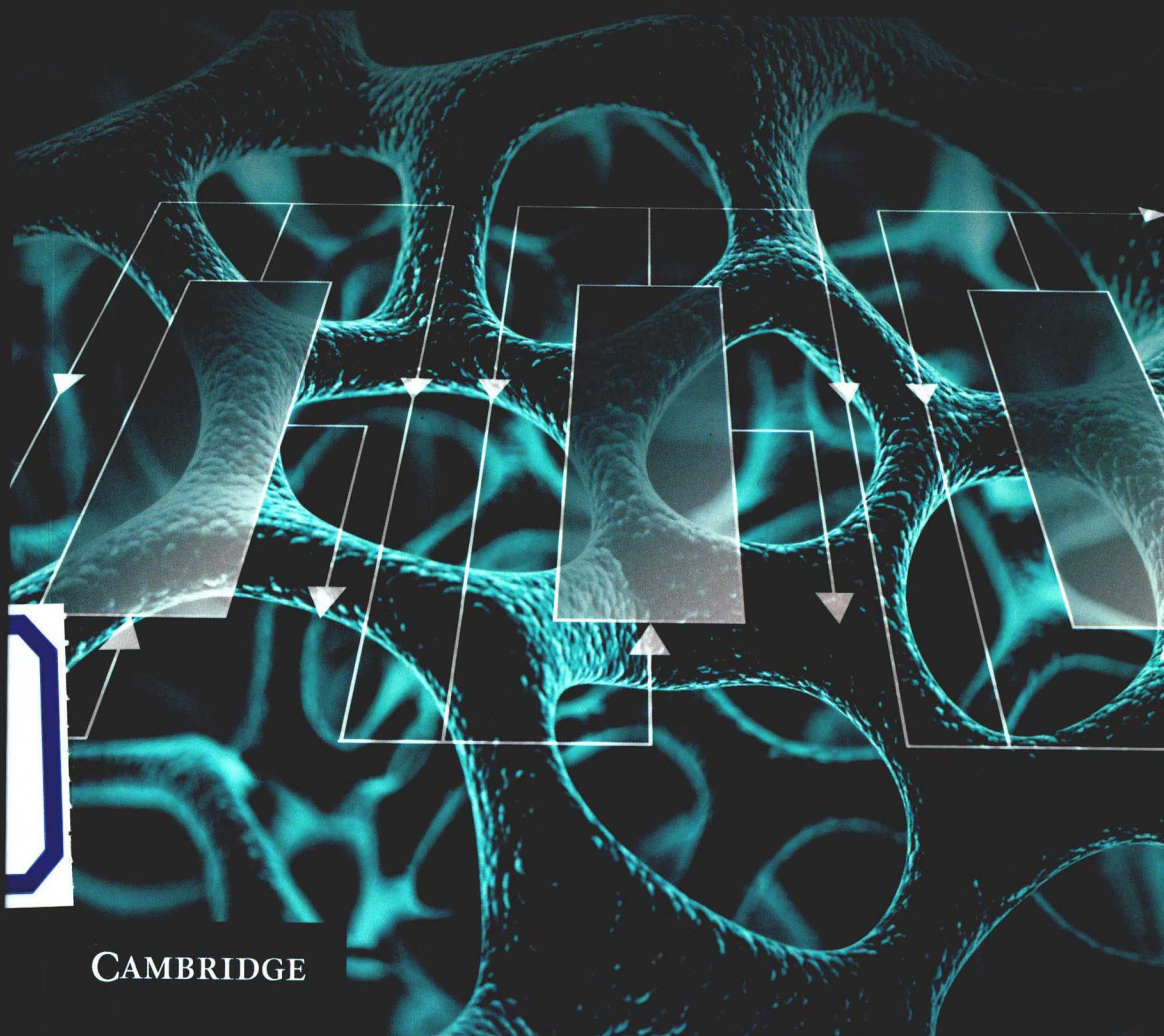


MEMBRANE FILTRATION

A Problem Solving Approach
with MATLAB[®]

GREG FOLEY



CAMBRIDGE

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Dublin City University



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Membrane Filtration

Focusing on the application of membranes in an engineering context, this hands-on computational guide makes previously challenging problems routine. It formulates problems as systems of equations solved with MATLAB, encouraging active learning through worked examples and end-of-chapter problems.

The detailed treatments of dead-end filtration include novel approaches to constant rate filtration and filtration with a centrifugal pump. The discussion of crossflow micro-filtration includes the use of kinetic and force balance models. Comprehensive coverage of ultrafiltration and diafiltration processes employs both limiting flux and osmotic pressure models. The effect of fluid viscosity on the mass transfer coefficient is explored in detail; the effects of incomplete rejection on the design and analysis of ultrafiltration and diafiltration are analysed; and quantitative treatments of reverse osmosis and nanofiltration process analysis and design are explored. Includes a chapter dedicated to the modelling of membrane fouling.

Greg Foley is a Lecturer at Dublin City University, with over 25 years' experience of teaching all aspects of chemical and bioprocess engineering. He is a dedicated educator, who has been nominated for the DCU President's Award for Excellence in Teaching on numerous occasions.

‘Membrane Filtration: A Problem Solving Approach with MATLAB’ is truly a most comprehensive, interesting and effective approach to this topic. The writing style is engaging, difficult concepts are explained with a clarity that is enviable and the coverage of the topics is quite comprehensive. When reading the text you can clearly hear Greg Foley speaking in every line.

I think the book is needed. It pulls together a diverse range of topics some of which are given only a cursory coverage in other texts and presents the topics in a connected manner with each chapter building on what was covered before. It is clearly a text aimed at teaching and the style will, I think, entice students to “learn by doing”. Some of the material, particularly the mathematical aspects of the subject will prove challenging for students who are weak in this area, however, Greg does offer a number of alternative solution methodologies which should overcome some of the complexities for those less interested in the mathematical “tricks” that often intrigue those of a more mathematical bent.

The book will also be very useful to research students and academic staff, like, myself who are involved in teaching modules with aspects of membrane separations. In fact this book proposal reminded me of those famous Lectures in Physics by Richard Feynman, in that while the original intention of that series of books was to introduce physics to undergraduates, that the main beneficiaries of the publications were Feynman’s colleagues, who gained fresh insights into their subject from the clarity of Feynman’s thought processes as revealed by his pedagogical style.

Dr Dermot M. Malone, University College Dublin

I enjoyed reading this book. Its style is rigorous yet easily understandable, with many clarifying examples and problems. Although it is primarily focused on students, it presents not only standard information but recent results in membrane engineering field as well.

Miroslav Fikar, Slovak University of Technology

For Julie and Leo

and

In memory of Tony Foley

Preface

It is often said, quite correctly, that to really learn a foreign language one needs to immerse oneself in it. For me, getting to grips with chemical and process engineering requires the same degree of immersion. It means formulating lots of problems using the core engineering skills of material and energy balancing. It also means using key mathematical and computational techniques on a regular basis. Having taught on an interdisciplinary degree programme for more than 20 years, it is clear to me that very few students can cope with doing a relatively small amount of engineering while at the same time studying lots of biology, or some other, less quantitative, subject. For most people, constant practice of engineering skills is required. The late and great golfer, Seve Ballesteros, once said: *To give yourself the best possible chance of playing to your potential, you must prepare for every eventuality. That means practice.* Engineering is similar to golf; the more time students spend solving practice problems, the better they become at *doing* engineering.

This book is all about immersing the reader in the field of *membrane filtration*. To use a football term, it is about encouraging the reader to ‘get stuck into’ quantitative aspects of membrane process design and analysis. As a consequence, the book is somewhat ‘equation heavy’ but that is the nature of engineering. In any event, none of the mathematics used is beyond what an undergraduate engineering student should be able to cope with.

In many ways, this is quite a conventional book, albeit heavily problem-based, and requires some effort on the part of the student. There is a tendency nowadays for teaching methods to make life almost too easy for the student. Consequently, there are lots of textbooks available that are beautifully produced and contain many coloured boxes and bulleted lists with headings like ‘*What you need to know*’, ‘*Learning outcomes*’, ‘*Key points to remember*’, etc. While superficially appealing, I think these encourage a very limited and examination-focused way of learning by students. To get the most out of this book, the reader needs to have pen and paper, and a computer, to hand. Active reading is required.

When I told friends and colleagues that I was writing a book on membrane filtration, they usually replied with something like: *How can you write a whole book on that?* This is a reasonable question. Superficially, membrane filtration is a simple process: a feed is presented with a barrier and some of the feed, a solid particle for example, is blocked by the barrier. When one thinks about a day-to-day activity like filtering coffee, things do indeed look very simple and hardly the subject worthy of a whole book. However, I say

two things to answer the question. First I say that membrane filtration spans everything from simple particulate filtration to the filtration of very small molecules such as salt ions. The underlying physics is quite different in each case. Second, I point out that engineers need to actually *calculate* things, whether it is to design a filtration system, analyse the performance of a system, or even optimise a system. All of these tasks require having a *mathematical description* of the process and therein lies the scope for a whole book. Problems must be formulated in the language of mathematics and solutions must be implemented.

The scope of the book is membrane filtration of *liquid systems* and includes dead-end filtration, crossflow microfiltration, ultrafiltration and diafiltration, nanofiltration and reverse osmosis. Each of these techniques is at a different stage of development, both from a practical and a theoretical point of view. Thus, a completely even treatment of each topic is impossible. The chapters on ultrafiltration and diafiltration (Chapters 4 to 8) form the heart of the book. I have chosen to have a separate chapter (Chapter 7) dealing with incomplete rejection because I think students learn better when going from the specific to the general rather than the other way around.

The emphasis throughout is on process design and analysis and there is very little coverage of membrane structure and membrane fabrication. In effect, this is very much a chemical engineering book. There are other fine books available that give broader coverage of the area but these can usually be classified as books on membrane *science*.

I always tell my students that solving problems in engineering has two parts; formulating the problem *and* actually doing the calculations. This book, therefore, places considerable emphasis on getting numerical predictions from the process models that are developed. Engineering calculations almost invariably require use of numerical methods. Most problems cannot be solved with neat, analytical solutions and the ability to solve non-linear algebraic and ordinary differential equations is a key skill for engineers. In the past, trial and error and graphical techniques abounded but these are largely obsolete as long as one formulates the problem appropriately. Nowadays, there are many options available for implementing numerical methods on personal computers but it is probably fair to say that MATLAB is the most widely used computation environment by engineers.

Getting to grips with MATLAB requires some effort and it would be preferable if the reader had some knowledge of the basics of this package. However, a lot can be achieved by simply copying the code of others and learning as one goes along. My own style of programming is very much influenced by my training in the classic language, FORTRAN – hence my frequent use of *global* variables (not necessarily best practice), which are essentially the same idea as the *common* statement in FORTRAN. Thankfully, none of the problems in this book is very demanding on computer power so the focus of the reader, at least in the short term, should be to get the correct answer and not worry too much about program elegance. That will come in time as one gets more experienced with using MATLAB.

All the codes in this book were written using MATLAB R2011b, kindly denoted by The Mathworks Inc. I also used the Optimisation and Symbolic Math toolboxes. All of the required functions that are used in the book are available in the Student Version of MATLAB. I would recommend that readers try to code their own solutions to the

worked examples before looking at my solution. Writing computer code is a great way to learn attention to detail.

When I originally set out to write this book, my vision was to create a textbook for an undergraduate module on *membrane engineering*. However, as I have progressed, the vision has broadened to providing a comprehensive source of material for anyone teaching membrane engineering at a variety of levels. Therefore, it is unlikely that the book will be used in its entirety for a single module. Rather, instructors can pick and choose from the book, depending on the time they have available and the overall structure of their programme. Thus, the book could quite easily be used at every level from the third year of a four-year engineering programme to master level, including interdisciplinary programmes like biotechnology. As I progressed through the writing of the book, it became clear to me that many of the topics within the book required me to venture into research territory. I needed to formulate process models that have not appeared in either textbooks or the research literature. Therefore, I would hope that the book will also be of interest to PhD students and more experienced researchers.

For the last 25 years, I have worked in the School of Biotechnology in Dublin City University. I would like to thank all my colleagues who have supported me or who have simply helped me to enjoy my 25 years working in education and research. The people in DCU whom I most need to thank, however, are my many students, although they probably had no idea that they were helping me! There is no better way to improve your understanding of a topic than having to teach it. On many occasions I have had flashes of understanding right in the middle of a lecture.

Most of all, I must thank all of my immediate and extended family for their fantastic support and love throughout the years, especially my wife, Julie, and my little boy, Leo. Without the love and support of everyone, especially Julie, this book would never have happened.

Greg Foley
Dublin, Ireland

Abbreviations

CFD	Computational fluid dynamics
CFMF	Crossflow microfiltration
CVD	Constant volume diafiltration
DAE	Differential algebraic equation
DDF	Discontinuous diafiltration
DEF	Dead-end filtration
DF	Diafiltration
LF	Limiting flux
MBR	Membrane bioreactor
MF	Microfiltration
MFCVD	(Crossflow) Microfiltration with constant volume diafiltration
MWCO	Molecular weight cut-off
NF	Nanofiltration
NLAE	Non-linear algebraic equation
ODE	Ordinary differential equation
OP	Osmotic pressure
PDE	Partial differential equation
PEG	Polyethylene glycol
RO	Reverse osmosis
UF	Ultrafiltration
UFCVD	Ultrafiltration with constant volume diafiltration
UFDF	Ultrafiltration with diafiltration
UFVVD	Ultrafiltration with variable volume diafiltration
VOP	Viscosity–osmotic pressure
VVD	Variable volume diafiltration

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