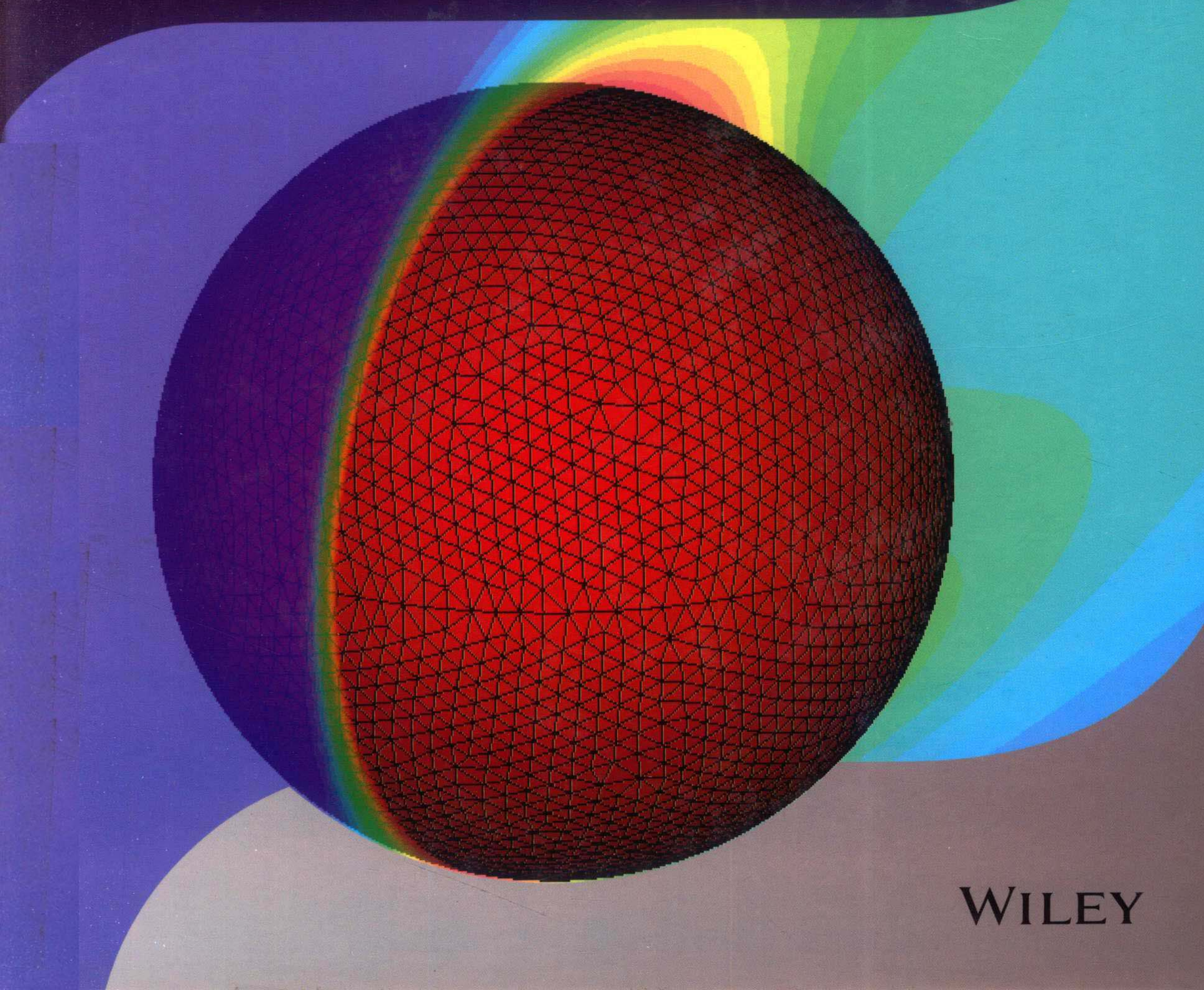
WILEY SERIES IN COMPUTATIONAL MECHANICS

Fundamentals of the Finite Element Method for Heat and Mass Transfer

Second Edition

P. Nithiarasu, R. W. Lewis,
and K. N. Seetharamu



WILEY

Fundamentals of the Finite Element Method for Heat and Mass Transfer

Second Edition

P. Nithiarasu
Zienkiewicz Centre for Computational Engineering
College of Engineering, Swansea University, UK

R. W. Lewis
Zienkiewicz Centre for Computational Engineering
College of Engineering, Swansea University, UK

K. N. Seetharamu
Department of Mechanical Engineering
PESIT, Bangalore, Karnataka, India

WILEY

This edition first published 2016 © 2016 by John Wiley & Sons, Ltd.
First edition published 2004 © 2004 by John Wiley & Sons, Ltd.

Registered office
John Wiley & Sons Ltd, The Atrium, Southern Gate, Chichester, West Sussex, PO19 8SQ, United Kingdom

For details of our global editorial offices, for customer services and for information about how to apply for permission to reuse the copyright material in this book please see our website at www.wiley.com.

Library of Congress Cataloging-in-Publication Data

Names: Nithiarasu, Perumal. | Lewis, R. W. (Roland Wynne) | Seetharamu, K. N.
| Lewis, R. W. (Roland Wynne). Fundamentals of the finite element method for heat and fluid flow.
Title: Fundamentals of the finite element method for heat and mass transfer.
Description: Second edition / P. Nithiarasu, R.W. Lewis, K.N. Seetharamu. | Chichester, West Sussex : John Wiley & Sons, Inc., 2016. | First edition: Fundamentals of the finite element method for heat and fluid flow / Roland W. Lewis, Perumal Nithiarasu, Kankanhalli N. Seetharamu (Hoboken, NJ : Wiley, 2004). | Includes bibliographical references and index.
Identifiers: LCCN 2015034600 | ISBN 9780470756256 (cloth : alk. paper)
Subjects: LCSH: Finite element method. | Heat equation. | Heat–Transmission.
| Fluid dynamics. | Mass transfer.
Classification: LCC QC20.7.F56 L49 2016 | DDC 530.15/5353–dc23 LC record available at http://lccn.loc.gov/2015034600

A catalogue record for this book is available from the British Library.

Set in 10/12.5pt Times by Aptara Inc., New Delhi, India.

Cover images: courtesy of the authors.

Printed and bound in Malaysia by Vivar Printing Sdn Bhd

1 2016

Preface to the Second Edition

In this second and enhanced edition of the book, we provide the readers with a detailed step-by-step application of the finite element method to heat and mass transfer problems. In addition to the fundamentals of the finite element method and heat and mass transfer, we have attempted to take the readers through some advanced topics of heat and mass transfer. The first edition of the book covered only the application of the finite element method to heat conduction and flow aided laminar heat convection. The second edition of the book has been enhanced further with turbulent flow and heat transfer, and mass transfer, in addition to advanced topics such as fuel cells.We believe that the second edition provides a comprehensive text for students, engineers and scientists who would like to pursue a finite element based heat transfer analysis. This textbook is suitable for beginners, senior undergraduate students, postgraduate students, engineers and early career researchers.

The first three chapters of the book deal with the essential fundamentals of both the heat conduction and the finite element method. In the first chapter, the fundamentals of energy balance and the standard derivations of relevant equations for the heat conduction analysis are discussed. Chapter 2 deals with the basic discrete systems which provide a basis for the finite element method formulations in the following chapters. The discrete system analysis is demonstrated through a variety of simple heat transfer and fluid flow problems. The third chapter gives a comprehensive account of the finite element method formulations and relevant history. Several examples and exercises included in Chapter 3 give the readers a complete overview of the theory and practice associated with the finite element method.

The application of the finite element method to heat conduction problems are discussed in detail in Chapters 4, 5 and 6. The conduction analysis starts with a simple one-dimensional steady-state heat conduction in Chapter 4 and is extended to multi-dimensions in Chapter 5. Chapter 6 gives the transient solution procedures for heat conduction problems.

Chapters 7, 8 and 9 deal with heat transfer by convection. In Chapter 7, heat transfer aided by the laminar motion of a single phase flow is discussed in detail. All the relevant differential equations are derived from first principles. All the three types of convection modes; forced, mixed and natural convection, are discussed in detail. Several examples and comparisons are provided to support the accuracy and flexibility of the finite element procedures discussed. In Chapter 8 the turbulent flow and heat transfer are discussed in some detail. Some examples and comparisons provide the readers a chance to assess the accuracy of the methods employed. Chapter 9 utilizes the finite element method developed in Chapters 1, 7 and 8 to provide a solution approach to flow and heat transfer in compact heat exchangers. Chapter 10 provides an introduction to the application of the finite element to problems of mass transfer. A detailed description of heat and mass transfer in porous media is then provide in Chapter 11. Two important applications of the finite element method for heat and mass transfer are explained in Chapters 12 and 13. Chapter 12 briefly introduces solidification problems using both heat conduction and convection approaches. Simple examples of solidification in this chapter may serve as a reference for students and researchers working in the area of solidification. In Chapter 13, we introduced a finite element solution approach to studying heat and mass transfer in fuel cells. Although the approach is only explained for solid oxide fuel cells, the method can be easily generalized to other types of fuel cells. Chapter 14 gives the reader sufficient information to understand the process of mesh generation. The main focus of this chapter is automatic and unstructured mesh generation. Some aspects of the adaptive mesh generation are also covered in this chapter. Finally, Chapter 15 briefly introduces the topic of computer implementation. The readers will be able to download the two-dimensional source codes and documentations from the website: **www.zetacomp.com**

Many people have assisted the authors either directly or indirectly during the preparation of this textbook. In particular, the authors wish to thank Dr Alessandro Mauro, Universitá degli Studi di Napoli Parthenope, for proofreading Chapter 13 and Dr Igor Sazonov, Swansea University, for helping the authors to put together part of Chapter 14. We would also like thank all our students, postdoctoral researchers and colleagues for providing help and support.

P. Nithiarasu, Swansea
R. W. Lewis, Swansea
K. N. Seetharamu, Bangalore

Series Editor's Preface

It is known that heat transfer provides a good context for teaching finite element methods and other computational mechanics topics. Fundamental concepts can be explained with such simple examples as heat conduction in 1D, then in 2D and 3D, and convective terms can be added to describe the special methods needed to deal with that class of partial differential equations. This book in our series does that, and with its distinguished, experienced authors, does it well. It not only teaches how to solve heat and mass transfer problems with finite element methods, but it also serves the purpose of teaching many different concepts in finite element methods. Readers from very diverse backgrounds will be able to benefit from this book. The book can be used by engineering undergraduate students to learn the fundamentals of heat and mass transfer and numerical methods, by graduate students in engineering and sciences to learn the advanced topics they need to know, and by practicing engineers and scientists as a good source and guide for research and development work in heat and mass transfer.

WILEY SERIES IN COMPUTATIONAL MECHANICS

Series Advisors:

René de Borst
Perumal Nithiarasu
Tayfun E. Tezduyar
Genki Yagawa
Tarek Zohdi

Fundamentals of the Finite Element Method for Heat and Mass Transfer	Nithiarasu, Lewis and Seetharamu	January 2016
Introduction to Computational Contact Mechanics: A Geometrical Approach	Konyukhov	April 2015
Extended Finite Element Method: Theory and Applications	Khoei	December 2014
Computational Fluid-Structure Interaction: Methods and Applications	Bazilevs, Takizawa and Tezduyar	January 2013
Introduction to Finite Strain Theory for Continuum Elasto-Plasticity	Hashiguchi and Yamakawa	November 2012
Nonlinear Finite Element Analysis of Solids and Structures, Second Edition	De Borst, Crisfield, Remmers and Verhoosel	August 2012
An Introduction to Mathematical Modeling: A Course in Mechanics	Oden	November 2011
Computational Mechanics of Discontinua	Munjiza, Knight and Rougier	November 2011
Introduction to Finite Element Analysis: Formulation, Verification and Validation	Szabó and Babuška	March 2011

Contents

1

Introduction

1.1 Importance of Heat and Mass Transfer

The subject of heat and mass transfer is of fundamental importance in many branches of engineering. A *mechanical engineer* may be interested to know the mechanisms of heat transfer involved in the operation of equipment, for example, boilers, condensers, air preheaters, economizers etc., in a thermal power plant in order to improve their performance. Nuclear power plants require precise information on heat transfer as safe operation is an important factor in their design. Refrigeration and air-conditioning systems also involve heat-exchanging devices, which need careful design. *Electrical engineers* are keen to avoid material damage in electric motors, generators and transformers due to hot spots, developed by improper heat transfer design. An *electronic engineer* is interested in knowing efficient methods of heat dissipation from chips and semi-conductor devices so that they function within safe operating temperatures. A *computer hardware engineer* is interested to know the cooling requirements of circuit-boards, as the miniaturization of computing devices is advancing at a rapid rate. *Chemical engineers* are interested in heat and mass transfer processes in various chemical reactions. A *metallurgical engineer* would be interested in knowing the rate of heat transfer required for a particular heat treatment process, e.g. the rate of cooling in a casting process has a profound influence on the quality of the final product. *Aeronautical engineers* are interested in knowing the heat transfer rate in rocket nozzles and in heat shields used in re-entry vehicles. An *agricultural engineer* would be interested in the drying of food grains, food processing and preservation. A *civil engineer* would need to be aware of the thermal stresses developed in quick setting concrete, the influence of heat and mass transfer on building and building materials as well as the effect of heat on nuclear containment and buildings etc. An *environmental engineer* is concerned with the effect of heat on dispersion of pollutants in air, transport of pollutants in soils, lakes and seas and their impact on life. A *bioengineer* is often interested in the heat and

Fundamentals of the Finite Element Method for Heat and Mass Transfer, Second Edition.
P. Nithiarasu, R. W. Lewis, and K. N. Seetharamu.

mass transfer processes, such as hypothermia and hyperthermia associated with the human body.

The above-mentioned applications are only a sample of heat and mass transfer applications. The solar system and the associated energy transfer from the sun are the principal factors for existence of life on Earth. It is not untrue to say that it is extremely difficult, often impossible, to avoid some form of heat transfer in any process on Earth.

The study of heat and mass transfer provides economical and efficient solutions for many critical problems encountered in diverse engineering items of equipment. For example, we can consider the development of heat pipes which can transport heat at a much greater rate than that of copper or silver rods of the same dimensions and even at almost isothermal conditions. The development of present-day gas turbine blades, where the gas temperature exceeds the melting point of the blade material, is possible by providing efficient cooling systems. This is another example of the success of heat transfer design methods. The design of computer chips, which encounter heat flux of the order occurring in re-entry-vehicles, especially when the surface temperature of the chips is limited to less than 100 °C, is again a success story of heat transfer design.

Although there are many successful heat transfer designs, further developments on heat and mass transfer studies are necessary in order to increase the life span and efficiency of the many devices discussed previously, which can lead to many more new inventions. Also, if we are to protect our environment, it is essential to understand the many heat and mass transfer processes involved and if necessary to take appropriate action.

1.2 Heat Transfer Modes

Heat transfer is that section of engineering science that studies the energy transport between material bodies due to temperature difference (Bejan 1993; Holman 1989; Incropera and Dewitt 1990; Sukhatme 1992). The three modes of heat transfer are:

(a) conduction

(b) convection and

(c) radiation.

The conduction mode of heat transport occurs either because of an exchange of energy from one molecule to another without actual motion of the molecules, or is due to the motion of free electrons if they are present. Therefore, this form of heat transport depends heavily on the properties of the medium and takes place in solids, liquids and gases if a difference in temperature exists.

Molecules present in liquids and gases have freedom of motion and by moving from a hot to a cold region, they carry energy with them. The transfer of heat from one region to another due to such macroscopic motion in a liquid or gas, added to the energy transfer by conduction within the fluid, is called heat transfer by convection. Convection may be either free, forced or mixed. When fluid motion occurs due to a density variation caused by temperature differences, the situation is said to be a free or natural convection. When the fluid motion is caused by an external force, such as pumping or blowing, the state is defined as being forced convection.

A mixed convection state is one in which both natural and forced convection are present. Convection heat transfer also occurs in boiling and condensation processes.

All bodies emit thermal radiation at all temperatures. This is the only mode which does not require a material medium for heat transfer to occur. The nature of thermal radiation is such that a propagation of energy, carried by *electromagnetic waves*, is emitted from the surface of the body. When these electromagnetic waves strike other body surfaces, a part is reflected, a part transmitted and the remaining part is absorbed.

All modes of heat transfer are generally present in varying degrees in a real physical problem. The important aspects in solving heat transfer problems are to identify the significant modes and to decide whether the heat transferred by other modes can be neglected.

1.3 The Laws of Heat Transfer

It is important to quantify the amount of energy being transferred per unit time and for that we require the use of rate equations. For heat conduction, the rate equation is known as *Fourier's law* (Fourier 1955) which is expressed for one dimension, as

$$q_x = -k\frac{dT}{dx}, \tag{1.1}$$

where q_x is the heat flux in the x direction (W/m^2); k is the thermal conductivity (W/mK, a property of the material, see Table 1.1) and dT/dx the temperature gradient (K/m).

Table 1.1 Typical values of thermal conductivity of some materials in W/mK at 20 °C.

Material	Thermal conductivity, k
Metals:	
Pure silver	410
Pure copper	385
Pure aluminium	200
Pure iron	73
Alloys:	
Stainless steel (18% Cr, 8% Ni)	16
Aluminium alloy (4.5% Cr)	168
Non metals:	
Plastics	0.6
Wood	0.2
Liquid:	
Water	0.6
Gasses:	
Dry air	0.025 (at atmospheric pressure)