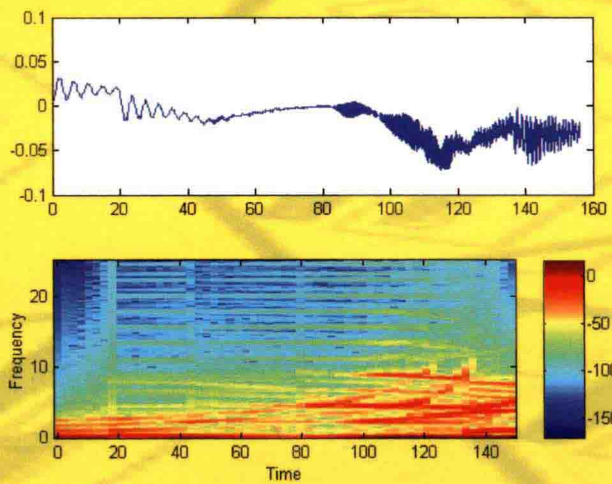
Dynamics and Control of Technical Systems



Edited by
José M Balthazar, Paulo B Gonçalves,
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Dynamics and Control of Technical Systems

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EDITORIAL

DYNAMICS AND CONTROL OF AEROSPACE AND VERTICAL TRANSPORTATION SYSTEMS

José M Balthazar, Paulo B Gonçalves, Stefan Kaczmarczyk, André Fenili, Marcos Silveira and Ignacio Herrera Navarro

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Introduction

This Special Issue presents a selection of papers initially presented at the 11th International Conference on Vibration Problems (ICOVP-2013), held from 9 to 12 September 2013 in Lisbon, Portugal. The main topics of this Special Issue are linear and, mainly, nonlinear dynamics, chaos and control of systems and structures and their applications in different field of science and engineering. According to the goal of the Special Issue, the selected contributions are divided into three major parts: "Vibration Problems in Vertical Transportation Systems", "Nonlinear Dynamics, Chaos and Control of Elastic Structures" and "New Strategies and Challenges for Aerospace and Ocean Structures Dynamics and Control".

The modern vertical transportation systems involve several technological and engineering challenges, especially in tall office and residential towers where the development and control of safe high-speed elevators and energy efficient escalator systems has become an important issue. In underground mining the safety of personnel and profitability rely on stable designs of hoisting installations. In space transportation technologies vertical propulsion systems based on various applications of momentum exchange and electrodynamics interactions with planetary magnetic fields are being proposed. The adverse dynamic responses and interactions pose serious operational and serviceability problems in these systems. So, the investigation of vibration phenomena in vertical transportation systems in the modern built environment and underground mining applications has become a fruitful research area in structural dynamics. Problems include lateral and longitudinal vibrations of hoist and compensating ropes as well as vertical and horizontal vibrations of the car/conveyance/counterweight assemblies and vertical motions of a compensating sheave. These vibrations are caused by various sources of excitation, including inertial loads due to the system acceleration/deceleration; periodic excitations caused by the building/host structure sway; excitations that originate at the drive sheave/hoist drum/drive machine due to torque ripple as well as at the car/conveyance/counterweight due to the guide rail imperfections and caused by aerodynamic effects.

Nonlinear dynamics and mechanics of structural elements such as beams, plates and shells is an important topic in applied mechanics and one of the most application-oriented fields in dynamics impacting several engineering fields, including civil, mechanical, naval, aero spatial, offshore and biomechanical engineering. So, research in this area has a positive impact in most technological applications including nowadays from large to micro and nano structures. These structural elements may experience large amplitude unwanted vibrations. Thus, vibration control is often necessary to obtain prescribed behaviors or to improve (and even optimize) the performance of structures. The application of different types of linear and nonlinear control strategies, including the control of chaos, is a flourishing research field in structural dynamics. Although chaos was detected by Poincaré in the 1880s, only in the last decades the analysis of chaotic systems has become an important research area in mathematics and physics. This has influenced the many recent developments in nonlinear dynamics and, consequently, the analysis of the nonlinear vibrations of slender structural systems. Finally, in recent years a large amount of research has been dedicated to new materials and their use in structural components. So research has been conducted in applications of these materials from micro and nano structures to large space structures. New phenomena in dynamics as well as new approaches to older ones are being discovered in the theoretical, numerical and experimental investigations of structures as shown herein.

The discussion of real problems in aerospace and how these problems can be understood and solved in view of numerical, computational, theoretical and experimental approaches is an important field in engineering science. The contributions pertaining this class of problems and methods associated to aerospace structures are an important part of this special issue. The contributions cover mathematical modeling, nonlinear analysis, control theory, nonlinear state estimation, experimental analysis and related topics associated to artificial satellites, airplanes, rockets, space tethers, helicopters and related systems.

Issue contents

This Special Issue includes 22 technical papers summarized below, plus one editorial paper.

Santhosh et al. studies the nonlinear dynamics of a shrouded turbine blade, considering a combination of impact and friction. For this, a one dimensional contact model which is capable of modeling the interface under constant and variable normal load is proposed. The steady state periodic solutions are obtained by multi-harmonic balance method (MHBM). Frequency response plots are generated for different values of normal load using the arc length continuation procedure. The MHBM solutions are validated using numerical integration.

Vibration phenomena taking place in lifting and hoisting installations may influence the dynamic performance of their components. For example, lateral and longitudinal vibrations of suspension ropes and compensating cables may result in an adverse dynamic behavior of the entire installation. Here, Kaczmarczyk and Mirhadizadeh present the modelling, simulation and experimental validation of the nonlinear dynamic interactions in an aramid suspension rope system. They develop a model of an aramid suspension rope system in order to predict nonlinear modal interactions taking place in the installation. Also, experimental tests have been conducted to validate the numerical results. They show that the nonlinear couplings may lead to adverse modal interactions in the system.

The helical buckling of an elastic beam confined in a cylinder is relevant to many applications, including oil drilling, medical catheters and even the conformation of DNA molecules. Silveira et al. focuses their work on the effects of nonlinear torsional stiffness on the dynamics of a slender elastic beam under torsional oscillations, which can be subject to helical buckling. They show that the formation of the helical configuration may be a result of only the torsional load, confirming that there is a different path to helical buckling which is not related to the sinusoidal buckling. In their low dimensional model hardening and softening characteristics are considered, as well as the effects of torsion and bending coupling.

The design of the satellite Attitude Control System (ACS) is a complex problem when the satellite structure has different type of components such as flexible solar panels, antennas, mechanical manipulators and tanks with fuel, due to dynamics interaction effects between these components. De Zouza and De Souza present the design of a satellite attitude control considering the interaction between fuel slosh and flexible dynamics in order to predict possible damage to the controller performance. The fuel slosh dynamics is modeled using its pendulum analogs mechanical system which parameters are identified using the Kalman filter technique. This information is used to designs and to compare the satellite attitude control system by the Linear Quadratic Regulator (LQR) and Linear Quadratic Gaussian (LQG) methods.

Fenili et al. presents the mathematical modelling of rotating nonlinear flexible beam-like wing with rectangular cross section. The structure is mathematically modeled considering linear curvature and clamped-free boundary conditions. Nonlinearities resulting from the coupling between the angular velocity of the rotating axis and the transversal vibration of the beam are considered. A drag force and a lift force acting along the beam length are also included in the mathematical model.

Thin walled shells are found in several modern industrial applications. Del Prado et al. study the effect of internal flowing fluid on the nonlinear behaviour of orthotropic circular cylindrical shells. To model the shell, the Donnell's non-linear shallow shell theory. The fluid is assumed to be incompressible and non-viscous and the flow to be isentropic and irrotational. The Galerkin method is applied to derive a precise low dimensional model. The results obtained show that the internal fluid and material properties have a significant influence on the vibration characteristics of the shell.

The behavior of vertical conveyors under periodic loads is an important problem which can be significantly affected by non-ideal excitation. Bayiroglu analyses here the nonlinear dynamics of unbalanced mass of a vertical conveyer with non-ideal exciters. The results of numerical simulation are plotted and Lyapunov exponents are calculated.

Müller et al. investigate the fatigue potential due to electrical and mechanical loads of electronic components of forklift trucks. A test of forklift truck is conducted to measure electrical and mechanical stresses and assess the fatigue potential of the operating load conditions. To compare the failure characteristics of the examined electronic components, fatigue tests were carried out on an electro-dynamic shaker. Additionally, the influence of vibration and shock excitation were checked for the control behavior of these components.

In prediction of the dynamic behavior of an elevator car system, it is important to take into account the influence of passengers' behavior in the car. Herrera et al. evaluate the influence of load occupancy ratio on the dynamic response of an elevator system under various load conditions. Experimental tests are conducted over a range of frequencies and amplitudes and excellent agreement between experimental tests and model predictions is achieved.

Alisverisçi carries out the nonlinear analysis of vibrational conveyers with non-ideal crank and rod exciters. The working range of vibrational conveyers with cubic nonlinear spring and non-ideal vibration exciter has been analyzed analytically for primary resonance by the method of multiple scales. Lyapunov exponents are numerically calculated and the results of numerical simulation are discussed.

Pielorz and Sado investigate the nonlinear vibrations in discrete-continuous mechanical systems consisting of rigid bodies connected by shafts torsionally deformed with local nonlinearities having hard or soft characteristics. The systems are loaded by an external moment harmonically changing in time. In this study the wave approach is used. Numerical results are presented for three-mass systems. Regular vibrations in the case of hard characteristic amplitude jumps are observed while in the case of soft characteristic an escape phenomenon is observed. Irregular vibrations, including chaotic motions, are found for selected parameters of the systems.

Alisverisçi et al. study the suppression of chaos in a nonideal double-well Duffing oscillator coupled to a rotor with limited power supply using an electromechanical damped device. The electromechanical damped device or electromechanical vibration absorber consists of an electrical system coupled magnetically to a mechanical structure (represented by the Duffing oscillator). Numerical simulations results are presented to demonstrate the effectiveness of the electromechanical vibration absorber. Lyapunov exponents are numerically calculated to prove the occurrence of a chaotic vibration in the non-ideal system and the suppressing of chaotic vibration in the system using the electromechanical damped device.

Follmann et al. discuss a method for measuring the phase of chaotic systems. This method has as input a scalar time series and operates by estimating a fundamental frequency for short windows along the whole extension of the signal. It minimizes the mean square error of fitting a sinusoidal function to the series segment. They demonstrate the applicability of the method on experimental time series obtained from two coupled Chua circuits.

Hamdia and Belhaq study the effect of time delayed feedback on stationary solutions in a van der Pol type system. The feedback gain is harmonically modulated with a resonant frequency. Perturbation analysis is conducted to obtain the modulation equations near primary resonance. It is shown that the modulated feedback gain position can influence significantly the steady state behavior of the delayed van der Pol oscillator. For appropriate values of the modulated delay parameters, limit cycles can be increased or quenched. New regions of quasi-periodic vibration may emerge for certain values of the modulated gain.

The ball and beam system is a common didactical experiment in control laboratories that can be used to illustrate many different closed-loop control techniques. Colón et al. studies the nonlinear ball and beam control system identification, in which the ball rolls with slipping and the friction force between the ball and the beam and collisions at the ends of the beam are considered. The actuator consists of a rubber belt attached at the free ends of the beam and connected to a DC motor. The elastic coefficients of the belt are experimentally identified, as well as the collision coefficients. The nonlinear behavior of the system is studied and a control strategy is proposed.

Fenili models a rigid-flexible two link rotating system is using Lagrange's equations. Nonlinearity arises in this problem from the coupling between the variable representing the angular velocity of the rotating axis connected to the flexible link and the variable representing the vibration of the flexible link. Sufficiently large angular velocities are considered in order to the system to undergo sufficiently strong nonlinear behavior. Vibration control is also investigated here. Results for different mathematical descriptions of the system are compared and discussed.

Comfort is an important topic in the elevator industry, and among the different factors which affect it, vibration in the car is one of the most important. Although technologies are available for reducing vibration in low speed elevators, the increased number of high buildings and sky-scrappers, forces the development of new technologies for medium and high speed elevators. Monge et al. proposes a design method of semi-active roller guides based on a magneto-rheological damper for high speed elevators. Different control strategies based on low cost acceleration sensors are also analyzed.

Silva et al. apply the least square method to the dynamics of rotational motion of artificial satellites, that is, its orientation (attitude) with respect to an inertial reference system, using real data of CBERS-2 (China Brazil Earth Resources Satellite). The attitude determination involves approaches of nonlinear estimation techniques, which knowledge is essential to the safety and control of the satellite and payload. Results show that one can reach accuracies in attitude determination within the prescribed requirements, besides providing estimates of the gyro drifts which can be further used to enhance the gyro error model.

Zanardi et al. analyze the disturbances in artificial satellites, related with the modeling of the Earth's gravitational potential as well as numerical implementation of a recursive algorithm to calculate the acceleration of the geopotential based on the Clenshaw summation. With this algorithm it is possible to calculate the geopotential acceleration for different orbits and different situations. This approach mitigates the numerical problems arising from the use of extended series expansion when computing recursively Legendre polynomials. The algorithm can be used in the solution of practical problems of orbital space mechanics and for the Brazilian Space Mission.

Reaction wheels are largely employed in satellite attitude control due to its large range of torque capability, small power consumption and high reliability. To achieve a good performance the RW design must deal with several restrictions and launch requirements. Carrara and Kuga present some approaches to the design and some experimental results for a reaction wheel (RW) current and speed control loops. The RW is controlled by speed reference, and a second speed mode control similar to the first one is implemented in an external computer. Both are then compared by means of the air-bearing attitude control performance during the wheel zero-speed crossing.

Finally, Chiaradia et al. are concerned with the determination of real time satellite orbit using short arcs of data, GPS signals and an Extended Kalman Filter (EKF). The algorithm uses the EKF to estimate the state vector and GPS receiver clock parameters with different step-sizes between the GPS measurements. The algorithm has been tested using raw single frequency pseudo range GPS measurements of the Topex/Poseidon satellite, and used as reference in this work.

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Chaotic Vibrations in Multi-Mass Discrete-Continuous Systems Torsionally Deformed with Local Nonlinearities

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Keywords: regular and irregular nonlinear vibrations, discrete-continuous systems, wave approach.

Abstract. The paper deals with nonlinear vibrations in discrete-continuous mechanical systems consisting of rigid bodies connected by shafts torsionally deformed with local nonlinearities having hard or soft characteristics. The systems are loaded by an external moment harmonically changing in time. In the study the wave approach is used. Numerical results are presented for three-mass systems. In the study of regular vibrations in the case of a hard characteristic amplitude jumps are observed while in the case of a soft characteristic an escape phenomenon is observed. Irregular vibrations, including chaotic motions, are found for selected parameters of the systems.

Introduction

Regular nonlinear vibrations in discrete-continuous mechanical systems are considered e.g. in [1,2]. Such systems consist of rigid bodies connected by elastic elements. Damping in the systems is taken into account by means of equivalent damping elements in selected cross-sections. In systems torsionally deformed, to the first rigid body a discrete element is attached. The spring in this element has nonlinear characteristics described by the polynomial of the third degree. The behavior of the systems in the case of the both types of characteristics, hard and soft, is discussed. The systems are loaded by an external moment represented by the function harmonically changing in time. In the considerations the wave approach, leading to solving equations with a retarded argument, is used.

Irregular vibrations, including chaos, are studied in the technical literature mainly for discrete models, [3-11]. In [12, 13] some attempts are done for the investigation of irregular vibrations in discrete-continuous systems after the adaptation the approach used in [8-10]. The aim of the present paper is to widen the results given in [12, 13] for the both types of the characteristics of local nonlinearities in multi-mass systems torsionally deformed.

Exemplary numerical results are presented for a three-mass system. From [1, 2] it follows that in the case of a hard characteristic amplitude jumps are observed while in the case of a soft characteristic an escape phenomenon is observed. These effects concern regular vibrations with large damping. Irregular and chaotic vibrations can occur for selected parameters. The possibilities of occurring of such vibrations are discussed on the basis of the Poincaré maps, the maximal exponents of Lyapunov and of bifurcation diagrams.

Assumptions and Governing Equations

Consider a multi-mass discrete-continuous system consisting of N shafts connected by rigid bodies. The i-th shaft, $i = 1,2,...,N$, is characterized by length l_i, density ρ, shear modulus G and polar moment of inertia I_{0i}, [1, 2]. The mass moment of inertia of rigid bodies, $i = 1,2,...N+1$, are J_i. The first rigid body J_1 is loaded by the moment $M(t)$. Equivalent external and internal damping, having coefficients d_i and D_i, are taking into account in appropriate cross-sections. The x-axis is parallel to the shafts axis and its beginning corresponds to the cross-sections where the first rigid body is located. The function $\theta_i(x,t)$ represents angular displacements in the cross-sections of the i-th shaft. It is assumed that displacements and velocities of the shaft cross-sections are equal to zero at time instant $t=0$.

A nonlinear discrete element is located in the cross-section $x=0$. The moment of forces acting in the nonlinear spring is described by the polynomial of the third degree

$$M_{sp} = k_1\theta_1 + k_3\theta_1^3 \tag{1}$$

where k_i are parameters standing by linear and nonlinear terms, correspondingly. The local nonlinearity can have the characteristic of a hard type or of a soft type.

It is convenient to introduce the following nondimensionless quantities

$$\begin{gathered}\bar{x}=x/l_1,\ \bar{t}=ct/l_1,\ \bar{\theta}_i=\theta_i/\theta_0,\ \bar{d}_i=d_il_1/(J_1c),\ \bar{D}_i=D_ic/l_1,\\ \bar{k}_1=k_1l_1^2/(J_1c^2),\ \bar{k}_3=k_3\theta_0^2l_1^2/(J_1c^2),\ K_r=I_{01}\rho l_1/J_1,\ E_i=J_1/J_i,\\ \bar{M}=Ml_1^2/(J_1c^2\theta_0),\ \bar{M}_{sp}=M_{sp}l_1^2/(J_1c^2\theta_0),\ \bar{l}_i=l_i/l_1,\ B_i=I_{0i}/I_{01},\end{gathered} \tag{2}$$

where $c=(G/\rho)^{1/2}$ is the speed of the torsional wave in shafts. Then, the determination of angular displacements and velocities leads to solving N equations of motion

$$\theta_{i,tt}-\theta_{i,xx}=0, \quad i=1,2,...,N \tag{3}$$

with zero initial conditions and with the following boundary conditions in appropriate cross-sections of the system

$$\begin{gathered}M(t)-\theta_{1,tt}+K_r(D_1\theta_{1,xt}+\theta_{1,x})-d_1\theta_{1,t}-M_{sp}(t)=0 \text{ for } x=0,\\ \theta_i(x,t)=\theta_{i+1}(x,t) \text{ for } x=\sum_{k=1}^{i}l_k,\ i=1,2,...,N-1,\\ -\theta_{i,tt}-K_rB_iE_{i+1}(D_i\theta_{i,xt}+\theta_{i,x})+K_rB_{i+1}E_{i+1}(D_{i+1}\theta_{i+1,xt}+\theta_{i+1,x})-E_{i+1}d_{i+1}\theta_{i,t}=0\\ \text{for } x=\sum_{k=1}^{i}l_k,\ i=1,2,...,N-1,\\ -\theta_{N,tt}-K_rB_NE_{N+1}(D_N\theta_{N,xt}+\theta_{N,x})-E_{N+1}d_{N+1}\theta_{N,t}=0 \text{ for } x=\sum_{k=1}^{N}l_k\end{gathered} \tag{4}$$

where comma denotes partial differentiation.

Eqs (3) are solved using the wave approach, i.e., looking for the solutions in the form

$$\theta_i(x,t)=f_i(t-x)+g_i(t+x-2\sum_{k=1}^{i-1}l_k),\ i=1,2,...,N \tag{5}$$

where the functions f_i and g_i represent the waves caused by the external loading $M(t)$ propagating in the i-th shaft in the positive and negative senses of the x-axis, respectively. These functions are continuous and equal to zero for negative arguments.

Substituting (5) into the boundary conditions (4) nonlinear ordinary differential equations with a retarded argument for the functions f_i and g_i are obtained. They have the following form

$$r_{N+1,1}g''_N(z)+r_{N+1,2}g'_N(z)=r_{N+1,3}f''_N(z-2l_N)+r_{N+1,4}f'_N(z-2l_N),$$

$$g_i(z)=f_{i+1}(z-2l_i)+g_{i+1}(z-2l_i)-f_i(z-2l_i),\quad i=1,2,....,N-1,$$

$$r_{11}f''_1(z)=M(z)+r_{12}g''_1(z)+r_{13}f'_1(z)+r_{14}g'_1(z)-k_1[f_1(z)+g_1(z)]$$
$$-k_3[f_1(z)+g_1(z)]^3, \tag{6}$$

$$r_{i1}f''_i(z)+r_{i2}f'_i(z)=r_{i3}g''_i(z)+r_{i4}g'_i(z)+r_{i5}f''_{i-1}(z)+r_{i6}f'_{i-1}(z),\quad i=2,3,...,N,$$

where

$$r_{11}=K_rD_1+1,\ r_{12}=K_rD_1-1,\ r_{13}=-K_r-d_1,\ r_{14}=K_r-d_1,\ r_{i1}=K_rE_i(B_iD_i+B_{i-1}D_{i-1})+1,$$
$$r_{i2}=E_i[K_r(B_i+B_{i-1})+d_i],\ r_{i3}=K_rE_i(B_iD_i-B_{i-1}D_{i-1})-1,\ r_{i4}=E_i[K_r(B_i-B_{i-1})-d_i],$$
$$r_{i5}=2K_rB_{i-1}E_iD_{i-1},\ r_{i6}=2K_rB_{i-1}E_i,\quad i=2,3,...,N,\ r_{N+1,1}=K_rB_NE_{N+1}D_N+1, \tag{7}$$
$$r_{N+1,2}=E_{N+1}(K_rB_N+d_{N+1}),\ r_{N+1,3}=K_rB_NE_{N+1}D_N-1,\ r_{N+1,4}=E_{N+1}(K_rB_N-d_{N+1}).$$

Eqs (6) are solved numerically using the Runge-Kutta method.

Numerical Calculations

Exemplary investigations are performed for a three-mass system, shown in Fig. 1, assuming the following dimensionless parameters, similarly as in [12, 13],

$$K_r=0.05,\ k_1=0.05,\ N=2,\ l_1=l_2=1,\ E_2=E_3=0.8,\ B_2=1. \tag{8}$$

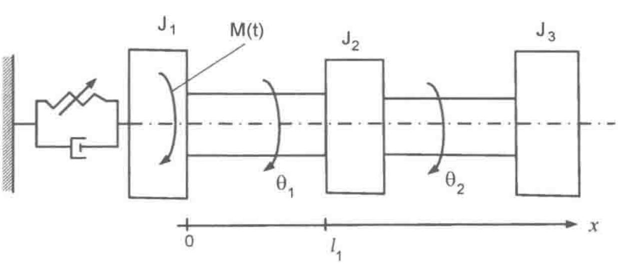

Fig. 1. Three-mass torsional system

The three-mass system is loaded by the following external moment

$$M(t)=M_0\sin pt \tag{9}$$

where M_0 is the amplitude and p is the frequency of the external moment. The numerical discussion is focused on the solutions in the steady states. Besides, all figures concern the cross-section $x = 1$.

The damping coefficients $d_0 = d_i = D_i$, the parameter k_3 and the amplitude M_0 of the external loading $M(t)$ can vary.

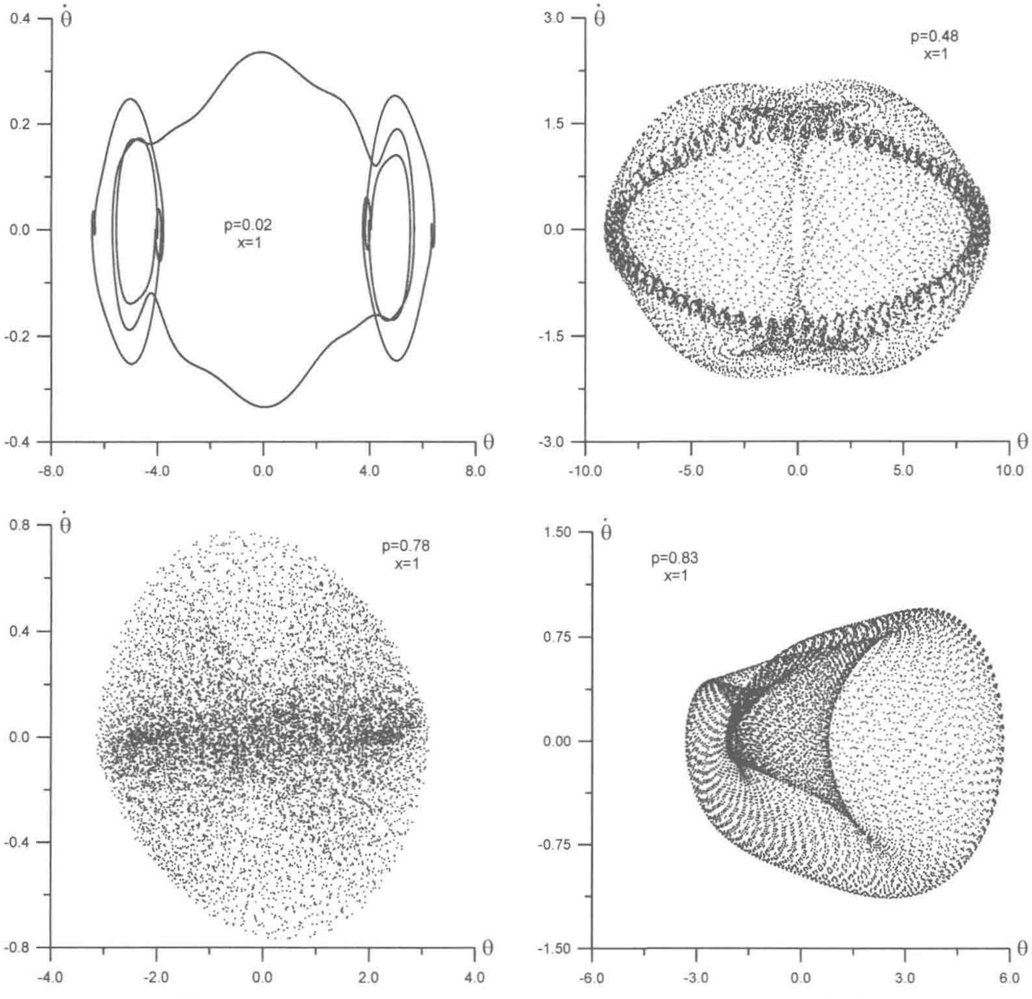

Fig. 2. Poincaré maps for $p = 0.02, 0.48, 0.78, 0.83$ with $k_3 = 0.005$

The characteristics of the local nonlinearity can be of a hard or of a soft type depending on the sign of the parameter k_3 standing by the nonlinear term in (1). Namely, for positive values for k_3 we deal with a hard characteristic and for negative values with a soft characteristic. We assume $k_3 = \pm 0.005$, so the both cases of the characteristics are considered in the present paper and some comparisons are performed.

Regular vibrations were studied in [1, 2] for the both types of nonlinear characteristics (1), i.e. with $k_3 = 0.005$ and $M_0 = 1$ for the hard characteristic case in [1] while with $k_3 = -0.005$ and $M_0 = 0.1$ for the soft characteristic case in [2]. It appears that in the case of a hard characteristic amplitude jumps are observed while in the remaining case the escape phenomenon occurs, [1, 2, 14]. The examples of these effects for amplitude-frequency curves are shown in appropriate figures given in [1, 2] for several values of the amplitudes M_0 and with damping coefficients $d_0 = d_i = D_i = 0.1$.

When damping is small, irregular vibrations including chaotic vibrations one can observe in the discussed system. The possibilities of occurring irregular vibrations are seen in bifurcations curves given for the three-mass system in papers [12, 13] for damping coefficients equal to $d_0 = d_i = D_i = 0.001$.

The studied system is a discrete-continuous system, so the required quantities can be determined in different cross-sections. In papers [12, 13] the numerical results are presented mainly for the cross-section $x = 0$, separately for the hard characteristics case and for the soft characteristics case. Here the results for the both types of the nonlinear characteristics are taking into account, however for the cross-section $x = 1$.

Diagrams in Figs 2 and 3 concern the hard characteristics case for $k_3 = 0.005$ and with $d_0 = 0.001$, $M_0 = 1$. From [13] it follows that chaotic vibrations can be expected for $p < 1.2$, so in Fig. 2 the Poincaré maps are plotted out for four selected values of the frequency p smaller than 1.2. Besides, maximal Lyapunov are determined for $p = 0.78, 0.83$, Fig. 3. They are positive, so this means that we deal with chaotic vibrations.

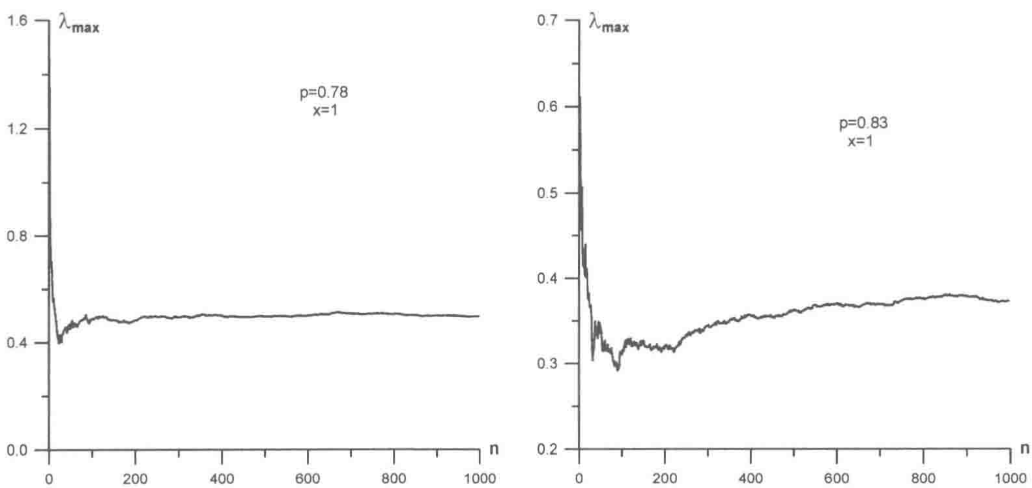

Fig. 3. Maximal Lyapunov exponents with $p = 0.78, 0.83$ and $k_3 = 0.005$

Diagrams in Figs 4 and 5 concern the system with the local nonlinearity having the characteristics of a soft type. In numerical analysis $k_3 = -0.005$, $d_0 = 0.001$, $M_0 = 0.4$ and $x = 1$ are assumed. From [12] it follows that chaotic vibrations can be observed for $0.8688 < p < 0.8776$.

Exemplary Poincaré maps are plotted out in Fig. 4 for $p = 0.7347, 0.876, 0.8763, 0.8775$. For the last two values of the frequency p the diagrams of the maximal Lyapunov exponents are presented in Fig. 5. They are also positive, so vibrations in these cases are chaotic.