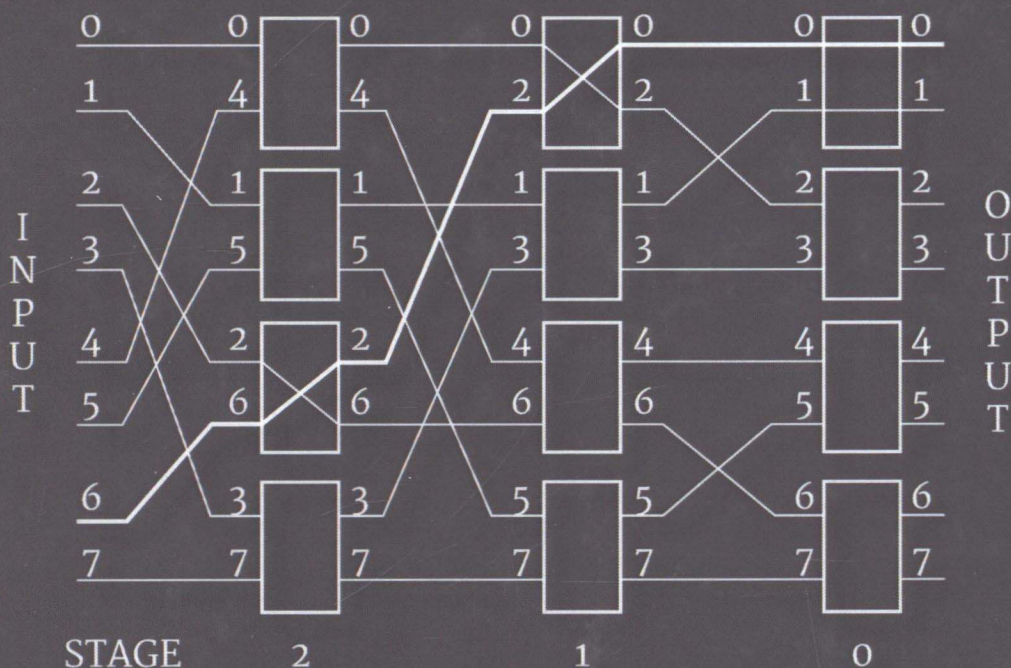


PERFORMABILITY ENGINEERING SERIES

Fundamentals of Reliability Engineering

Applications in Multistage Interconnection Networks

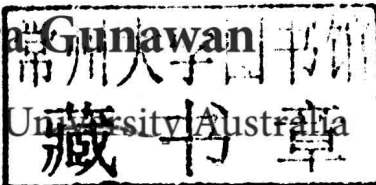


Indra Gunawan

Fundamentals of Reliability Engineering

Applications in Multistage
Interconnection Networks

Indra Gunawan
Federation University Australia



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Performability Engineering Series

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Scope: A true performance of a product, or system, or service must be judged over the entire life cycle activities connected with design, manufacture, use and disposal in relation to the economics of maximization of dependability, and minimizing its impact on the environment. The concept of performability allows us to take a holistic assessment of performance and provides an aggregate attribute that reflects an entire engineering effort of a product, system, or service designer in achieving dependability and sustainability. Performance should not just be indicative of achieving quality, reliability, maintainability and safety for a product, system, or service, but achieving sustainability as well. The conventional perspective of dependability ignores the environmental impact considerations that accompany the development of products, systems, and services. However, any industrial activity in creating a product, system, or service is always associated with certain environmental impacts that follow at each phase of development. These considerations have become all the more necessary in the 21st century as the world resources continue to become scarce and the cost of materials and energy keep rising. It is not difficult to visualize that by employing the strategy of dematerialization, minimum energy and minimum waste, while maximizing the yield and developing economically viable and safe processes (clean production and clean technologies), we will create minimal adverse effect on the environment during production and disposal at the end of the life. This is basically the goal of performability engineering.

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Preface

The purpose of this book is to provide readers with fundamentals of reliability engineering and demonstrate reliability approaches for evaluating system reliability with case studies in multistage interconnection networks.

The book can be used as an introductory book in reliability engineering for undergraduate/graduate students in Industrial/Electrical/Computer Engineering as well as engineers, researchers or managers. Practical applications are included to describe the importance of reliability measurement to achieve better systems.

In the first part of the book (chapters 1-5), it introduces the concept of reliability engineering, elements of probability theory, probability distributions, availability and data analysis.

The second part of the book (chapters 6-11) provides an overview of parallel/distributed computing, network design considerations, classification of multistage interconnection networks, network reliability evaluation methods, and reliability analysis of multistage interconnection networks including reliability prediction of distributed systems using Monte Carlo method.

It covers comprehensive reliability engineering methods and practical aspects in interconnection network systems. Students, engineers, researchers, managers will find this book as a valuable reference source.

The main key features of this book include:

- Fundamental of reliability engineering.
- Elements of probability and probability distributions.
- Classification of network systems.
- Reliability evaluation methods.
- Reliability analysis of multistage interconnection network systems is illustrated as practical applications of reliability methods including reliability prediction of distributed systems using Monte Carlo method.

I would like to express my gratitude to Prof. K.B. Misra for his kind assistance in reviewing the book.

Finally, my heartfelt thanks go to my wife Donna, daughters Jessica and Cynthia for their continuous support and my parents Suwita and Effie Gunawan for their motivation and encouragement.

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1

Introduction to Reliability Engineering

Reliability is defined as the probability that a system (part or component) can perform its intended task under specified conditions and time interval. It is used normally as the quantitative measure of the performance of a designed part, component or system. Reliability is also a design parameter which can be improved by design modification, redesign, elimination of deficiencies, and addition of redundant components or units.

The first part of this book (chapters 1–5) describes fundamentals of reliability engineering and the second part (chapters 6–11) presents reliability methods and its applications in Multistage Interconnection Networks (MIN). Chapter 9–11 discusses in details reliability analysis of network systems. Reliability of MIN is an important parameter that can be used as a measure on how reliable the interconnected components in network systems.

1.1 The Logic of Certainty

Event is a statement that can be true or false. “It may rain today” is not an event. According to our current state of knowledge, we may say that

an event is true, false, or possible (uncertain). Eventually, an event will be either true or false.

Sample Space is the set of all possible outcomes of an experiment [1–4]. Each elementary outcome is represented by a sample point. Examples: there are six possible outcomes/numbers {1, 2, 3, 4, 5, 6} from tossing a die; the failure time of a component is $\{0, \infty\}$. A collection of sample points is an event.

Indicator variables for events can be written in the following form. If an event i is true then $X_i = 1$ and if an event i is false then $X_i = 0$. Two basic operations, Union (OR) and Intersection (AND) are discussed.

1.2 Union (OR) operation

Suppose there are two events, A and B in the sample space. The equations below represent C as a union of the two events. $X_C = 1$ means that an event C is true when either event A or B is true.

$$A \cup B = C \quad (1.1)$$

$$X_C = 1 - (1 - X_A)(1 - X_B) \quad (1.2)$$

$$X_C \equiv \coprod X_j \quad (1.3)$$

Diagram Venn and fault tree for union (OR) operations are shown in Figure 1.1 below.

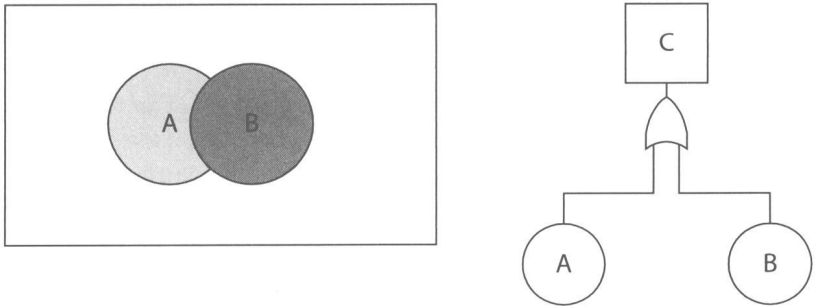


Figure 1.1 Diagram Venn and Fault Tree for Union (OR) Operation.

1.3 Intersection (AND) operation

The equations below represent C as an intersection of A and B. $X_C = 1$ means that an event C is true when both the events are true.

$$A \cap B = C \quad (1.4)$$

$$X_C = X_A X_B \quad (1.5)$$

$$X_C \equiv \prod X_j \quad (1.6)$$

Diagram Venn and fault tree for intersection (AND) operations are shown in Figure 1.2 below.

In A and B are mutually exclusive events (they are independent to each other) then

$$A \cap B = \emptyset \quad (1.7)$$

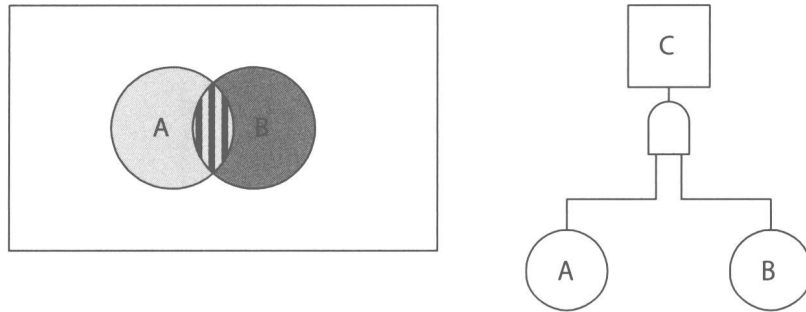


Figure 1.2 Diagram Venn and Fault Tree for Intersection (AND) Operation.

1.4 Series systems

Structure function of system failure and success in series systems can be defined as follows:

System failure:

$$X = 1 - \prod_{j=1}^N (1 - X_j) \equiv \prod_{j=1}^N X_j \quad (1.8)$$

System success:

$$Y = \prod_{j=1}^N Y_j \quad (1.9)$$

Where $X_j = 1$ or $Y_j = 1$ represent when the component j is failed or working.

Reliability block diagram and fault tree for series systems are shown in Figure 1.3 below.

The system reliability R_s is the product of the individual element reliabilities:

$$R_s = R_1 \times R_2 \times R_3 \times \dots \times R_N \quad (1.10)$$

If we assume that each of the elements has a constant failure rate, then the reliability of the i_{th} element is given by the exponential relation:

$$R_i = e^{-\lambda_i t} \quad (1.11)$$

Thus,

$$R_s = e^{-\lambda_1 t} e^{-\lambda_2 t} \dots e^{-\lambda_i t} \dots e^{-\lambda_N t}$$

$$R_s = e^{-\lambda_s t} = e^{-(\lambda_1 + \lambda_2 + \dots + \lambda_i + \dots + \lambda_N) t} \quad (1.12)$$

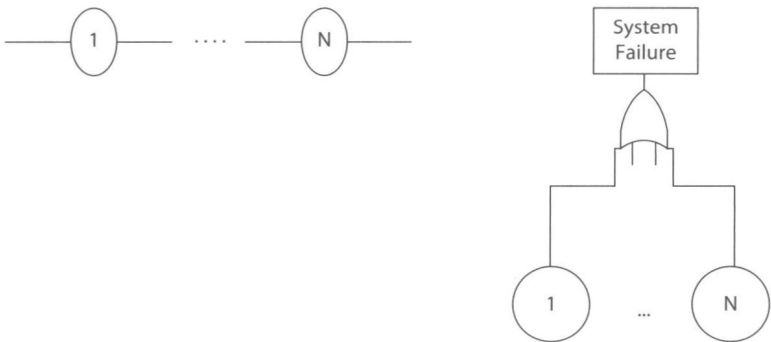


Figure 1.3 Reliability Block Diagram and Fault Tree for Series Systems.

and failure rate of system of N elements in series

$$\lambda_s = \lambda_1 + \lambda_2 + \dots + \lambda_i + \dots + \lambda_N \quad (1.13)$$

Since $R_s = 1 - F_s$ and $R_i = 1 - F_i$

Then

$$1 - F_s = (1 - F_1) (1 - F_2) \dots (1 - F_i) \dots (1 - F_N)$$

$$= 1 - (F_1 + F_2 + \dots + F_i + \dots + F_N) + \text{products of the } F\text{'s}$$

If the individual F_i are small, i.e. $F_i \ll 1$,

$$F_s \approx F_1 + F_2 + \dots + F_i + \dots + F_N \quad (1.14)$$

1.5 Parallel systems

Structure function of system failure and success in parallel systems can be defined as follows:

System failure:

$$X = \prod_{j=1}^N X_j \quad (1.15)$$

System success:

$$Y = \prod_{j=1}^N Y_j \quad (1.16)$$

Where $X_j = 1$ or $Y_j = 1$ represent when the component j is failed or working.

Reliability block diagram and fault tree for parallel systems are shown in Figure 1.4 below.

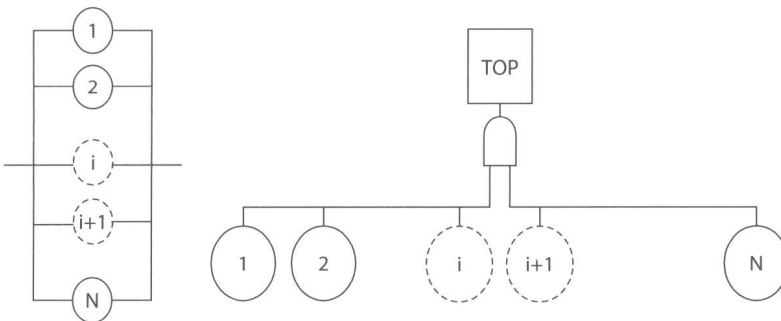


Figure 1.4 Reliability Block Diagram and Fault Tree for Parallel Systems.

The unreliability of parallel system is given by:

$$F_s = F_1 F_2 \dots F_i \dots F_N \quad (1.17)$$

If the individual elements are identical:

$$F_1 = F_2 = \dots = F_i = \dots = F_N = F \quad (1.18)$$

This gives:

$$F_s = F^n$$

1.6 General Series-Parallel System

A general series-parallel system consists of n identical subsystems in parallel and each subsystem consists of m elements in series.

If R_{ji} is the reliability of the i th elements in the j th subsystem, then the reliability of the j th subsystem is:

$$R_j = R_{j1} R_{j2} \dots R_{ji} \dots R_{jm} = \prod_{i=1}^{i=m} R_{ji} \quad (1.19)$$

The corresponding unreliability of the j th subsystem is:

$$F_j = 1 - \prod_{i=1}^{i=m} R_{ji} \quad (1.20)$$

The overall system unreliability is:

$$F_o = \prod_{j=1}^{j=n} \left[1 - \prod_{i=1}^{i=m} R_{ji} \right] \quad (1.21)$$

1.7 Active Redundancy

A system is referred to as “ k out of n ” if the overall system will continue to function correctly when only k ($k \leq n$) of the n elements/systems are working normally; the remaining $(n - k)$ elements/systems ensure extra reliability.

In 2 out of 4 system, the overall system unreliability is:

$F = \text{Prob. (A,B,C,D fail)} + \text{Prob. (A,B,C fail)} + \text{Prob. (B,C,D fail)} + \text{Prob. (A,C,D fail)} + \text{Prob. (A,B,D fail)}$