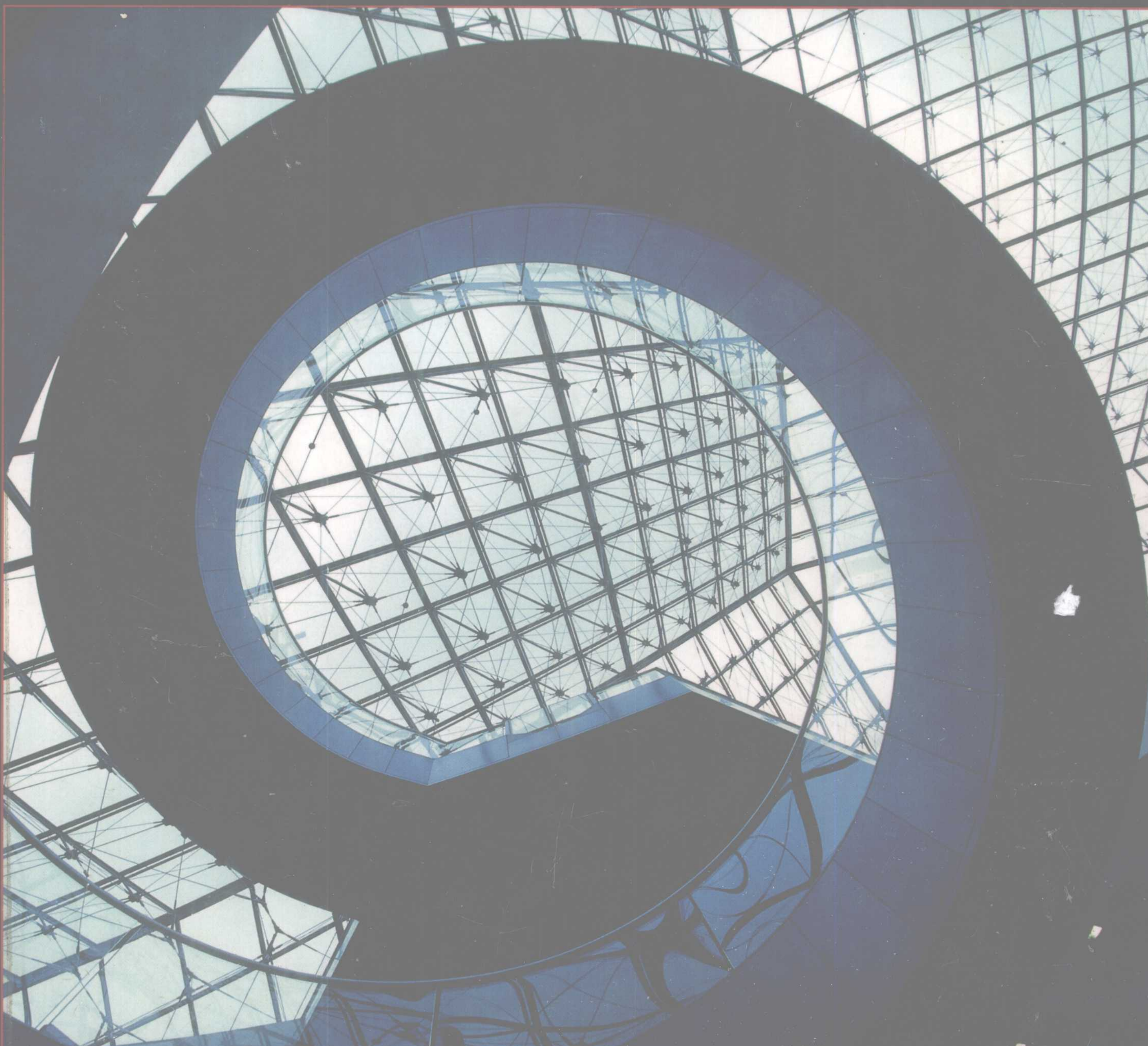
Calculus

3RD EDITION



Strauss · Bradley · Smith

CALCULUS

THIRD EDITION

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Preface

This text was developed to blend the best aspects of calculus reform with the reasonable goals and methodology of traditional calculus. It achieves this middle ground by providing sound development, stimulating problems, and well-developed pedagogy within a framework of a traditional structure. "Think, then do," is a fair summary of our approach.

New to this Edition

The acceptance and response from our first two editions has been most gratifying. For the third edition, we wanted to take a good book and make it even better. If you are familiar with the previous editions, the first thing you will notice is that we have added a new coauthor, Monty J. Strauss. His added expertise, and his attention to accuracy and detail, as well as his many years of experience teaching calculus, have added a new dimension to our exposition. Here is what is new in the third edition:

Organization

- In this edition we introduce e^x and $\ln x$ in Chapter 2 after we have defined the notion of a limit. This is beneficial because it allows the number e to be properly defined using limits. We also assume a knowledge of the conic sections and their graphs. A free *Student Mathematics Handbook* is available that provides review and reference material on these transcendental functions.
- l'Hôpital's Rule is now covered earlier in Chapter 4. This placement allows instructors to explore more interesting applications like curve sketching.
- A new section covering applications to business, economics, and the life sciences has been added to Chapter 6 on Applications of the Integral. This new material is designed to help students see how calculus relates to and is used in other disciplines.
- The chapter on polar coordinates and parametric forms has been distributed to other chapters in the book. The polar coordinate system and graphing in polar forms is in Chapter 6 in the context of the integration topic of finding areas. Parametric representation of curves now appears in the book where it is first needed, in Chapter 9.
- Modeling continues as a major theme in this edition. Modeling is now introduced in Section 3.4, and then appears in almost every section of the book. These applications are designated MODELING PROBLEMS. Some authors use the words "Modeling Problem" to refer to any applied problem. In the third edition of *Calculus*, we make a distinction between *modeling problems* and *application problems* by defining a modeling problem as follows. A **modeling problem** is a problem that requires that the reader make some assumptions about the real world in order to derive or come up

with the necessary mathematical formula or mathematical information to answer the question. These problems also include real-world examples of modeling by citing the source of the book or journal that shows the modeling process.

Problem Sets

- We have added a new major category of problems, called **counterexample problem**. A *counterexample* is an example that disproves a proposition or theorem, and in mathematics we are often faced with a proposition that is true or false, and our task is to prove the proposition true or to find a counterexample to disprove the proposition. In the third edition of *Calculus*, we attempt to build the student's ability with this type of situation to mean that the student must either find justification that the proposition is true or else find a counterexample. We believe this new form of problem to be important for preparing the student for future work in not only advanced mathematics courses, but also for analytically oriented courses.
- **Exploration Problems** explore concepts which may prove true or false and provide opportunities for innovative thinking.
- **Interpretation Problems** require exposition that requires a line of thinking that is not directly covered in the textbook.

Supplements

- **Interactive CD** (free with every new copy). The new CD-ROM is designed to enhance students' computational and conceptual understanding of calculus. This CD-ROM is not an add-on of extra material to the text but rather an incredibly useful expansion of the text. See the media supplement portion of the Walk-Through for a complete description of each CD.
- **TestGen-EQ.** This easy to use test generator contains all of the questions from the printed Test Item File.
- **Prentice Hall Online Homework Grader.** For more details, see the PH Homework Grader section in the Walk-Through.

Hallmark Features

Some of the distinguishing characteristics of the earlier editions are continued with this edition:

- It is possible to begin the course with either Chapter 1 or Chapter 2 (where the calculus topics begin).
- We believe that students *learn* mathematics by *doing* mathematics. Therefore, the **problems and applications** are perhaps the most important feature of any calculus book. You will find that the problems in this book extend from routine practice to challenging. The problem sets are divided into *A* Problems (routine), *B* Problems (requiring independent thought), and *C* Problems (theory problems). You will find the scope and depth of the problems in this book to be extraordinary while engineering and physics examples and problems play a prominent role, we include applications from a wide variety of fields, such as biology, economics, ecology, psychology, and sociology. In addition, the chapter summaries provide not only topical review, but also many miscellaneous exercises. Although the chapter reviews are typical of examinations, the miscellaneous problems are not presented as graded problems, but rather as a random list of problems loosely tied to the ideas of that chapter. In addition, there are cumulative reviews located at natural subdivision points in the text: Chapters 1–5, Chapters 6–8, Chapters 1–10, and Chapters 11–13. For a full description of each type of problem available, see pages xviii through xxi of Walk-Through.
- We understand that students often struggle with prerequisite material. Further, it is often frustrating for instructors to have to reteach material from previous courses. As

a result, we have created a unique **Student Mathematics Handbook**. This handbook functions as a **"Just-in-Time" Precalculus Review** that provides precalculus drill/review material, a catalog of curves, analytic geometry, and integral tables. Students are guided through the text to this handbook by a SMH symbol located in the text margin. This guide is entirely author-written and offered FREE with every new copy of the text.

SMH

- We have taken the introduction of **differential equations** seriously. Students in many allied disciplines need to use differential equations early in their studies and consequently cannot wait for a postcalculus course. We introduce differential equations in a natural and reasonable way. Slope fields are introduced as a geometric view of antidifferentiation in Section 5.1, and then are used to introduce a graphical solution to differential equations in Section 5.6. We consider separable differential equations in Chapter 5 and first-order linear equations in Chapter 7, and demonstrate the use of both modeling a variety of applied situations. Exact and homogeneous differential equations appear in Chapter 14, along with an introduction to second-order linear equations. The "early and often" approach to differential equations is intended to illustrate their value in continuous modeling and to provide a solid foundation for further study.
- **Visualization** is used to help students develop better intuition. Much of this visualization appears in the wide margins to accompany the text. Also, since tough calculus problems are often tough geometry (and algebra) problems, this emphasis on graphs will help students' problem-solving skills. Additional graphs are related to the student problems, including answer art.
- We have included dozens of "TECHNOLOGY NOTES" devoted to the use of technology. We strive to keep such references "platform neutral" because specific calculators and computer programs frequently change and are better considered in separate technology manuals. These references are designed to give insight into how technological advances can be used to help understand calculus. Problems requiring a graphing calculator or software and computer also appear in the exercises. On the other hand, problems that are not specially designated may still use technology (for example, to solve a higher-degree equation). Several technology manuals are also available at a discount price. See Instructor/Student Supplement section in the Walk-Through for details.
- **Guest essays** provide alternate viewpoints. The questions that follow are called MATHEMATICAL ESSAYS and are included to encourage individual writing assignments and mathematical exposition. We believe that students will benefit from individual writing and research in mathematics. Another pedagogical feature is the **"What this says:"** box in which we rephrase mathematical ideas in everyday language. In the problem sets we encourage students to summarize procedures and processes or to describe a mathematical result in everyday terms.
- **Group research projects**, each of which appears at the end of a chapter and involves intriguing questions whose mathematical content is tied loosely to the chapter just concluded. These projects have been developed and class-tested by their individual authors, to whom we are greatly indebted. Note that the complexity of these projects increases as we progress through the book and the mathematical maturity of the student is developed.
- We continue to utilize the **humanness** of mathematics. History is not presented as additional material to learn. Rather we have placed history into *problems* that lead the reader from the development of a concept to actually participating in the discovery process. The problems are designated as Historical Quest problems. The problems are not designed to be "add-on or challenge problems," but rather to become an integral part of the assignment. The level of difficulty of Quest problems ranges from easy to difficult. An extensive selection of biographies of noted mathematicians can be found on the internet site accompanying this text (`www.prenhall.com/strauss`)

Accuracy and Error Checking

Because of careful checking and proofing by several people besides each of the three authors, the authors and publisher believe this book to be substantially error free. For any errors remaining, the authors would be grateful if they were sent to Monty J. Strauss at **M.Strauss@ttu.edu** or Monty J. Strauss, Department of Mathematics and Statistics, Texas Tech University, Lubbock, TX 79409-1042.

Acknowledgments

The writing and publishing of a calculus book is a tremendous undertaking. We take this responsibility very seriously because a calculus book is instrumental in transmitting knowledge from one generation to the next. We would like to thank the many people who helped us in the preparation of this book. First, we thank our editor George Lobell, who led us masterfully through the development and publication of this book. We sincerely appreciate Henri Feiner, who not only worked all of the problems but also read and critiqued each word of the manuscript, and Carol Williams who transcribed our manuscript into TEX. We would like to thank Dennis Kletzing for his work in transforming the manuscript into finished pages. Finally, we would like to thank Lynn Savino Wendel, who led us through the production process.

Of primary concern is the accuracy of the book. We had the assistance of many: Henri Feiner, who read the entire manuscript and offered us many valuable suggestions; and Nancy and Mary Toscano, who were meticulous in their checking of our manuscript. Thanks also to the accuracy checkers of the previous editions, Jerry Alexanderson, Mike Ecker, Ken Sydel, Diana Gerardi, Kurt Norlin, Terri Bittner, Nancy Marsh, and Mary Toscano. We would also like to thank the following readers of the text for the many suggestions for improvement:

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CALCULUS

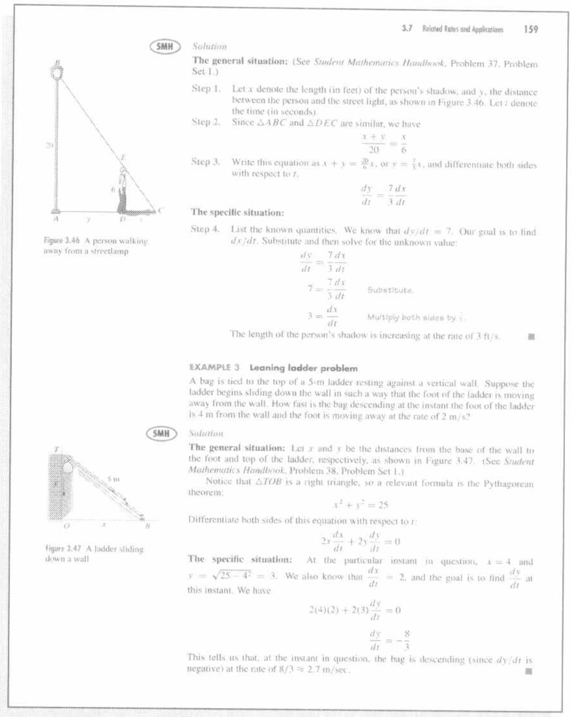
A Guide to Using this Text

HALLMARK FEATURES

Unique Student Mathematics Handbook ▶

(Included on the Student CD-ROM as well as in an optional print supplement)

This "Just-in-Time Precalculus Review" provides precalculus drill/review material, a catalogue of curves, analytic geometry, and integral tables. Students are guided from the text to this handbook by a (SMH) symbol located in the text margin. This guide is entirely text-author written. See the description of the Student CD-ROM for other features.



3.7 Related Rates and Applications 159

SMH *Solution*

The general situation: (See *Student Mathematics Handbook*, Problem 37, Problem Set 1.)

Step 1. Let x denote the length (in feet) of the person's shadow, and y, the distance between the person and the street light, as shown in Figure 3.46. Let t denote the time (in seconds).

Step 2. Since $\triangle ABC$ and $\triangle DEC$ are similar, we have

$$\frac{x+y}{20} = \frac{x}{6}$$

Step 3. Write this equation as $x + y = \frac{20}{6}x$, or $y = \frac{7}{3}x$, and differentiate both sides with respect to t.

$$\frac{dy}{dt} = \frac{7}{3}\frac{dx}{dt}$$

The specific situation:

Step 4. List the known quantities. We know that $dy/dt = 7$. Our goal is to find dx/dt. Substitute and then solve for the unknown value:

$$\frac{dy}{dt} = \frac{7}{3}\frac{dx}{dt}$$

$$7 = \frac{7}{3}\frac{dx}{dt} \quad \text{Substitute.}$$

$$3 = \frac{dx}{dt} \quad \text{Multiply both sides by } \tfrac{3}{7}.$$

The length of the person's shadow is increasing at the rate of 3 ft/s. ■

Figure 3.46 A person walking away from a streetlamp

EXAMPLE 3 Leaning ladder problem

A bag is tied to the top of a 5-m ladder resting against a vertical wall. Suppose the ladder begins sliding down the wall in such a way that the foot of the ladder is moving away from the wall. How fast is the bag descending at the instant the foot of the ladder is 4 m from the wall and the foot is moving away at the rate of 2 m/s?

SMH *Solution*

The general situation: Let x and y be the distances from the base of the wall to the foot and top of the ladder, respectively, as shown in Figure 3.47. (See *Student Mathematics Handbook*, Problem 38, Problem Set 1.)

Notice that $\triangle TOB$ is a right triangle, so a relevant formula is the Pythagorean theorem:

$$x^2 + y^2 = 25$$

Differentiate both sides of this equation with respect to t:

$$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$$

The specific situation: At the particular instant in question, $x = 4$ and $y = \sqrt{25 - 4^2} = 3$. We also know that $\frac{dx}{dt} = 2$, and the goal is to find $\frac{dy}{dt}$ at this instant. We have

$$2(4)(2) + 2(3)\frac{dy}{dt} = 0$$

$$\frac{dy}{dt} = -\frac{8}{3}$$

This tells us that, at the instant in question, the bag is descending (since dy/dt is negative) at the rate of $8/3 \approx 2.7$ m/sec. ■

Figure 3.47 A ladder sliding down a wall

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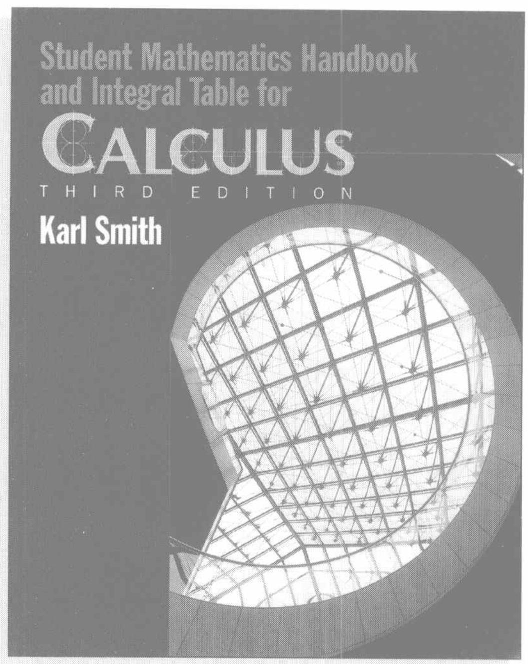
CD-ROM included free in both the Instructor and Student Editions.



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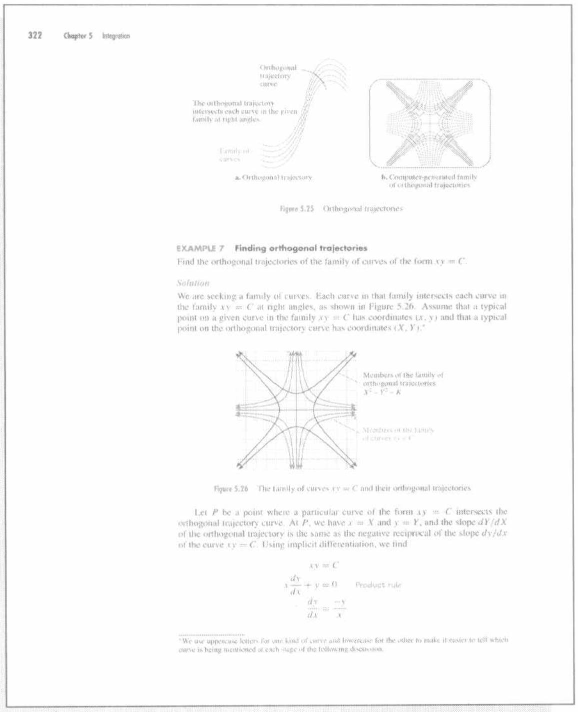

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Orthogonal trajectory curve

The orthogonal trajectory intersects each curve in the given family at right angles.

Family of curves

a. Orthogonal trajectory

b. Computer-generated family of orthogonal trajectories

Figure 5.25 Orthogonal trajectories

EXAMPLE 7 Finding orthogonal trajectories

Find the orthogonal trajectories of the family of curves of the form $xy = C$.

Solution

We are seeking a family of curves. Each curve in that family intersects each curve in the family $xy = C$ at right angles, as shown in Figure 5.26. Assume that a typical point on a given curve in the family $xy = C$ has coordinates (x, y) and that a typical point on the orthogonal trajectory curve has coordinates (X, Y).*

Members of the family of orthogonal trajectories $X^2 - Y^2 = K$

Members of the family of curves $xy = C$

Figure 5.26 The family of curves $xy = C$ and their orthogonal trajectories

Let P be a point where a particular curve of the form $xy = C$ intersects the orthogonal trajectory curve. At P, we have $x = X$ and $y = Y$, and the slope dY/dX of the orthogonal trajectory is the same as the negative reciprocal of the slope dy/dx of the curve $xy = C$. Using implicit differentiation, we find

$$xy = C$$

$$x\frac{dy}{dx} + y = 0 \quad \text{Product rule}$$

$$\frac{dy}{dx} = \frac{-y}{x}$$

*We use uppercase letters for one kind of curve and lowercase for the other to make it easier to tell which curve is being mentioned at each stage of the following discussion.

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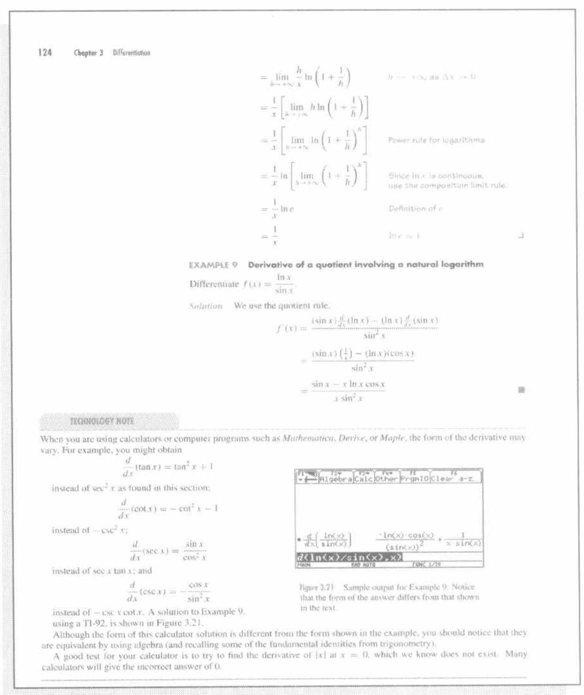
◀ Emphasis on visualization

Over 1900 graphs and pictures are featured throughout the text. Designed to supplement the text prose, the art program helps student to develop mathematical intuition and problem-solving skills.

Effective use of technology ▶

Graphing calculators and computer algebra systems are carefully integrated throughout the text. The "Technology Notes" feature gives students insight into how technological advances can be used to help understand calculus. Instructors can also encourage students to use technology to complete exercise sets. Laboratory manuals are available at a discount when packaged with the text. See supplements section of a complete list of available technology.



124 Chapter 3 Differentiation

$$= \lim_{h\to+\infty} \frac{h}{x}\ln\left(1+\frac{1}{h}\right) \quad h \to +\infty \text{ as } \Delta x \to 0$$

$$= \frac{1}{x}\left[\lim_{h\to+\infty} h\ln\left(1+\frac{1}{h}\right)\right]$$

$$= \frac{1}{x}\left[\lim_{h\to+\infty} \ln\left(1+\frac{1}{h}\right)^h\right] \quad \text{Power rule for logarithms}$$

$$= \frac{1}{x}\ln\left[\lim_{h\to+\infty}\left(1+\frac{1}{h}\right)^h\right] \quad \text{Since } \ln x \text{ is continuous, use the composition limit rule.}$$

$$= \frac{1}{x}\ln e \quad \text{Definition of } e$$

$$= \frac{1}{x} \quad \ln e = 1$$

EXAMPLE 9 Derivative of a quotient involving a natural logarithm

Differentiate $f(x) = \dfrac{\ln x}{\sin x}$.

Solution We use the quotient rule.

$$f'(x) = \frac{(\sin x)\frac{d}{dx}(\ln x) - (\ln x)\frac{d}{dx}(\sin x)}{\sin^2 x}$$

$$= \frac{(\sin x)\left(\frac{1}{x}\right) - (\ln x)(\cos x)}{\sin^2 x}$$

$$= \frac{\sin x - x\ln x\cos x}{x\sin^2 x}$$

TECHNOLOGY NOTE

When you are using calculators or computer programs such as *Mathematica*, *Derive*, or *Maple*, the form of the derivative may vary. For example, you might obtain

$$\frac{d}{dx}(\tan x) = \tan^2 x + 1$$

instead of $\sec^2 x$ as found in this section;

$$\frac{d}{dx}(\cot x) = -\cot^2 x - 1$$

instead of $-\csc^2 x$;

$$\frac{d}{dx}(\sec x) = \frac{\sin x}{\cos^2 x}$$

instead of $\sec x \tan x$; and

$$\frac{d}{dx}(\csc x) = -\frac{\cos x}{\sin^2 x}$$

instead of $-\csc x \cot x$. A solution to Example 9, using a TI-92, is shown in Figure 3.21.

Figure 3.21 Sample output for Example 9. Notice that the form of the answer differs from that shown in the text.

Although the form of this calculator solution is different from the form shown in the example, you should notice that they are equivalent by using algebra (and recalling some of the fundamental identities from trigonometry).

A good test for your calculator is to try to find the derivative of $|x|$ at $x = 0$, which we know does not exist. Many calculators will give the incorrect answer of 0.

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PROBLEM SETS

"What Does This Say?" Problems ▶

These problems ask students to respond to questions or to rephrase sentences in their own words. Designed to help students develop the essential skill of communicating mathematical ideas.

> 4.4 Curve Sketching with Asymptotes: Limits Involving Infinity 227
>
> **4.4 PROBLEM SET**
>
> Ⓐ **1. WHAT DOES THIS SAY?** Outline a method for curve sketching.
>
> **2. WHAT DOES THIS SAY?** What are critical numbers? Discuss the importance of critical numbers in curve sketching.
>
> **45.** The *ideal speed* v for a banked curve on a highway is modeled by the equation
>
> $$v^2 = gr \tan \theta$$

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Interpretation Problems ▶

These problems require exposition that requires a line of thinking that is not directly covered in the textbook. Encourages students to think critically about calculus concepts.

> **58. Interpretation Problem** Evaluate $\lim_{x\to 0}\left[x^2 - \frac{\cos x}{1{,}000{,}000{,}000}\right]$. Explain why a calculator solution may lead to an incorrect conclusion about the limit.

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21. $\lim_{x\to 0^+} \frac{\sin x}{\sqrt{x}}$	**22.** $\lim_{x\to 0^+} \frac{1-\cos\sqrt{x}}{x}$
23. $\lim_{x\to 0} \frac{\sin 2x}{x}$	**24.** $\lim_{x\to 0} \frac{\sin 4x}{9x}$
25. $\lim_{t\to 0} \frac{\tan 5t}{\tan 2t}$	**26.** $\lim_{x\to 0} \frac{\cot 3x}{\cot x}$
27. $\lim_{x\to 0} \frac{1-\cos x}{\sin x}$	**28.** $\lim_{x\to 0} \frac{\sin^2 x}{2x}$
29. $\lim_{x\to 0} \frac{\sin^2 x}{x^2}$	**30.** $\lim_{x\to 0} \frac{x^2\cos 2x}{1-\cos x}$
31. $\lim_{x\to 0} \frac{\sec x - 1}{x \sec x}$	**32.** $\lim_{x\to\pi/4} \frac{1-\tan x}{\sin x - \cos x}$

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◀ Computational Problems

Graded exercise sets that are progressively more challenging and grouped into three categories. 'A' problems are routine, 'B' problems require independent thought, and 'C' problems focus on theory. Makes assigning exercises easier and allows students to gain confidence as they progress through the exercises.

20. Modeling Problem Two towns A and B are 12.0 mi apart and are located 5.0 and 3.0 mi, respectively, from a long, straight highway, as shown in Figure 4.62.

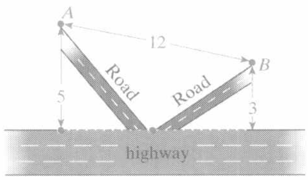

Figure 4.62 Building the shortest road

A construction company has a contract to build a road from A to the highway and then to B. Analyze a model to determine the length (to the nearest tenth of a mile) of the *shortest* road that meets these requirements.

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Modeling Problems

Designed to help students see the applications of calculus to multiple disciplines and real-world situations.

Counterexample Problems

These problems help students develop the ability to construct counterexamples. Students are asked to formulate an example that satisfies certain conditions.

67. Counterexample Problem When the line segment $y = x - 2$ between $x = 1$ and $x = 3$ is revolved about the x-axis, it generates part of a cone. The formula obtained in Section 6.4 says that the surface area of the cone is

$$\int_1^3 2\pi(x-2)\sqrt{2}\,dx$$

but this integral is equal to 0. What is wrong? What is the actual surface area?

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37. Exploration Problem Suppose that $f(t)$ is continuous for all t and that for any number x it is known that the average value of f on $[-1, x]$ is

$$A(x) = \sin x$$

Use this information to deduce the identity of f.

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Exploration Problems

These problems explore concepts which may prove true or false and provide opportunities for innovative thinking.

PROBLEM SETS

Spy Problems ▶

A student favorite, these problems which run throughout the text like a movie serial, trace the heroic events of an international spy. His survival requires the students' help by successfully answering calculus questions.

54. Spy problem Just as the Spy is about to catch up with Scélérat (Problem 70 of Chapter 10 Supplementary Problems), the snow gives way and he falls into a cavern. He staggers to his feet and removes his skis. Why is it so warm? Good grief—the cave is a large roasting oven! Fortunately, he is wearing his heat-detector ring, which indicates the direction of greatest temperature decrease. Suppose the bunker is coordinatized so that the temperature at each point (x, y) on the floor of the bunker is given by

$$T(x, y) = 3(x-6)^2 + 1.5(y-1)^2 + 41$$

degrees Fahrenheit, where x and y are in feet. The Spy begins at the point $(1, 5)$ and stumbles across the room at the rate of 4 ft/min, always moving in the direction of maximum temperature decrease. But he can last no more than 2 minutes under these conditions! Assuming that there is an escape hole at the point where the temperature is minimal, does he make it or is the Spy toast at last?

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◀ Historical Quest Problems

These problems help students to place calculus in a historical context, and further encourages them to take part in the mathematical discovery process.

81. HISTORICAL QUEST

Ramanujan was that rarest of mathematicians, an instinctive genius with virtually no formal training. Like his Hindu predecessor, Bhaskara (see the Historical Quest in Problem 55 of Section 4.4), Ramanujan had an uncanny instinct for numerical "truth" and conceived of his results in much the same way that a sculptor "sees" a statue in a raw block of stone. In his short life, he initiated new ways of thinking about number theory and made conjectures that are the subject of mathematical inquiry to this day.

SRINIVASA RAMANUJAN
1887–1920

A story related by his friend and mentor, the eminent British number theorist G. H. Hardy (1877–1947), serves to illustrate the resonance between Ramanujan's mind and the concept of number. Ramanujan was ill and Hardy came to visit him in the hospital. At a loss for how to begin the conversation, Hardy idly remarked that he had arrived in a taxi with the "dull" number 1729. Ramanujan immediately grew excited and exclaimed, "No, Hardy, it is a very interesting number, for it is the smallest integer that can be expressed as the sum of cubes in two different ways." ($1729 = 1^3 + 12^3 = 9^3 + 10^3$.)

In a letter from Ramanujan to G. H. Hardy at Cambridge University,* Ramanujan stated

$$1 - 5\left(\frac{1}{2}\right)^3 + 9\left(\frac{1\cdot 3}{2\cdot 4}\right)^3 - 13\left(\frac{1\cdot 3\cdot 5}{2\cdot 4\cdot 6}\right)^3 + \cdots = \frac{2}{\pi}$$

Hardy spent a great deal of time wondering how this sum could be equal to $2/\pi$. ■

a. Find a_k so this sum can be expressed as $\sum_{k=1}^{\infty} a_k$.

b. There is no elementary way to prove this identity. Use technology to check that this formula is valid.

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◀ Journal Problems

Journal problems are reprinted from leading mathematics journals. Citations are provided if students wish to locate additional information. A great resource for further exploration.

60. Journal Problem *Crux,* problem by John A. Winterink* ■ Prove the validity of the following simple method for finding the center of a conic: For the central conic,

$$\phi(x, y) = ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$$

$ab - h^2 \neq 0$, show that the center is the intersection of the lines $\partial\phi/\partial x = 0$ and $\partial\phi/\partial y = 0$.

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END-OF-CHAPTER PROBLEMS

CHAPTER 8 REVIEW

Proficiency Examination

CONCEPT PROBLEMS

1. What is a sequence?
2. What is meant by the limit of a sequence?
3. Define the convergence and divergence of a sequence.
4. Explain each of the following terms:
   a. bounded sequence
   b. monotonic sequence
   c. strictly monotonic sequence
5. State the BMCT.
6. What is an infinite series?
7. Compare or contrast the convergence and divergence of sequences and series.
8. What is a telescoping series?
9. Describe the harmonic series. Does it converge or diverge?
10. Define a geometric series and give its sum.
11. State the divergence test.
12. State the integral test.
13. State the p-series test.
14. State the direct comparison test.
15. State the limit comparison test.
16. State the zero-infinity limit comparison test.
17. State the ratio test.
18. State the root test.
19. What is the alternating series test?
20. How do you make an error estimate for an alternating series?
21. State the absolute convergence test.
22. What is meant by absolute and conditional convergence?
23. What is the generalized ratio test?
24. What is a power series?
25. How do you find the convergence set of a power series?
26. What are the radius of convergence and interval of convergence for a power series?
27. What is a Taylor polynomial?
28. State Taylor's theorem.
29. What are a Taylor series and a Maclaurin series?
30. State the binomial series theorem.

PRACTICE PROBLEMS

31. Find the limit of the sequence $\left\{\left(1+\frac{1}{n}\right)^n\right\}$
32. a. Find the limit of the sequence $\left\{\frac{e^n}{n!}\right\}$.
    b. Test the series $\sum_{k=1}^{\infty}\frac{e^k}{k!}$ for convergence.
    c. Discuss the similarities and/or differences of parts a and b.

In Problems 33–37, test the given series for convergence.

33. $\sum_{k=2}^{\infty}\frac{1}{k\ln k}$ 34. $\sum_{k=1}^{\infty}\frac{\pi^k k!}{k^k}$

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◀ Concept Problems

Concept problems are designed to develop students' conceptual understanding of calculus by learning how to think and draw conclusions about calculus topics.

◀ Practice Problems

Practice problems are specifically designed to help students develop their computational skills.

▼ Putnam Examination Problems

These challenging problems are provided to give insight into the types of problems that are asked in mathematical competitions, like the Mathematical Association of America's annual competition.

97. Putnam Examination Problem Let f be a real-valued function having partial derivatives defined for $x^2+y^2<1$ that satisfies $|f(x,y)|\leq 1$. Show that there exists a point (x_0, y_0) in the interior of the unit circle such that $[f_x(x_0,y_0)]^2+[f_y(x_0,y_0)]^2\leq 16$.

98. Putnam Examination Problem Find the smallest volume bounded by the coordinate planes and a tangent plane to the ellipsoid

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$$

99. Putnam Examination Problem Find the shortest distance between the plane $Ax+By+Cz+1=0$ and the ellipsoid

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}\leq 1$$

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Supplementary Problems

Describe the domain of each function given in Problems 1–4.

1. $f(x,y)=\sqrt{16-x^2-y^2}$ 2. $f(x,y)=\dfrac{x^2-y^2}{x-y}$
3. $f(x,y)=\sin^{-1}x+\cos^{-1}y$ 4. $f(x,y)=e^{x+y}\tan^{-1}\left(\dfrac{y}{x}\right)$

Find the partial derivatives f_x and f_y for the functions defined in Problems 5–10.

5. $f(x,y)=\dfrac{x^2-y^2}{x+y}$ 6. $f(x,y)=x^3e^{3y/(2x)}$
7. $f(x,y)=x^2y+\sin\dfrac{y}{x}$ 8. $f(x,y)=\ln\left(\dfrac{xy}{x+2y}\right)$
9. $f(x,y)=2x^3y+3xy^2+\dfrac{y}{x}$ 10. $f(x,y)=xye^{xy}$

For each function given in Problems 11–15, describe the level curve or level surface $f=c$ for the given values of the constant c.

11. $f(x,y)=x^2-y$; $c=2$, $c=-2$
12. $f(x,y)=6x+2y$; $c=0$, $c=1$, $c=2$
13. $z=f(x,y)=\begin{cases}\sqrt{x^2+y^2} & \text{if } z\geq 0\\ |y| & \text{if } z<0\end{cases}$
    $c=0$, $c=1$, $c=-1$
14. $f(x,y,z)=x^2+y^2+z^2$; $c=16$, $c=0$, $c=-25$
15. $f(x,y,z)=x^2+\dfrac{y^2}{2}+\dfrac{z^2}{9}$; $c=1$, $c=2$

Evaluate the limits in Problems 16 and 17, assuming they exist.

16. $\displaystyle\lim_{(x,y)\to(1,1)}\frac{xy}{x^2+y^2}$ 17. $\displaystyle\lim_{(x,y)\to(0,0)}\frac{x+ye^{-x}}{1+x^2}$

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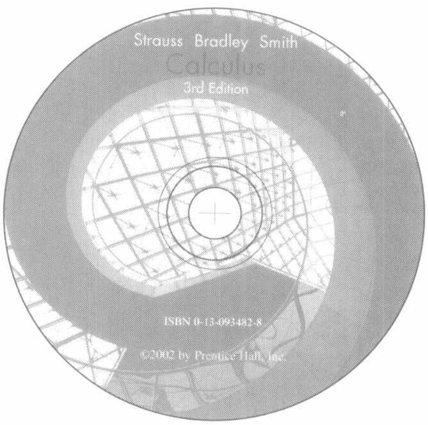
▲ Supplemental Problems

Extensive sets of problems (usually about 100) occur at the end of each chapter. These problems are based on randomly selected topics from the complete chapter. Also helps students to check their comprehension of the entire chapter.

MEDIA SUPPLEMENTS



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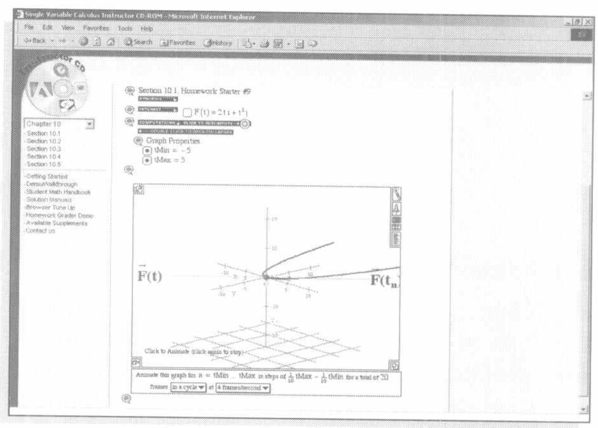

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