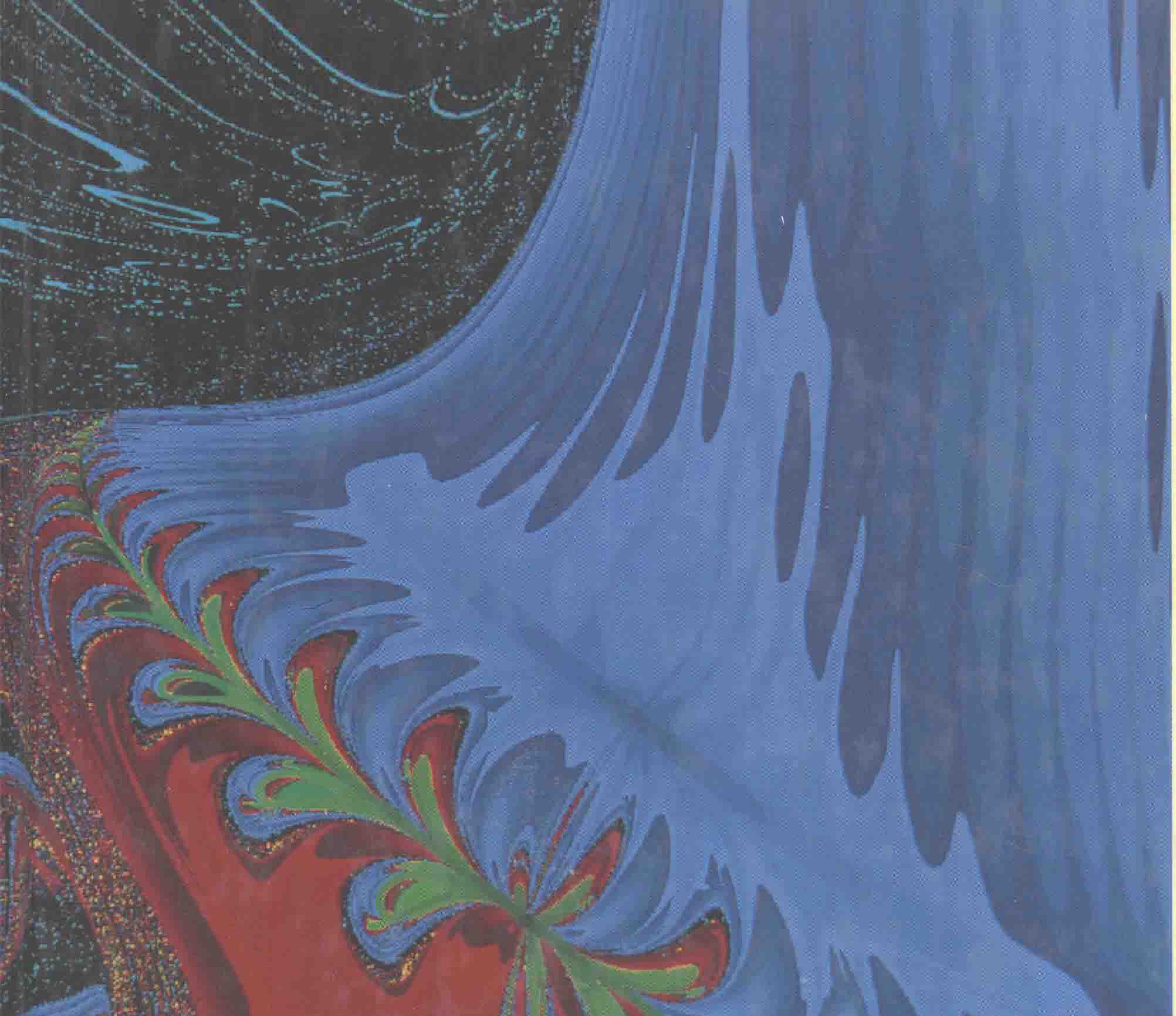

FIFTH EDITION

PRECALCULUS

DAVID COHEN

PRECALCULUS

A PROBLEMS-ORIENTED APPROACH

Fifth Edition

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The image is obtained in analyzing a numerical method of solution for a system of nonlinear equations arising in an engineering design problem. Iteration of functions, a topic new to this edition of *Precalculus,* plays a key role in generating the solutions and the resulting image.

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FOR MY SON ANDY

PREFACE TO THE FIFTH EDITION

This text is for students who are preparing to take calculus or other courses requiring a background in precalculus mathematics. As in the earlier editions, my goal has been to create a book that is *accessible* to the student. The presentation is student-oriented in three specific ways. First, I've tried to talk to, rather than lecture at, the student. Second, examples are consistently used to introduce, to explain, and to motivate concepts. And third, all of the initial exercises for each section are carefully correlated with the worked examples in that section.

AUDIENCE In writing *Precalculus,* I have assumed that the students have been exposed to intermediate algebra, but that they have not necessarily mastered that subject. Also, for many precalculus students, there may be a gap of several years between their last mathematics course and the present one. For these reasons, the review material in parts of the first two chapters is unusually thorough.

CURRICULUM REFORM This new edition of *Precalculus* reflects several of the major themes that have been developed in the curriculum reform movement of this decade. Overall, there is an increased emphasis upon graphs and visualization, and there is a tighter linkage between the graphical, numerical, and algebraic viewpoints. (See, for example, Section 4.3, *More on Iteration. Quadratics and Population Growth;* or Section 5.5, *Equations and Inequalities with Logs and Exponents.*)

TECHNOLOGY In the following discussion and throughout this text, the term *graphing utility* refers to either a graphics calculator or a computer with software for graphing and analyzing functions.

Over the past five to ten years, all of us in the mathematics teaching community have become increasingly aware of the *graphing utility* and its potential for making a positive impact in our teaching. While it is clear that more reforms lie ahead, many of the specific details have already evolved in workshops and in classrooms across the country. At present, however, even within a given school, some instructors teach the course with the graphing utility, while others do not. Consequently, this 1997 edition of *Precalculus* allows for both approaches to instruction. In addition to the regular exercise sets at the end of every section, there are also graphing utility exercises for the appropriate sections in Chapters 2 through 12.

FEATURES

1. *Word problems and applications.* Word problems and strategies for solving them are explained and developed throughout the book. Maximum-minimum problems relating to quadratic functions are discussed in detail in Section

4.5. The preceding section introduces some strategies for approaching these problems. To ensure that precalculus students gain appropriate practice and experience with these important strategies, the initial exercises in Section 4.4 make specific references to the corresponding worked examples in the text. In general, applications are integrated throughout the text. Two complete sections (5.6 and 5.7) are devoted to applications of the exponential function.

2. *Emphasis on graphing.* Graphs and techniques for graphing are developed throughout the text, and graphs are used to explain and reinforce algebraic concepts. See, for example, Sections 2.6 (inequalities), 3.3 (graphing techniques), and 7.4 (sine and cosine).
3. *Calculator exercises.* There are two broad categories of calculator exercises in this text:
   (i) GRAPHING UTILITY EXERCISES
   Following the regular exercise sets, there are graphing utility exercises for the appropriate sections in Chapters 2 through 12. These exercises reinforce and supplement the core material. Many of the exercises are divided into parts and ask the student to investigate a question from each of the three perspectives—graphical, numerical, and algebraic. (See, for example, Exercises 21 and 22 on pages 208–209.)
   (ii) (ORDINARY) CALCULATOR EXERCISES
   As in the previous edition, there are numerous calculator exercises integrated throughout the regular exercise sets. Just as with the graphing utility exercises, these exercises reinforce or supplement the core material. Where appropriate, real-life data is used. See, for example, Exercises 30 and 33 on page 184. Some of the calculator exercises contain surprising results that motivate subsequent theoretical questions; and a few of these exercises demonstrate that the use of a calculator cannot replace thinking or the need for mathematical proofs.
4. *End-of-chapter material.* Each chapter concludes with a detailed chapter summary, a *Writing Mathematics* section, an extensive Chapter Review exercise set, and a Chapter Test. In the *Writing Mathematics* questions, the student is asked to organize his or her thoughts and respond in complete sentences. Some of the questions are simply true-or-false questions that can be explained in just a sentence or two. In other cases, some study, perhaps some group work, and a more elaborate written response is required. For example, in Chapter 1 (pages 30–32), the *Writing Mathematics* section describes ancient Babylonian, Greek, and Arab techniques for solving quadratic equations; the student is then asked to demonstrate and explain the method in her or his own words. As another example, in Chapter 9, the *Writing Mathematics* section describes a new proof for the law of cosines from the *College Mathematics Journal,* and the student is asked to fill in the details. (See page 594.)

CHANGES IN THIS EDITION

Comments and suggestions from students, instructors, and reviewers have helped me to revise this text in a number of ways that I believe will make the book more useful to the instructor and more accessible to the student. The major changes occur in the following areas.

1. The material on inequalities, an important subject for both precalculus and calculus, has been moved from Chapter 1 to Chapter 2 (Sections 2.5 and

2.6). This allows the use of graphs to help explain and interpret the solutions of inequalities. This is one aspect of the increased emphasis on graphs and visualization in this new edition.

2. A new feature, the Graphical Perspective begins in Chapter 2. (See, for example, pages 41 and 50.) These graphs supplement the text discussion and help the student with visualization. Of course, any graph in any math book can be said to do this. But the Graphical Perspectives are designed so that students who have a graphing utility can (and should be encouraged to) produce their own versions of the picture. Experience shows that this involves the students in a more active way in reading the text. (For students without the technology, there is no problem; they simply get a book with more pictures and more emphasis on graphical interpretation of algebraic results.)
3. As described earlier, beginning in Chapter 2 there are graphing utility exercises, at the ends of appropriate sections, following the regular exercise sets. In contrast to the previous edition, these graphing utility exercises are not tied to a specific brand or model of calculator. (Keystroke-level instructions with additional exercises are, however, provided in the *Graphing Utilities Manual* that is available with this text. Additionally, Appendix A.1 at the end of this book provides a brief overview of the use of graphing utilities in the context of precalculus.)
4. In Chapter 3 (*Functions*), Section 3.3 has been expanded to include a discussion of the order in which translations and reflections are to be carried out. In Section 3.4, text and exercises on iteration of functions have been added. Graphical iteration (the ''cobweb diagram'') is explained in detail. This forms the background for a major new section in Chapter 4, *More on Iteration. Quadratics and Population Growth.* This section (4.3) introduces fixed points, both algebraically and geometrically, and uses them in a very informal graphical introduction to population growth and dynamical systems. For students (and instructors) who wish to pursue these topics further, but still at the precalculus level, I recommend the excellent paperback by Robert L. Devaney, *Chaos, Fractals, and Dynamics* (Menlo Park, California: Addison-Wesley, 1990).
5. In Chapter 5, a new section (5.5) has been added, *Equations and Inequalities with Logs and Exponents.* (A portion of this material on equations was covered only briefly in the previous edition.) Graphs and Graphical Perspectives are used extensively in this section to interpret the examples and solutions.

   In Section 5.7, the data in the exercises on exponential growth and decay has been extensively updated and (as in Chapter 4) there is an increased emphasis on modeling real-life data (with linear, quadratic, and exponential functions). There are new exercises asking the student to compare the results obtained by using different models for the same data set. Exercises on the logistic growth curve have also been added to the section on exponential growth and decay.
6. Chapter 7 in the previous edition (*Trigonometric Functions of Real Numbers*) is now split into two chapters: Chapter 7, *Graphs of the Trigonometric Functions;* and Chapter 8, *Analytical Trigonometry.*

   In Chapter 7, the initial section on radian measure from the previous edition has been expanded slightly and divided into two sections. Section 7.1 now contains material on the definition of radian measure and the evaluation of the trigonometric functions only. The geometric material on arc length, sector area, and angular speed is now covered in Section 7.2. Note that none of these three topics involves the trigonometric functions in its definition. Thus, moving these topics out of the key Section 7.1 sharpens the focus of 7.1. Additionally, the geometry in Section 7.2 is enriched with the discussion of the area of a segment of a circle and exercises involving elements of gothic architecture and Hippocrates' results on the area of a lune. (See Exercises 29–36 on pages 423–426.)

The introductory section on the graphs of sine and cosine (7.3 in the previous edition, 7.4 in the present one) now includes an extended example on solving trigonometric equations of the form $\cos x = k$. Classroom experience has shown that this is an excellent way to emphasize the properties of the cosine graph (and, likewise in exercises, the sine graph) and to integrate the graphical and algebraic viewpoints. This material also helps to prepare the student for sections in the next chapter on trigonometric equations and inverse functions.

As an application in analyzing functions of the form $y = A \sin(Bx - C)$ and $y = A \cos(Bx - C)$, there is a new section (7.5) on simple harmonic motion.

7. Chapter 8, *Analytical Trigonometry,* incorporates the addition formulas for sine, cosine, and tangent in one section (8.1). Section 8.3 on the product-to-sum and the sum-to-product formulas is new.
8. Two new sections on partial fractions have been added in Chapter 12 (*Roots of Polynomial Equations*). Together, they form a comprehensive treatment of the subject, more complete than that offered in many calculus texts. If the instructor wishes, however, only the first section can be covered; it is self-contained and omits the theory.

SUPPLEMENTARY MATERIALS

1. The *Student's Solutions Manual,* by Ross Rueger, contains complete solutions for the odd-numbered exercises and for the test questions at the end of each chapter.
2. The *Instructor's Solutions Manual,* by Ross Rueger, contains answers or solutions for every even-numbered exercise in *Precalculus.*
3. The *Graphing Utilities Manual* covers each of the six most popular graphing calculators and the following three computer programs: Mathematica, MAPLE, and DERIVE. It also provides documentation and laboratory exercises for *GraphToolz* (described in No. 8).
4. The *Computer-Generated Testing Program,* Westest 3.2, is available to schools adopting *Precalculus.* (There are versions for both Macintosh and Windows.)
5. The *Test Bank,* by Charles Heuer, is a package that contains three different chapter tests, as well as two multiple-choice versions, for each chapter in *Precalculus.*
6. West's *Math Tutor* software, by Mathens, for DOS, Windows, and Macintosh, is an algorithmically based tutorial, keyed to the text, that allows students to work problems on the computer. The software has been completely revised based on extensive reviews, making it much easier to use.
7. *Cohen's Precalculus Video Series* are video tapes available to qualified adopters. Each tape is keyed to a particular section of *Precalculus.*
8. *Graph Toolz,* by Tom Saxton, is a software program for graphing and evaluating functions. This easy-to-use software is free to qualified adopters. Doc-

umentation and laboratory exercises are found in the *Graphing Utilities Manual.* (It is available for the Macintosh family of computers.)

9. *Transparency masters* for many of the key figures or tables appearing in the text are available to schools adopting *Precalculus.*

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David Cohen
Lunada Bay, California, 1997

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CHAPTER 1

ALGEBRA BACKGROUND FOR PRECALCULUS

There are topics [in algebra] *whose consideration prepares a student for a deeper understanding.*

Leonhard Euler (1707–1783) in *Introduction to Analysis of the Infinite,* translated by John D. Blanton (New York: Springer-Verlag, 1988)

Perhaps Pythagoras was a kind of magician to his followers because he taught them that nature is commanded by numbers. There is a harmony in nature, he said, a unity in her variety, and it has a language: numbers are the language of nature.

Jacob Bronowski in *The Ascent of Man* (Boston: Little, Brown and Co., 1973)

INTRODUCTION

In Chapters 1 and 2 we review several key topics from algebra and coordinate geometry that form the foundation for our work in precalculus. Although you are probably already familiar with some of this material from previous courses, don't be lulled into a false sense of security. Now, in your second exposure to these topics, you really have the opportunity to master them. Take advantage of this opportunity; it will pay great dividends both in this course and in calculus.

1.1 SETS OF REAL NUMBERS

What secrets lie hidden in decimals?

Stephen P. Richards in *A Number for Your Thoughts* (New Providence, NJ: S. P. Richards, 1982)

Natural numbers have been used since time immemorial; fractions were employed by the ancient Egyptians as early as 1700 B.C.; and the Pythagoreans, in ancient Greece, about 400 B.C., discovered numbers, like $\sqrt{2}$, which cannot be fractions.

Stefan Drobot in *Real Numbers* (Englewood Cliffs, NJ: Prentice-Hall, Inc., 1964)

Here, as in your previous mathematics courses, most of the numbers we deal with are *real numbers.* These are the numbers used in everyday life, in the sciences, in industry, and in business. Perhaps the simplest way to define a real number is this: A **real number** is any number that can be expressed in decimal form. Some examples of real numbers are

$$7(=7.000\ldots) \qquad -\tfrac{2}{3}(=-0.\overline{6}) \qquad \sqrt{2}\,(=1.4142\ldots)$$

(The bar above the 6 in the decimal $-0.\overline{6}$ indicates that the 6 repeats indefinitely.)

Certain sets of real numbers are referred to often enough to be given special names. These are summarized in the box that follows.

PROPERTY SUMMARY SETS OF REAL NUMBERS

NAME	DEFINITION AND COMMENTS	EXAMPLES
Natural numbers	These are the ordinary counting numbers: 1, 2, 3, and so on.	1, 4, 29, 1066
Integers	These are the natural numbers along with their negatives and zero.	−26, 0, 1, 1776
Rational numbers	As the name suggests, these are the real numbers that are *ratios* of two integers (with nonzero denominators, of course). It can be proved that a real number is rational if and only if its decimal expansion terminates (e.g., 3.15) or repeats (e.g., $2.\overline{43}$).	$4\ (=\frac{4}{1})$, $-\frac{2}{3}$, $1.7\ (=\frac{17}{10})$, $4.\overline{3}$, $4.1\overline{73}$
Irrational numbers	These are the real numbers that are not rational. Section A.4 of the Appendix contains a proof of the fact that the number $\sqrt{2}$ is irrational. The proof that π is irrational is more difficult. The first person to prove that π is irrational was the Swiss mathematician J. H. Lambert (1728–1777).	$\sqrt{2}$, $3+\sqrt{2}$, $3\sqrt{2}$, π, $4+\pi$, 4π

As you've seen in previous courses, the real numbers can be represented as points on a **number line,** as shown in Figure 1. As indicated in Figure 1, the point associated with the number zero is referred to as the **origin.**

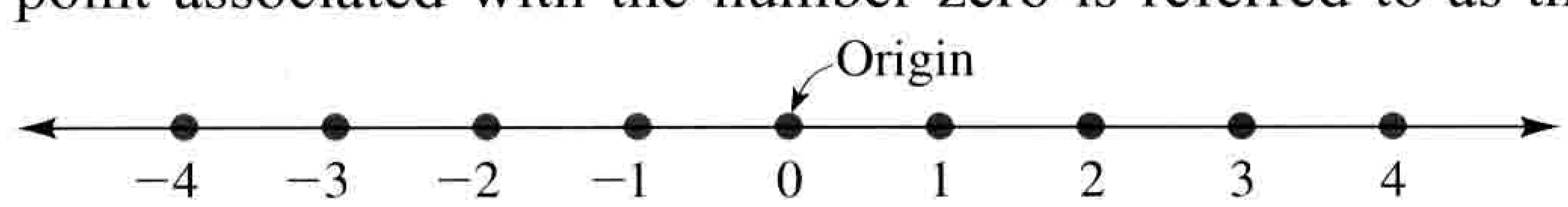


FIGURE 1

The fundamental fact here is that there is a **one-to-one correspondence** between the set of real numbers and the set of points on the line. This means that each real number is identified with exactly one point on the line; conversely, with each point on the line we identify exactly one real number. The real number associated with a given point is called the **coordinate** of the point. As a practical matter, we're usually more interested in relative locations than precise locations on a number line. For instance, since π is approximately 3.1, we show π slightly to the right of 3 in Figure 2. Similarly, since $\sqrt{2}$ is approximately 1.4, we show $\sqrt{2}$ slightly less than halfway from 1 to 2 in Figure 2.

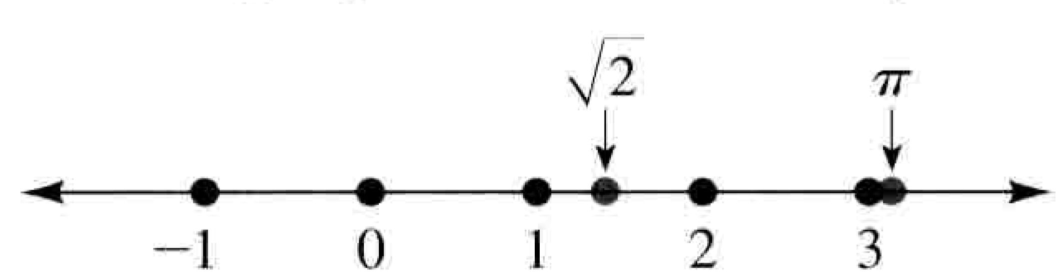


FIGURE 2

It is often convenient to use number lines that show reference points other than the integers used in Figure 2. For instance, Figure 3(a) displays a number line with reference points that are multiples of π. In this case, it is the integers that we then locate approximately. For example, in Figure 3(b) we show the approximate location of the number 1 on such a line.

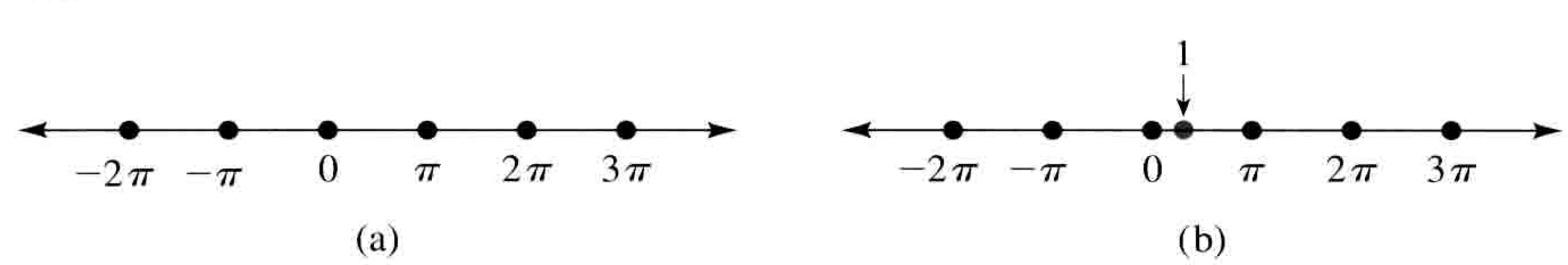


FIGURE 3

Two of the most basic relations for real numbers are **less than** and **greater than,** symbolized by $<$ and $>$, respectively. For ease of reference, we review these and two related symbols in the box that follows.

PROPERTY SUMMARY NOTATION FOR LESS THAN AND GREATER THAN

NOTATION	DEFINITION	EXAMPLES
$a < b$	a is less than b. On a number line, oriented as in Figure 1, 2, or 3, the point a lies to the left of b.	$2 < 3$; $-4 < 1$
$a \leqslant b$	a is less than or equal to b.	$2 \leqslant 3$; $3 \leqslant 3$
$b > a$	b is greater than a. On a number line oriented as in Figure 1, 2, or 3, the point b lies to the right of a. [$b > a$ is equivalent to $a < b$.]	$3 > 2$; $0 > -1$
$b \geqslant a$	b is greater than or equal to a.	$3 \geqslant 2$; $3 \geqslant 3$

In general, relationships involving real numbers and any of the four symbols $<$, $\leqslant$, $>$, and $\geqslant$ are called **inequalities.** One of the simplest uses of inequalities is in defining certain sets of real numbers called *intervals.* Roughly speaking, any uninterrupted portion of the number line is referred to as an **interval.** In the definitions that follow, you'll see notations such as $a < x < b$. This means that *both* of the inequalities $a < x$ and $x < b$ hold; in other words, the number x is between a and b.

DEFINITION: Open Intervals and Closed Intervals

The **open interval** (a, b) consists of all real numbers x such that $a < x < b$. See Figure 4(a).
The **closed interval** $[a, b]$ consists of all real numbers x such that $a \leqslant x \leqslant b$. See Figure 4(b).

(a) The open interval (a, b) contains all real numbers from a to b, excluding a and b.

(b) The closed interval $[a, b]$ contains all real numbers from a to b, including a and b.

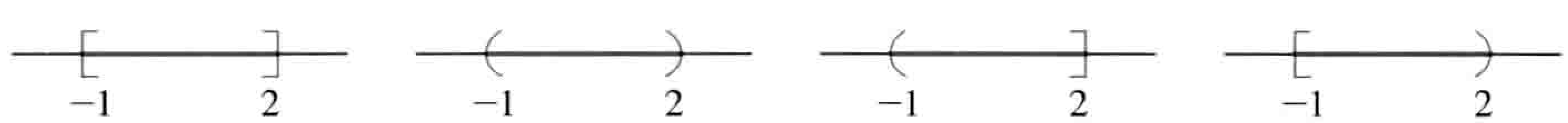
FIGURE 4

Notice that the brackets in Figure 4(b) are used to indicate that the numbers a and b are included in the interval $[a, b]$, whereas the parentheses in Figure 4(a) indicate that a and b are excluded from the interval (a, b). At times you'll see notation such as $[a, b)$. This stands for the set of all real numbers x such that $a \leqslant x < b$. Similarly, $(a, b]$ denotes the set of all numbers x such that $a < x \leqslant b$.

EXAMPLE 1 Show each interval on a number line, and specify inequalities describing the numbers x in each interval.

$$[-1, 2] \qquad (-1, 2) \qquad (-1, 2] \qquad [-1, 2)$$

Solution See Figure 5.



$[-1, 2]$	$(-1, 2)$	$(-1, 2]$	$[-1, 2)$
$-1 \leq x \leq 2$	$-1 < x < 2$	$-1 < x \leq 2$	$-1 \leq x < 2$

FIGURE 5

In addition to the four types of intervals shown in Figure 5, we can also consider **unbounded intervals.** These are intervals that extend indefinitely in one direction or the other, as shown, for example, in Figure 6. We also have a convenient notation for unbounded intervals; for example, we indicate the unbounded interval in Figure 6 with the notation $(2, \infty)$.

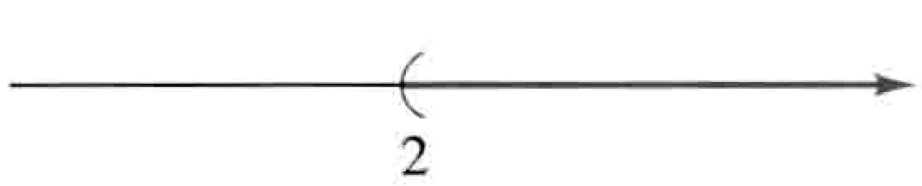


FIGURE 6
The set of all real numbers x such that $x > 2$.