
What Is a Matrix? / Purpose of Studying Matrix Algebra	
6.2 Special Types of Matrices	193
Vectors / Square Matrices / Transpose of a Matrix	
6.3 Matrix Operations	196
Matrix Addition and Subtraction / Scalar Multiplication / The Inner Product / Matrix Multiplication / Representation of an Equation / Representation of Systems of Equations	
6.4 The Determinant	206
The Determinant of a (1×1) Matrix / The Determinant of a (2×2) Matrix / The Determinant of a (3×3) Matrix / The Method of Cofactors (Optional) / Properties of Determinants	
6.5 The Inverse of a Matrix	214
Determining the Inverse / Finding the Inverse Using Cofactors (Optional) / The Inverse and Systems of Equations	
6.6 Selected Applications	222
<i>Minicase A: Brand Switching</i>	242
<i>Minicase B: Matrix Inversion Computerized</i>	243
 CHAPTER 7	
LINEAR PROGRAMMING: AN INTRODUCTION	245
7.1 Linear Programming	246
Introduction / A Scenario / Structural Constraints and Nonnegativity Constraints / The Generalized LP Model	
7.2 Some Applications of Linear Programming	248
Diet-Mix Models / Transportation Models / Capital Budgeting Models / Blending Models	
7.3 Graphical Solutions	258
The Graphics of Linear Inequalities / Systems of Linear Inequalities / Area of Feasible Solutions / Incorporating the Objective Function / Corner-Point Solutions / Alternative Optimal Solutions / No Feasible Solution / Unbounded Solutions	
7.4 Computer Solution Methods	274
Requirements of the Simplex Method / Basic Feasible Solutions and the Simplex / An Illustration of an LP Package / Shadow Prices / Sensitivity Analysis	
<i>Minicase: Contract Awards</i>	295

CHAPTER 8	
THE SIMPLEX METHOD	297
8.1 The Simplex Procedure	297
Overview of the Simplex Procedure / Solution by Enumeration /	
The Algebra of the Simplex Procedure / Incorporating the Objective	
Function / Summary of Simplex Procedure / Maximization	
Problems with Mixed Constraints / Minimization Problems	
8.2 Special Phenomena	312
Alternative Optimal Solutions / No Feasible Solution /	
Unbounded Solutions	
8.3 The Dual Problem	317
Formulation of the Dual / Primal-Dual Solutions / Epilogue	
 CHAPTER 9	
INTRODUCTION TO PROBABILITY THEORY	327
9.1 Permutations and Combinations	328
Permutations / Combinations	
9.2 Basic Probability Concepts	331
Experiments, Outcomes, and Events / Probabilities and Odds /	
Some Additional Rules of Probability / Statistical Independence /	
Statistical Dependence	
9.3 Mathematical Expectation	349
Random Variables / Probability Distributions / Mean and	
Standard Deviation / Expected Monetary Value	
9.4 The Binomial Probability Distribution	356
Bernoulli Processes / The Binomial Distribution / Mean and	
Standard Deviation of the Binomial Distribution	
9.5 Continuous Probability Distributions	363
The Normal Probability Distribution	
<i>Minicase: The Birthday Problem</i>	379
 CHAPTER 10	
NONLINEAR FUNCTIONS	381
10.1 When Linear Functions Are Inappropriate	381
10.2 Quadratic Functions and Their Characteristics	385
Mathematical Form / Graphical Representation / Special Insights	
to Sketching Quadratic Functions / Determining the Equation of	
Quadratic Functions	
10.3 Quadratic Functions: Applications	393
10.4 Other Nonlinear Functions	400
Polynomial Functions / Rational Functions	
<i>Minicase: Nonlinear Break-Even and Profit Analysis</i>	409

CHAPTER 11	
DIFFERENTIATION	411
11.1 Limits and Continuity	412
Limits of Functions / Some Properties of Limits /	
Limits and Infinity / Continuity	
11.2 Average Rate of Change	426
Average Rate of Change and Slope	
11.3 The Derivative	434
Instantaneous Rate of Change / The Limit Approach to Finding	
the Derivative / Instantaneous Rate of Change Revisited	
11.4 Differentiation	445
Rules of Differentiation / Instantaneous Rate of Change—Again	
11.5 Higher-Order Derivatives	452
The Second Derivative / Third and Higher-Order Derivatives	
11.6 Differentiation of Special Functional Forms	455
(Optional)	
Chain Rule / Other Derivatives	
CHAPTER 12	
OPTIMIZATION: METHODOLOGY	469
12.1 Derivatives: Additional Interpretations	469
The First Derivative / The Second Derivative / Concavity and	
Inflection Points / Concavity from a Different Perspective	
12.2 Identification of Maxima and Minima	478
Relative Extrema / Critical Points / The First-Derivative Test / The	
Second Derivative Test / When the Second-Derivative Test Fails /	
Higher-Order Derivative Test (Optional)	
12.3 Restricted-Domain Considerations	493
When the Domain Is Restricted	
12.4 Curve Sketching	496
Key Data Points	
CHAPTER 13	
OPTIMIZATION: APPLICATIONS	505
13.1 Revenue, Cost, and Profit Applications	505
Revenue Applications / Cost Applications / Profit Applications	
13.2 Marginal Approach to Profit Maximization	518
Marginal Revenue / Marginal Cost / Marginal Profit Analysis	
13.3 Additional Applications	525
<i>Minicase: The EOQ Model</i>	543

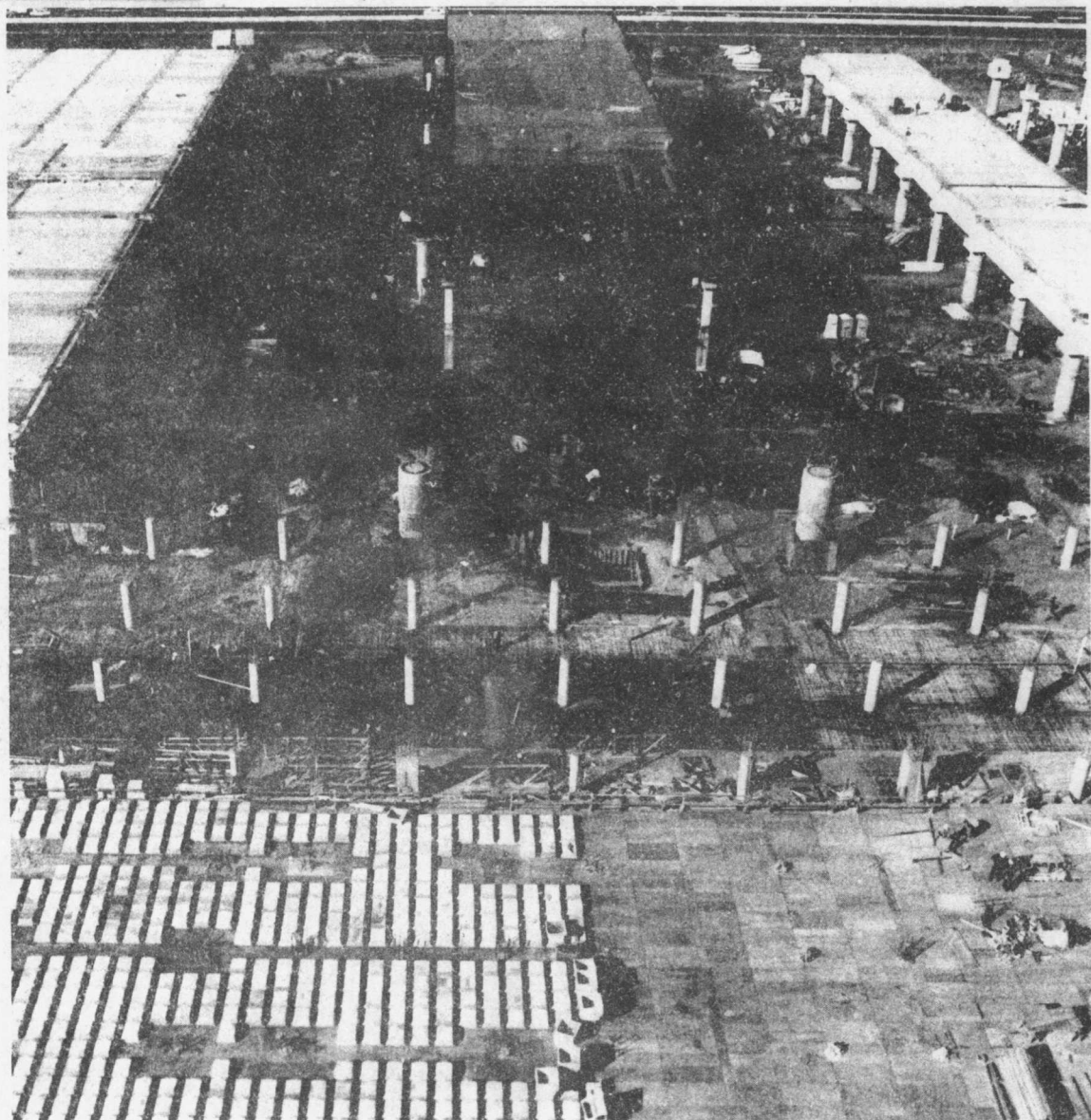
CHAPTER 14	
EXPONENTIAL AND LOGARITHMIC FUNCTIONS	547
14.1 Characteristics of Exponential Functions	547
Characteristics of Exponential Functions / Base- e Exponential Functions / Conversion to Base- e Functions	
14.2 Applications of Exponential Functions	555
14.3 Derivatives and Their Application	565
Derivatives of Exponential Functions	
14.4 Logarithmic Functions and Their Derivatives	570
Logarithms / Properties of Logarithms / Solving Logarithmic and Exponential Equations / Logarithmic Functions / Derivatives of Logarithmic Functions	
14.5 Applications of Logarithmic Functions	579
<i>Minicase: Time of Death?</i>	589
 CHAPTER 15	
OPTIMIZATION: FUNCTIONS OF SEVERAL VARIABLES	591
15.1 Graphical Representation of Bivariate Functions	592
Graphical Representation / Sketching Bivariate Functions	
15.2 Partial Derivatives	596
Derivatives of Bivariate Functions / Interpreting Partial Derivatives / Second-Order Derivatives	
15.3 Optimization of Bivariate Functions	607
Critical Points / Distinguishing among Critical Points	
15.4 Applications of Bivariate Optimization	620
15.5 n-Variable Optimization (Optional)	632
Necessary Condition for Relative Extrema / Sufficient Conditions	
15.6 Optimization Subject to Constraints (Optional)	636
The Lagrange Multiplier Method (Equality Constraint) / Sufficient Condition / n -Variable Single-Equality Constraint Case / Interpreting λ	
<i>Minicase: Back-Order Inventory Model</i>	648
 CHAPTER 16	
INTEGRAL CALCULUS: AN INTRODUCTION	651
16.1 Antiderivatives	652
The Antiderivative Concept / Revenue and Cost Functions	
16.2 Rules of Integration	656
Integration / Rules of Integration	
16.3 Additional Rules of Integration	661
16.4 Differential Equations	666
Classifications of Differential Equations / Solutions of Ordinary Differential Equations / Extensions of Differential Equations	

16.5 Other Techniques of Integration (Optional)	672
Integration by Parts / Integration by Partial Fractions / Table of Integrals	
CHAPTER 17	
INTEGRAL CALCULUS: APPLICATIONS	689
17.1 Definite Integrals	689
The Definite Integral / Evaluating Definite Integrals / Properties of Definite Integrals	
17.2 Definite Integrals and Areas	698
Areas between a Function and the x Axis / Finding Areas between Curves	
17.3 Applications of Integral Calculus	707
17.4 Integral Calculus and Probability (Optional)	721
<i>Minicase: The Social Security Dilemma: A Problem of Solvency</i>	728
CHAPTER 18	
MATHEMATICS OF FINANCE	731
18.1 Interest and Its Computation	731
Simple Interest / Compound Interest	
18.2 Single-Payment Computations	735
Compound Amount / Present Value / Other Applications of the Compound-Amount Formula / Effective Interest Rates	
18.3 Annuities and Their Future Value	745
The Sum of an Annuity / Determining the Size of an Annuity	
18.4 Annuities and Their Present Value	751
The Present Value of an Annuity / Determining the Size of the Annuity / Mortgages	
18.5 Cost-Benefit Analysis	758
Discounted Cash Flow / Extensions of Discounted Cash Flow Analysis	
<i>Minicase: XYZ Corporation</i>	768
APPENDIX A SUMMATION NOTATION	A.1
SELECTED ANSWERS	A.3
INDEX	I.1

**APPLIED
MATHEMATICS**
for Business,
Economics,
and the
Social Sciences



1



A REVIEW OF ALGEBRA AND SET THEORY (OPTIONAL)

1.1 THE REAL NUMBER SYSTEM	1.7 SETS DEFINED
1.2 POLYNOMIALS	1.8 SET OPERATIONS
1.3 FACTORING	1.9 SAMPLE APPLICATIONS
1.4 FRACTIONS	KEY TERMS AND CONCEPTS
1.5 EXPONENTS AND RADICALS	ADDITIONAL EXERCISES
1.6 EQUATIONS	CHAPTER TEST

CHAPTER OBJECTIVES

- Review the fundamentals of algebra which are necessary for the study of the material in the remainder of this text
 - Provide an overview of the nature of sets and their representation, logic, and algebra
 - Illustrate the application of set theory
-

Algebra is the only prerequisite for using this text. The first part of this chapter provides a brief review of the elements of algebra which the author believes are important in studying the material in the remainder of the chapter. To guide you in your review of algebra it is suggested that you take the following self-correcting algebra test. Its purpose is to help you diagnose those areas in which you need more review. The results of the test can guide you in your review of Secs. 1.1 to 1.6.

Algebra Pretest

	CORRESPONDING SECTION IN CHAPTER
1 $ -10 =$	1.1
2 $x^3 \cdot x^4 =$	1.2
3 $[(x^3)^2]^3 =$	1.2
4 $x^5/x^3 =$	1.2
5 $(4x - 2y + z) - (-3x + 4y - 2z) =$	1.2
6 $\frac{2x^2(3x^3)}{(-2x^2)^2} =$	1.2
7 Factor $2a^3b^2c + 4a^2bc^2$.	1.3